

Meta-extract: Using Existing Facts in Meta-reasoning

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ACL2 has long supported user-defined simplifiers, so-called *metafunctions* and *clause processors*, which are installed when corresponding rules of class `:meta` or `:clause-processor` are proved. Historically, such simplifiers could access the logical world at execution time and could call certain built-in proof tools, but one could not assume the soundness of the proof tools or the truth of any facts extracted from the world or context when proving a simplifier correct. Starting with ACL2 Version 6.0, released in December 2012, an additional capability was added which allows the correctness proofs of simplifiers to assume the correctness of some such proof tools and extracted facts. In this paper we explain this capability and give examples that demonstrate its utility.

1 Introduction

The meta rule and clause processor facilities of the ACL2 theorem proving system [6] are designed to allow users to write custom proof routines which, once proven correct, can be called by the ACL2 prover¹. These facilities descend from the meta reasoning capability provided by earlier Boyer-Moore provers [3]. Relevant background can be found in earlier ACL2 papers on meta reasoning [5] and clause processors [7], along with ACL2 documentation [1].

Such meta-level proof routines have historically been limited in their use of ACL2’s database of stored facts and its built-in prover functions. For example, a metafunction could call the ACL2 rewriter, but when proving this metafunction correct, the result of calling the rewriter could not be assumed to be equivalent to the input — that is, the rewriter could not be assumed to be correct. Similarly, metafunctions and clause processors could both examine the ACL2 world (the logical state of the prover, including the database of stored facts), but in proving the correctness of these functions, facts extracted from the world could not be assumed to be correct.

Meta-extract is an ACL2 feature first introduced in Version 6.0 (December, 2012). It allows for certain facts stored in the ACL2 world and certain ACL2 prover routines to be assumed correct when proving the correctness of metafunctions and clause processors. In particular, additional *meta-extract hypotheses* that capture the correctness of such routines and facts are allowed to be present in these correctness theorems. These additional hypotheses preserve the soundness of metareasoning in ACL2 because they encode assumptions that we are already making — that the facts stored in the ACL2 logical world are true and that ACL2’s prover routines are sound.

Note that meta-extract hypotheses can be used to help *prove* theorems to be stored as meta rules or as clause-processor rules, but they have no effect on how those rules are *applied* during subsequent proofs. For that, we refer readers to existing papers [5, 7] as well as documentation [1] topics including META, CLAUSE-PROCESSOR, and EXTENDED-METAFUNCTIONS. Also see the topic META-EXTRACT

¹ Clause processors may alternatively be “trusted” — used without proof — but at the cost of a *trust tag* marking this use as a source of potential unsoundness. [7]

for additional user-level documentation. Note that references to documentation topics, such as those above, are underlined to denote hyperlinks to topics in the online [documentation](#) [1] for ACL2 and its books.

This paper primarily addresses metafunctions rather than always referring to both metafunctions and clause processors. The correctness arguments are analogous. The meta-extract functionality available for clause processors is a subset of the functionality available for metafunctions: the *global facts* discussed below are available for both, but *contextual facts* need contexts that are only available below the clause level.

1.1 Meta-extract Hypotheses

We next introduce the various meta-extract hypotheses and explain informally why it is sound to include them. See the *Essay on Correctness of Meta Reasoning* in the ACL2 source code for a rigorous mathematical argument.

Correctness theorems for metafunctions are stated using a function that we will call a *pseudo-evaluator*, typically defined via the [defevaluator](#) macro. (Elsewhere in the ACL2 literature this is simply referred to as an evaluator; however, we want to emphasize the difference between a pseudo-evaluator and a real term evaluator.) A pseudo-evaluator is a constrained function about which only certain facts are known; these facts are ones that would also be true of a “real” evaluator capable of fully interpreting any ACL2 term. The intention is for these facts to suffice for proving a metafunction correct, without the need for a real term evaluator². To prove a metafunction correct the user must show that the pseudo-evaluation of the term output by the metafunction is equal (or equivalent) to the pseudo-evaluation of the input term. Intuitively, if this can be proved of a pseudo-evaluator, and the only facts known about the pseudo-evaluator are ones that are also true of the real evaluator of ACL2 terms, then this must also be true of the real evaluator: that is, the evaluations of the input and output terms of the metafunction are equivalent, and thus the metafunction is correct.

