CS 383C/CAM 383C/M 383E **Homework 1** **Due: Sept 18, 2002**

1. (MATLAB Assignment) The MATLAB command "hilb(n)" generates an $n \times n$ Hilbert matrix, which we denote by H_n in the following:

   a Solve:

   $$H_n x_n = b_n$$

   for x_n, where $b_n = H_n * ones(n, 1)$ (ones(n,1) is a MATLAB command).
   Try $n = 3, 10, 20$.
   Use the MATLAB command "\" to solve the above system. (See "help mldivide").

   b How close is x_n to the exact solution? Comment.

   c Does the MATLAB command "\" do pivoting? If so, what pivoting? Give reasons for your answer.
   NOTE: You are required to print out and submit the MATLAB session that accomplished the above (see "help diary").

2. Let A be an $n \times n$ nonsingular matrix. Prove that:

   a A has a unique LU decomposition, where L is unit lower triangular and U is upper triangular.

   b $A = L_1 U_1$ is unique, where L_1 is lower triangular, and U_1 is unit upper triangular.

   c Is there a contradiction in a and b?

   d Prove that $A = L_1 U_1$ exists if and only if all leading principal submatrices of A are nonsingular. Note that L_1 and U_1 are as in b above.
   NOTE: For a and b, you may assume that all leading principal submatrices of A are nonsingular.

3. (MATLAB Assignment) Let A be an $n \times n$ matrix. Assume that no pivoting is needed in the following:

   a Write `matlab` code to compute A^{-1}. (Note: Write your own program to compute the inverse, do not use inbuilt `matlab` functions to do your work.)

   b Compute the exact number of floating point operations (division, multiplications, subtractions) required by your code. Include n^2 and lower order terms in $O(n^2)$.

   c Do you recommend computing A^{-1} to solve $Ax = b$?

   d Compare results obtained by your program with those given by the matlab command: `inv(A)`. Please include some examples (for e.g. try computing the inverse of H_3, H_{10} the Hilbert matrices as defined in Problem 1 above).
   NOTE: You are required to submit your MATLAB code along with some example computations for this Problem.