CS 383C/CAM 383C/M 383E **Homework 2** **Due: Sept 30 2002**

1. (Triangle inequality) Prove that for the vector norm $\|\cdot\|_2$ the triangle inequality holds. That is, prove:

$$\|\mathbf{x}+\mathbf{y}\|_2 \leq \|\mathbf{x}\|_2 + \|\mathbf{y}\|_2 \quad \mathbf{x},\mathbf{y} \in \mathbb{C}^d$$

2. (Backward Error Analysis) Do Question 1.11 from the textbook.

3. Let A and X be nonsingular $n \times n$ matrices. Prove that:

   a If $\|X\| < 1$, then $I + X$ is nonsingular.

   b If $\frac{\|\delta A\|}{\|A\|} < \frac{1}{\kappa(A)}$, then $A + \delta A$ is nonsingular.

   NOTE: You may assume that $\|\cdot\|$ is an induced matrix norm (see Definition 1.9 in Textbook.)

4. (Norms)

   a Prove that $\|A\|_1 \equiv \max_{x\neq 0} \frac{\|Ax\|_1}{\|x\|_1} = \max_j \sum_{i=1}^n |a_{ij}|$.

   b What is the vector $x^\star$ (with $\|x^\star\|_1 = 1$) such that $\|A\|_1 = \|Ax^\star\|_1$.

   c Consider $Ax = b$ and $A(x+\delta x) = b + \delta b$, where:

$$A = \begin{bmatrix} 1000 & 999 \\ 999 & 998 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -998 & 999 \\ 999 & -1000 \end{bmatrix}$$

   Choose x and δb, both nonzero, such that equality holds in the bound

$$\frac{\|\delta x\|_1}{\|x\|_1} \leq \kappa_1(A) \cdot \frac{\|\delta b\|_1}{\|b\|_1}$$

   HINT: Examine the proof of the above inequality.

5. (IEEE Arithmetic) Consider IEEE double precision arithmetic where each normalized floating point number is interpreted as $(-1)^s \cdot (1+f) \cdot 2^{e-1023}$.

   a What are s, f, and e for representing the number 3? Give binary or decimal values.

   b What are s, f, and e for $fl(\frac{1}{3})$? Is $fl(\frac{1}{3}) = \frac{1}{3}$? Give binary values for s, f and e.

   c What is $fl((\frac{1}{3}) \cdot 3)$? Does the answer equal 1? Try it in MATLAB.

6. (Partial Pivoting Can Fail) Consider matrices $W_n(x)$ of the form:

$$W_5(x) = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1+x \end{pmatrix}, \qquad 0 \leq x \leq 2.$$

$W_n(x)$ is a very well-conditioned matrix.

<u>a</u> What is the LU decomposition of $W_n(x)$? Does partial pivoting make any difference?

<u>b</u> Consider single precision arithmetic in which $fl(2^{24}+1) = 2^{24}$. Find the smallest value of n for which Gaussian Elimination will return the solution of $W_n(0)y = e_1$, when we asked for $W_n(1)y = e_1$? What about in IEEE double precision arithmetic?

<u>c</u> Can you now tell if the MATLAB command "\" does partial or complete pivoting?

ANSWERS to <u>a</u> and <u>b</u> should use pen and paper only.