The meta-extract feature allows certain additional hypotheses in the statement of the correctness theorem of a metafunction. These meta-extract hypotheses are applications of the pseudo-evaluator to calls of either the function `meta-extract-global-fact+`, its less general version `meta-extract-global-fact`, or the function `meta-extract-contextual-fact`. These functions produce various sorts of terms by extracting facts from the ACL2 world and calling ACL2 prover subroutines, constructed so that if ACL2 and its logical state are sound, the terms produced should always be true. For example, these functions can produce:

- the body of a previously proven theorem;
- the definitional equation of a previously defined function;
- a term equating a call of a function on quoted constants to the quoted value of that call;
- a term equating some term *a* to the result of rewriting *a* using [mfc-rw](#); or
- a term describing the type of *a*, according to ACL2’s [type-set](#) reasoning.

When we use such facts in our metafunction, meta-extract hypotheses allow us to assume that they are true according to the pseudo-evaluator while doing the correctness proof.

² It would be unsound to define what we are calling a real evaluator in ACL2 — in particular, one that could evaluate terms containing calls of the evaluator itself, or functions that call the evaluator. In order to fix our handwavy argument, you could instead think of introducing an evaluator that can interpret all functions that are defined at the point when the metafunction is being applied.

Intuitively, adding meta-extract hypotheses to a metafunction’s correctness theorem is allowable because we expect the (real) evaluation of any term produced by one of the meta-extract functions to return true. If we prove the pseudo-evaluator theorem with meta-extract hypotheses and, as before, reason that since the theorem is true of the pseudo-evaluator, it is also true of the real evaluator, then the final step is to observe that meta-extract hypotheses are true using the real evaluator (or else ACL2 is already unsound). Therefore, even with meta-extract hypotheses, we can still conclude that the evaluations of the metafunction’s output and input terms are equivalent.

Why not somehow axiomatize the idea that the contents of the ACL2 world are correct? Simply put, we don’t see how to do that, and ideas we have heard along those lines have been unsound. At the least, one would seem to need to formalize the complex notion of a valid world.

1.2 Organization of This Paper

In Section 2 we provide examples of meta-extract hypotheses and summary documentation of their various forms. Section 3 presents a community book that provides a convenient way to use the meta-extract facility. Next we present applications in Section 4 and finally, we conclude with Section 5.

2 Meta-extract

Next we explain meta-extract by first giving two examples and then summarizing the general forms of meta-extract hypotheses.

2.1 Tutorial Examples

We present two examples for the two kinds of meta-extract hypotheses, corresponding to evaluation of calls of `meta-extract-contextual-fact` and of `meta-extract-global-fact`. (Since a call `(meta-extract-global-fact obj state)` is an abbreviation for `(meta-extract-global-fact+ obj state state)`, we are thus effectively illustrating `meta-extract-global-fact+` as well.)

2.1.1 Meta-extract-contextual-fact

Our first example is intentionally contrived and quite trivial, intended only to provide an easy introduction to meta-extract. It illustrates the use of `meta-extract-contextual-fact`. The intent is to simplify any term of the form `(nth x lst)`, when `x` is easily seen by ACL2 to be a symbol in the current context, to `(car lst)`.

In ACL2, the use of metafunctions is always supported by an evaluator, called a *pseudo-evaluator* in the preceding section. Let us introduce an evaluator that “knows” about the functions relevant to this example.

```
(defevaluator nthmeta-ev nthmeta-ev-lst
  ((typespec-check ts x)
   (nth n x)
   (car x)))
```

Next we define a metafunction, intended to replace any term `(nth n x)` by a corresponding term `(car x)` when `n` is known to be a symbol using type-set reasoning.³

³For details such as the meaning of `:forcep nil`, see the documentation for [meta-extract](#).

```
(defun nth-symbolp-metafn (term mfc state)
  (declare (xargs :stobjs state))
  (case-match term
    (('nth n x)
      (if (equal (mfc-ts n mfc state :forcep nil)
                  *ts-symbol*)
          (list 'car x)
          term))
    (& term)))
```

When the input term matches `(nth n x)`, this calls `mfc-ts` to deduce the possible types of n . If that type-set equals `*ts-symbol*` then that term must evaluate to a symbol, and the term can be reduced to `(car x)`.⁴

