CS 383C/CAM 383C/M 383E # Homework 1 **Due: Sept 14, 1999**

1. (MATLAB Assignment) The MATLAB command "hilb(n)" generates an $n \times n$ Hilbert matrix, which we denote by H_n in the following:

   <u>a</u> Solve:
   $$H_n x_n = b_n$$
   for x_n, where $b_n = H_n * ones(n, 1)$ (ones(n,1) is a MATLAB command).
   Try $n = 3, 10, 20$.
   Use the MATLAB command "\" to solve the above system. (See "help mldivide").

   <u>b</u> How close is x_n to the exact solution? Comment.

   <u>c</u> Does the MATLAB command "\" do pivoting? If so, what pivoting? Give reasons for your answer.
   NOTE: You are required to print out and submit the MATLAB session that accomplished the above (see "help diary").

2. Let A be an $n \times n$ nonsingular matrix. Prove that:

   <u>a</u> A has a unique LU decomposition, where L is <u>unit</u> lower triangular and U is upper triangular.

   <u>b</u> $A = L_1 U_1$ is unique, where L_1 is lower triangular, and U_1 is <u>unit</u> upper triangular.

   <u>c</u> Is there a contradiction in <u>a</u> and <u>b</u>?

   <u>d</u> Prove that $A = L_1 U_1$ exists if and only if all leading principal submatrices of A are nonsingular. Note that L_1 and U_1 are as in <u>b</u> above.
   NOTE: For <u>a</u> and <u>b</u>, you may assume that all leading principal submatrices of A are nonsingular.

3. Let A be an $m \times n$ matrix, with $m \neq n$.

   <u>a</u> Does $A = LU$ exist? If so, under what conditions?

   <u>b</u> What are the shapes (sizes) of L and U? An illustration by pictures is acceptable.

   <u>c</u> What are the number of parameters in L and U? In A?

   <u>d</u> When does $A = PLU$ exist? P is any $m \times n$ permutation matrix.