(a) (4 points)
Write a MATLAB function \([L,U] = MyLU(A)\) which computes the \(LU\) factorization of the input matrix \(A\). Write it as 3 nested loops.

(b) (2 points)
Write the function \(y = UTriSol(U,x)\) which solves for \(y\) such that \(Uy = x\) (given \(U\) and \(x\)); write the function \(x = LTriSol(L, b)\) which solves for \(x\) such that \(Lx = b\) (given \(L\) and \(b\)).

(c) (5 points)
Write a function \(x = LUSolve(A, b)\). You should use \(MyLU\), \(UTriSol\) and \(LTriSol\).

(d) (6 points)
Use the above to solve for \(x\) when

(i)
\[
A = \begin{bmatrix}
-2 & 4 & -1 & -1 & 3 \\
4 & -9 & 0 & 5 & 3 \\
-4 & 5 & -5 & 5 & 3 \\
-8 & 8 & -23 & 20 & 3 \\
-1 & 1 & 2 & 3 & 3 \\
\end{bmatrix},

b = \begin{bmatrix}
12 \\
-32 \\
3 \\
-13 \\
-8 \\
\end{bmatrix}
\]

(1)

(ii)
\[
A = \begin{bmatrix}
10^{-16} & 1 \\
1 & 1 \\
\end{bmatrix},

b = \begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
\]

(2)

(iii)
\[
A = \begin{bmatrix}
0 & 1 & 1 \\
1 & 1 & 2 \\
2 & 2 & 6 \\
\end{bmatrix},

b = \begin{bmatrix}
2 \\
4 \\
10 \\
\end{bmatrix}
\]

(3)

(e) (2 points)
Comment on the accuracy of \(x\) in (1), (2) and (3).

(f) (5 points)
Incorporate partial pivoting in \(MyLU\), i.e., write the function \([P,L,U] = MyPLU(A)\). Note that \(PA\) must be equal to \(LU\).

(g) (6 points)
Use \(MyPLU\) to solve \(Ax = b\) when \(A\) and \(b\) are as in (1), (2) and (3). Comment on the accuracy of the solution.