Now we can present a meta rule with a meta-extract hypothesis. Without that hypothesis the formula below is not a theorem, because the function `mfc-ts` has no axiomatic properties; all we know about it below is what we are told by the meta-extract hypothesis, as discussed further below.

```
(defthm nth-symbolp-meta
  (implies (nthmeta-ev (meta-extract-contextual-fact '(:typeset ,(cadr term))
                                                       mfc
                                                       state)
                        a)
    (equal (nthmeta-ev term a)
            (nthmeta-ev (nth-symbolp-metafn term mfc state) a)))
  :rule-classes ((:meta :trigger-fns (nth))))
```

To see what the meta-extract hypothesis above gives us, consider the following theorem provable by ACL2.

```
(equal (meta-extract-contextual-fact '(:typeset ,x)
                                       mfc
                                       state)
  (list 'typespec-check
        (list 'quote
              (mfc-ts x mfc state :forcep nil))
        x))
```

At a high level, this theorem shows us that `meta-extract-contextual-fact` returns a term of the form `(typespec-check (quote  $ts$ )  $x$ )`, which asserts that the term x belongs to the set of values represented by ts . The meta-extract hypothesis applies the pseudo-evaluator to this term, and since `typespec-check` is one of its known functions, the hypothesis reduces to

```
(typespec-check (mfc-ts (cadr term) mfc state :forcep nil)
  (nthmeta-ev (cadr term) a)).
```

The interesting case in proving our metafunction correct is when this `mfc-ts` call equals `*ts-symbol*`. In this case the hypothesis becomes

⁴Requiring the type-set to equal `*ts-symbol*` is an unnecessarily strong check, chosen merely for ease of presentation: it requires that ACL2's determination of the term's possible types include all three of the basic types T, NIL, and non-Boolean symbols.

```
(typespec-check *ts-symbol* (nthmeta-ev (cadr term) a))
```

which, when expanded, implies that the evaluation of `(cadr term)` must be a symbol, which enables the proof of `nth-symbolp-meta`.

Recall that meta-extract hypotheses do not affect the *applications* of meta rules; they only support their proofs. Therefore, the following test of the example above yields no surprises.

```
(defstub foo (x) t)
(thm (implies (symbolp (foo x))
              (equal (nth (foo x) y) (car y))))
:hints (("Goal" :in-theory '(nth-symbolp-meta))))
```

2.1.2 Meta-extract-global-fact

Our second example is from community book “`demos/nth-update-nth-meta-extract.lisp`”, which uses `meta-extract-global-fact`. Let us begin by considering what problem this book is attempting to solve.

Consider a `defstobj` event, `(defstobj st fld1 fld2 ... fldn)`. A common challenge in reasoning about stobjs is the simplification of *read-over-write* terms, of the form `(fldi (update-fldj v st))`, which indicate that we are to read `fldi` after updating `fldj`. That term simplifies to `(fldi st)` when $i \neq j$, and otherwise it simplifies to `v`. How do we get ACL2 to do such simplification automatically? The following two approaches are standard.

- Disable the stobj accessors and updaters after proving rewrite rules to simplify terms of the form `(fldi (update-fldj val st))`, to `val` if $i = j$ and to `(fldi st)` if $i \neq j$.
- Let the stobj accessors and updaters remain enabled, relying on a rule such as the built-in rewrite rule `nth-update-nth` to rewrite terms, obtained after expanding calls of the accessors and updaters, of the form `(nth i (update-nth j val st))`.

The first of these requires n^2 rules, which is generally feasible but can perhaps get somewhat unwieldy. The second of these provides a simple solution, but when proofs fail, the resulting checkpoints can be more difficult to comprehend.

Here, we outline a solution that addresses both of these concerns: a macro that generates a suitable meta rule. Details of this proof development may be found in community book “`demos/nth-update-nth-meta-extract.lisp`”. First we introduce our metafunction. Next, we prove a meta rule for a specific stobj. Finally, we mention a macro that generates a version of this rule that is suitable for an arbitrary specified stobj.

Our metafunction returns the input term unchanged unless it is of the form `(r (w v x))`, where r and w are *reader* (accessor) and *writer* (updater) functions defined to be calls of `nth` and `update-nth`, on explicit indices: `(r x) = (nth 'i x)` and `(w v x) = (update-nth 'i v x)`. In that case, the function `nth-update-nth-meta-fn-new-term` computes a new term: `v` if $i = j$, and otherwise, `(r x)`.

```
(defun nth-update-nth-meta-fn (term mfc state)
  (declare (xargs :stobjs state)
            (ignore mfc))
  (or (nth-update-nth-meta-fn-new-term term state)
      term))
```

Notice below that in computing the new term, the definitions of the reader and writer are extracted from the logical world using the function, `meta-extract-formula`, which returns the function's definitional equation. For example:

```
ACL2 !>(meta-extract-formula 'atom state)
(EQUAL (ATOM X) (NOT (CONSP X)))
ACL2 !>
```

We thus rely on the correctness of `meta-extract-formula` for the equality of the input term and the term returned by the following function.

```
(defun nth-update-nth-meta-fn-new-term (term state)
  (declare (xargs :stobjs state))
  (case-match term
    ((reader (writer val x))
     (and (not (eq reader 'quote))
          (not (eq writer 'quote))
          (let* ((reader-formula (and (symbolp reader)
                                      (meta-extract-formula reader state)))
                 (i-rd (fn-nth-index reader reader-formula)))
            (and
             i-rd ; the body of reader is (nth 'i-rd ...)
             (let* ((writer-formula (and (symbolp writer)
                                         (meta-extract-formula writer state)))
                    (i-wr (fn-update-nth-index writer writer-formula)))
               (and
                i-wr ; the body of writer is (update-nth 'i-wr ...)
                (if (eql i-rd i-wr)
                    val
                    (list reader x))))))))))
    (& nil)))
```

Next we introduce a (pseudo-)evaluator to use in our meta rule.

```
(defevaluator nth-update-nth-ev nth-update-nth-ev-lst
  ((nth n x)
   (update-nth n val x)
   (equal x y)))
```

We need to define one more function before presenting our meta rule. It takes as input a term ($f\ t_1 \dots t_n$) with list of formals ($v_1 \dots v_n$), and builds an alist that maps each v_i to the value of t_i in a given alist. Like our metafunction, it consults `meta-extract-formula` to obtain the formal parameters of f . An earlier attempt to use the function, `formals`, failed: the meta rule's proof needs these formals to connect to those found by our metafunction.

```
(defun meta-extract-alist-rec (formals actuals a)
  (cond ((endp formals) nil)
        (t (acons (car formals)
                   (nth-update-nth-ev (car actuals) a)
                   (meta-extract-alist-rec (cdr formals) (cdr actuals) a)))))

(defun meta-extract-alist (term a state)
```

```
(declare (xargs :stobjs state :verify-guards nil))
(let* ((fn (car term))
      (actuals (cdr term))
      (formula (meta-extract-formula fn state)) ; (equal (fn ...) ...)
      (formals (cdr (cadr formula))))
  (meta-extract-alist-rec formals actuals a)))
```

We now define a stobj and prove a corresponding meta rule.

```
(defstobj st
  fld1 fld2 fld3 fld4 fld5 fld6 fld7 fld8 fld9 fld10
  fld11 fld12 fld13 fld14 fld15 fld16 fld17 fld18 fld19 fld20)

(defthm nth-update-nth-meta-rule-st
  (implies
    (and (nth-update-nth-ev ; (f (update-g val st))
      (meta-extract-global-fact (list :formula (car term)) state)
      (meta-extract-alist term a state))
      (nth-update-nth-ev ; (update-g val st)
      (meta-extract-global-fact (list :formula (car (cadr term)))
        state)
      (meta-extract-alist (cadr term) a state))
      (nth-update-nth-ev ; (f st) -- note st is (caddr (cadr term))
      (meta-extract-global-fact (list :formula (car term)) state)
      (meta-extract-alist (list (car term)
        (caddr (cadr term)))
        a state)))
    (equal (nth-update-nth-ev term a)
      (nth-update-nth-ev (nth-update-nth-meta-fn term mfc state) a)))
  :hints ...
  :rule-classes ((:meta :trigger-fns (fld1 fld2 ... fld20))))
```

The proof of the theorem below takes no measurable time, and applies the metafunction proved correct above.

```
(in-theory (disable fld1 ... fld20 update-fld1 ... update-fld20))

(defthm test1
  (equal (fld3 (update-fld1 1
    (update-fld2 2
      (update-fld3 3
        (update-fld4 4
          (update-fld3 5
            (update-fld6 6 st)))))))
    3))
```

Notice that there is nothing about the meta rule above that is specific to the particular stobj, *st*, except for the `:trigger-fns` that it specifies. In the community book mentioned above (“demos/nth-update-nth-meta-extract.lisp”), we define a macro that automates the generation of such a meta rule for an arbitrary stobj. Our macro takes the name of a stobj, *s*, and does two things: it disables all of the stobj’s accessors and updaters, and it proves a meta rule that simplifies every term of the form $(r\ (w\ v\ s))$, where *r* and *w* are an accessor and updater, respectively, for the stobj *s*.

The meta rule above uses three `meta-extract-global-fact` hypotheses corresponding to uses of three particular facts. Enumerating all of the facts potentially used in the metafunction in this way could easily become unwieldy; in Section 3 we discuss a utility that solves this problem, allowing one `meta-extract-global-fact` hypothesis and one `meta-extract-contextual-fact` hypothesis to cover all of the facts that might be used.

2.2 General Forms

In this section we summarize briefly the forms of meta-extract hypotheses. Additional details may be found in the documentation for `meta-extract`.

Below, let `evl` be the pseudo-evaluator (see Section 1.1) used in a meta rule. The two primary forms of meta-extract hypotheses that can be used in a meta rule are as follows. In the first, `aa` represents an arbitrary term; in the second, `a` must be the second argument of the two calls of the pseudo-evaluator in the conclusion of the theorem.

```
(evl (meta-extract-global-fact obj state) aa)
(evl (meta-extract-contextual-fact obj mfc state) a)
```

The second form above is only legal for meta rules about extended metafunctions (which take arguments `mfc` and `state`). The first form above is actually equivalent to the first form below, which in turn is a special case of the second form below.

```
(evl (meta-extract-global-fact+ obj state state) aa)
(evl (meta-extract-global-fact+ obj st state) aa)
```

The last form supports clause processors that modify state as long as they do not change the logical world; it produces the same value as the previous form as long as both `st` and `state` have equal world fields.

If the arguments to the `meta-extract-*` function are somehow malformed, then it returns the trivial term `'T`, which is not of any use in proving a metafunction correct. Otherwise, each invocation produces a term that states the “correctness” of an invocation of some utility, such as `mfc-rw` or `meta-extract-formula`. The terms produced for different kinds of invocation vary according to the particular concept of correctness appropriate to the utility in question.

We now describe the allowed `obj` arguments to `meta-extract-global-fact` and the terms they produce. The documentation for `extended-metafunctions` explains meta-level functions discussed below, such as `mfc-rw` and `mfc-ap`.

- `(:formula f)` produces the value of `(meta-extract-formula f state)`, which allows metafunctions to assume that any invocation of `meta-extract-formula` produces a true formula. `Meta-extract-formula` looks up various kinds of formulas from the world:
 - the body of f if it is a theorem name
 - the definitional equation of f if it is a defined function
 - the constraint of f if it is a constrained function
 - the `defchoose` axiom of f if it is a `defchoose` function.
- `(:lemma f n)` produces a term corresponding to the n th rewrite rule stored in the `lemmas` property of function f , which allows any such rule to be assumed correct in a metafunction. The term returned is the value of

```
(rewrite-rule-term (nth 'n (getpropc 'f 'lemmas nil (w state))))),
```

assuming that n is a valid index (does not exceed the length of the indicated list). Here `rewrite-rule-term` transforms a `rewrite-rule` record structure into a term such as

```
(implies hyps (equiv lhs rhs)).
```

The rewrite rules stored in `:lemmas` are not simply theorem bodies, which could be accessed using `meta-extract-formula`, but rewrite rule structures, which separately store the left-hand side, right-hand side, equivalence relation, and hypotheses, and also contain heuristic information such as the backchain limits and match-free mode.

- `(:fncall f L)` produces a term `(equal c 'v)`, where c is the call that applies f to the quotations of the values in argument list L , and v is the value of that call computed by `magic-ev-fncall`. This allows metafunctions to assume that `magic-ev-fncall` correctly evaluates function applications.

The allowed `obj` arguments to `meta-extract-contextual-fact` are as follows.

- `(:typeset term)` produces a term stating the correctness of the type-set produced by `mfc-ts` for *term*. Specifically, it produces the term `(typespec-check 'ts term)`, where ts is the result of `(mfc-ts term mfc state :forcep nil :ttreep nil)` and `(typespec-check ts val)` is true when val 's actual type is in the type-set ts .
- `(:rw+ term alist obj equiv)` produces a term stating that `mfc-rw+` correctly rewrites *term* under substitution *alist* with objective *obj* under equivalence relation *equiv*. The form of the term produced is

```
(equiv term' rw)
```

where *term'* is the new term formed by substituting *alist* into *term* and *rw* is the result of the call `(mfc-rw+ term alist obj equiv mfc state :forcep nil :ttreep nil)`. The *equiv* argument may also be `T`, meaning IFF, or `NIL`, meaning EQUAL.

- `(:rw term obj equiv)` is similar to the `:rw+` form above, but instead of `mfc-rw+` it supports `mfc-rw`, which takes no *alist* argument. Instead, `NIL` is used for the substitution.
- `(:ap term)` uses `mfc-ap` to derive a linear arithmetic contradiction indicating that *term* is false, and produces `(not term)` if that is successful, that is, if `(mfc-ap term mfc state :forcep nil)` returns true; otherwise it just produces `'T`.
- `(:relieve-hyp hyp alist rune target backptr)` uses `mfc-relieve-hyp` to attempt to prove that *hyp* holds under substitution *alist*, and produces the substitution of *alist* into *hyp* if successful, that is, if `(mfc-relieve-hyp hyp alist rune target backptr mfc state :forcep nil :ttreep nil)` returns true; otherwise, `'T`.

We have seen there are two forms of meta-extract hypotheses: `meta-extract-contextual-fact` and `meta-extract-global-fact+` (and its less general form, `meta-extract-global-fact`). We could have split each of these into several forms, resulting in eight forms for the eight kinds of values of *obj* listed above. However, in the next section we describe a utility that essentially generalizes the two supported forms, which eliminates the need for the user to think about the precise values of *obj*; it was convenient to generalize two supported forms rather than eight.

3 Using “meta-extract-user.lisp”

The community book “clause-processors/meta-extract-user.lisp” is designed to allow more convenient use of the meta-extract facility. The main contribution of this book is in the event-generating macro `def-meta-extract`. For a given pseudo-evaluator `evl`, `def-meta-extract` produces macros `evl-meta-extract-contextual-facts` and `evl-meta-extract-global-facts` that expand to meta-extract hypotheses where the `obj` argument is a call of a “bad-guy” (Skolem) function. This essentially universally quantifies the `obj` argument: informally, the term

$$obj_0 = (\text{evl-meta-extract-contextual-bad-guy } a \text{ mfc state})$$

is some `obj` for which the formula

$$\phi = (\text{evl } (\text{meta-extract-contextual-fact } obj \text{ mfc state}) \text{ a})$$

is false, if any such `obj` exists; so by asserting that ϕ is true for obj_0 , we assert $(\forall obj) \phi$. Therefore,

```
(evl (meta-extract-contextual-fact
      (evl-meta-extract-contextual-bad-guy a mfc state)
      mfc state)
  a)
```

implies that for any `obj`,

```
(evl (meta-extract-contextual-fact obj mfc state) a).
```

The `def-meta-extract` utility also proves several theorems about the pseudo-evaluator. They are proven by functional instantiation of similar theorems proved in “meta-extract-user.lisp” about a base evaluator that only supports the six functions `typespec-check`, `if`, `equal`, `not`, `iff`, and `implies`; this means that `evl` must also support at least these six functions for the utility to work.

The theorems proved by `def-meta-extract` obviate the need for the user to reason about the specifics of the definitions of `meta-extract-contextual-fact` and `meta-extract-global-fact+` and the proper construction of their `obj` arguments, while still supporting all the facilities listed in Section 2.2. For example, this rule shows that `(evl-meta-extract-global-facts)` implies the correctness of `meta-extract-formula` (and makes no reference to the form of the `obj` argument to `meta-extract-global-fact`):

```
(defthm evl-meta-extract-formula
  (implies (and (evl-meta-extract-ev-global-facts)
                (equal (w st) (w state))))
    (evl (meta-extract-formula name st) a)))
```

This rule shows that `(evl-meta-extract-contextual-facts a)` implies the correctness of `mfc-rw+` (specifically, when `nil`, meaning `equal`, is given as the equivalence relation argument):

```
(defthm evl-meta-extract-rw+-equal
  (implies (evl-meta-extract-contextual-facts a)
    (equal (evl (mfc-rw+
                  term alist obj nil
                  mfc state :forcep nil)
            a)
      (evl (sublis-var alist term) a))))
```

The above rule involves the system function `sublis-var`, which substitutes `alist` into `term` but reduces ground calls of primitive functions⁵ to their values. In order to reason about `sublis-var`, the pseudo-evaluator should support all of the primitive functions that it treats specially. The community book “`clause-processors/sublis-var-meaning.lisp`” defines a pseudo-evaluator `cterm-ev` that supports exactly these functions and proves the following theorem that describes how `sublis-var` evaluates:

```
(defthm eval-of-sublis-var
  (implies (and (pseudo-term x)
                (not (assoc nil alist)))
    (equal (cterm-ev (sublis-var alist x) a)
           (cterm-ev x (append (cterm-ev-alist alist a) a))))))
```

To reason about `mfc-rw+`, `mfc-rw`, and `mfc-relieve-hyp`, whose meta-extract assumptions all involve `sublis-var`, one can define a pseudo-evaluator that supports both the functions required for `def-meta-extract` and for `sublis-var`. Community book “`centaur/misc/context-rw.lisp`”, for example, defines a pseudo-evaluator supporting all these functions, uses `def-meta-extract`, and functionally instantiates the above theorem about `sublis-var` to allow it to reason about `mfc-rw+`.

4 Applications

4.1 GL Symbolic Interpreter

The GL framework for bit-level symbolic execution [8, 9] is an important tool for hardware verification efforts at Centaur Technology [2]. To prove a theorem, GL attempts to symbolically interpret the conclusion and render it into a Boolean formula which can be proved via a satisfiability check. Given a bit-level representation of each variable, the symbolic interpreter recursively computes a bit-level representation of each subterm. It expands function definitions as needed, down to certain primitive functions for which support is built in. Recent versions can also apply rewrite rules. It is implemented as a clause processor and uses `meta-extract` to look up function definitions and rewrite rules from the world and to evaluate ground terms using `magic-ev-fncall`.

GL was originally written before `meta-extract`, and used two tricks to replace its functionality. The complexity of these tricks reflects the utility of `meta-extract`, since it allows GL to now avoid these desperate measures.

- In order to justify the correctness of function definitions, GL would keep track of all definitions that were used, and return each definitional equation as an additional proof obligation from the clause processor. GL proof hints were orchestrated so as to use `:by` hints to attempt to cheaply discharge these obligations. GL did not yet use rewrite rules, but they could have been handled similarly.
- In order to apply functions to concrete arguments, each GL clause processor had an `apply` function that could call a fixed set of functions by name using a `case` statement. GL provided automation for defining new such clause processors so that users could build in their own set of functions.

⁵By “primitive functions” we mean built-in functions such as `binary-+` that do not have defining events — that is, those found in the ACL2 constant `*primitive-formals-and-guards*`.

4.2 Rewrite-bounds

The community book “centaur/misc/bound-rewriter.lisp” provides a tool for solving certain inequalities: it replaces subterms of an inequality with known bounds if those subterms are in monotonic positions. For example, the term $a - b$ monotonically decreases as b increases, so if we wish to prove $c < a - b$ and we know $B \geq b$, then it suffices to prove $c < a - B$. While this example would be easily handled by ACL2’s linear arithmetic solver, there are problems that the bound rewriter can handle easily that overwhelm ACL2’s nonlinear solver and cannot be solved with linear arithmetic – e.g.,

```
(implies (and (rationalp a) (rationalp b) (rationalp c)
              (<= 0 a) (<= 0 b) (<= 1 c)
              (<= a 10) (<= b 20) (<= c 30))
          (<= (+ (* a b c) (* a b) (* b c) (* a c))
              (+ (* 10 20 30) (* 10 20) (* 20 30) (* 10 30))))
```

This formula can’t be solved with linear arithmetic, because it is not a linear problem. (If each product of variables is replaced by a fresh variable, the result is clearly not a theorem.) Moreover, the hint `:nonlinearp t` causes ACL2 to hang indefinitely. However, the hint

```
(rewrite-bounds ((<= a 10)
                 (<= b 20)
                 (<= c 30)))
```

solves it instantaneously by replacing upper-boundable occurrences of a by 10, b by 20, and c by 30. The same results are obtained — a quick proof using `rewrite-bounds` but an indefinite hang using nonlinear arithmetic — if the arithmetic expression on the last line of the theorem is replaced by its value, 7100.

The bound rewriter tool is implemented as a meta rule and uses meta-extract extensively. To determine which subterms are in monotonic positions, it uses type-set reasoning to determine the signs of subterms. For example, $a \cdot b$ increases as b increases if a is nonnegative and decreases as b increases if a is nonpositive; if we can’t (weakly) determine the sign of a , then we can’t replace b or any subterm with a bound. To determine whether a proposed bound of a subterm is (contextually) true, it uses `mfc-relieve-hyp` to show it by rewriting, and if that fails, `mfc-ap` to show it by linear arithmetic reasoning. The correctness of these uses of ACL2 reasoning utilities are justified by meta-extract-contextual-fact hypotheses.

4.3 Context-rw

A meta rule for context-sensitive rewriting, accomplishing something similar to Greve’s “Nary” framework [4], is defined in “centaur/misc/context-rw.lisp” (see [contextual-rewriting](#)). Like Nary, it supports an analogue of congruence reasoning using contexts that are more general than equivalence relations. An example of its use is to allow, e.g.,

```
(logbitp 4 (logand (logior a b c (logapp 6 d e)) f g))
```

to be simplified to

```
(logbitp 4 (logand (logior a b c d) f g))
```

without defining a rewrite rule specifically for that case. The context rewriter uses certain theorems to justify rewriting subterms under new contexts, that is, with new calls wrapped around them. In this case we might have a rule that says that `(logbitp 4 ...)` induces a `(logand 16 ...)` context; this context can then be propagated through the `logand` and `logior` down to the `logapp` call, which simplifies under that context to just `d`, using a traditional rewrite rule such as the following:

```
(implies (and (syntaxp (and (quotep n) (quotep m)))
              (equal (logtail m n) 0))
          (equal (logand n (logapp m a b))
                  (logand n a)))
```

When interpreted as a context rule, the following says that `logbitp` induces a `logand` context, by directing replacement of an instance of the right-hand side of the equality by the corresponding instance of the left-hand side.

```
(implies (syntaxp (quotep n))
          (equal (logbitp n (logand (ash 1 (nfix n)) m))
                  (logbitp n m)))
```

And the following rule says that `logior` propagates such a `logand` context onto its second argument:

```
(implies (syntaxp (quotep n))
          (equal (logand n (logior a (logand n b)))
                  (logand n (logior a b))))
```

A syntactic requirement for context rules is that the left- and right-hand sides must be identical except that some subterm of the LHS is replaced by a variable in the RHS (in this case `b`), and that is the only occurrence of that variable in the RHS. That variable corresponds to the subterm onto which the context will be propagated, and the corresponding subterm of the LHS reflects the context that will be propagated onto it. So the rule immediately above says:

When `logior` occurs under a `(logand n ...)` context where `n` is a constant, propagate the `(logand n ...)` context onto the second argument of the `logior`.

Experienced ACL2 users might note that the roles of the left- and right-hand sides are essentially reversed in this usage; this is because often the reverse of a good context-propagation rule is also a good rewrite rule.

The context rewriter uses `meta-extract` in order to trust rewrite rules extracted from the world and results from `mfc-rw+` and `mfc-relieve-hyp`, which it uses, respectively, to simplify subterms under new contexts and to discharge hypotheses necessary for applying the context rules.

4.4 Others

A few other utilities from the community books use `meta-extract` solely to be able to extract a formula from the world (using `meta-extract-formula`) and assume it to be true. For example, “`clause-processors/witness-cp.lisp`” provides a framework for reasoning about quantification (see `witness-cp`); it uses `meta-extract-formula` to look up a stored fact showing that a term representing a universal quantification implies any instance of the quantified formula. A second, “`clause-processors/just-expand.lisp`”, provides a clause processor and meta rule that force expansion of certain terms, somewhat similar to the `:expand` hint. A third, “`clause-processors/replace-equalities.lisp`”, provides a tool for replacing known equalities in ways that the rewriter can’t, e.g., replacing a variable with a term. For example, the following is not a valid rewrite rule because its left-hand side is a variable, but it could be a good `replace-equalities` rule:

```
(implies (matches pattern x)
          (equal x
                  (patternsubst pattern (sigma pattern x))))
```

5 Conclusion

This paper explains meta-extract hypotheses and shows how they can be put to good use, either directly or by way of the `def-meta-extract` utility to obtain two simple, general meta-extract hypotheses. The implementation and logical justification for meta-extract are delicate, so it made sense to implement only the primitive notions of Section 2 in ACL2 and then introduce `def-meta-extract` in a book, with less trusted code as a result.

The meta-extract facility has been successfully used to create specialized proof tools that are admitted by ACL2 as fully verified metafunctions or clause-processors, including the GL bit-blasting framework, which is in daily use at Centaur Technology. We hope that this paper contributes to wider successful use of the meta-extract feature.

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