

L11: Landmark Mapping

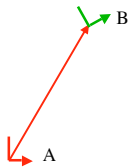
CS 344R/393R: Robotics
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Landmark Map

- Locations and uncertainties of n landmarks, with respect to a specific frame of reference.
 - World frame: fixed origin point
 - Robot frame: origin at the robot
- **Problem:** how to combine new information with old to update the map.

A Spatial Relationship is a Vector

- A spatial relationship holds between two poses: the position and orientation of one, in the frame of reference of the other.

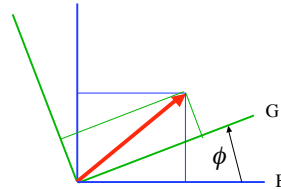


$$\mathbf{x}_{AB} = \begin{bmatrix} x_{AB} \\ y_{AB} \\ \phi_{AB} \end{bmatrix}$$

$$\mathbf{x}_{AB} = [x_{AB} \quad y_{AB} \quad \phi_{AB}]^T$$

Rotation Matrix

$$\begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} x_G \\ y_G \end{bmatrix} = \begin{bmatrix} x_B \\ y_B \end{bmatrix}$$



Inverse:

$$\begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}$$

Relation to Affine Transformations

- The spatial relationship

$$\mathbf{x}_{AB} = [x_{AB} \quad y_{AB} \quad \phi_{AB}]^T$$

- is equivalent to the affine transformation

$$\begin{bmatrix} \cos \phi_{AB} & -\sin \phi_{AB} & x_{AB} \\ \sin \phi_{AB} & \cos \phi_{AB} & y_{AB} \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation plus translation

Uncertain Spatial Relationships

$$\hat{\mathbf{x}} = E[\mathbf{x}]$$

$$\mathbf{C}(\mathbf{x}) = E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]$$

- An *uncertain* spatial relationship is described by a probability distribution of vectors, with a *mean* and a *covariance matrix*.

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{\phi} \end{bmatrix} \quad \mathbf{C}(\mathbf{x}) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\phi} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\phi} \\ \sigma_{x\phi} & \sigma_{y\phi} & \sigma_\phi^2 \end{bmatrix}$$

A Map with n Landmarks

- Concatenate n vectors into one big state vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} \quad \hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \\ \vdots \\ \hat{\mathbf{x}}_n \end{bmatrix}$$

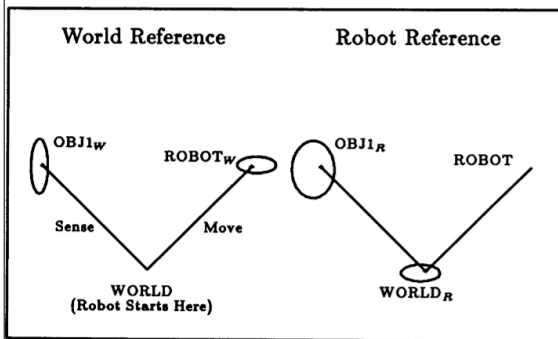
- And one big $3n \times 3n$ covariance matrix.

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} \mathbf{C}(\mathbf{x}_1) & \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2) & \cdots & \mathbf{C}(\mathbf{x}_1, \mathbf{x}_n) \\ \mathbf{C}(\mathbf{x}_2, \mathbf{x}_1) & \mathbf{C}(\mathbf{x}_2) & \cdots & \mathbf{C}(\mathbf{x}_2, \mathbf{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}(\mathbf{x}_n, \mathbf{x}_1) & \mathbf{C}(\mathbf{x}_n, \mathbf{x}_2) & \cdots & \mathbf{C}(\mathbf{x}_n) \end{bmatrix}$$

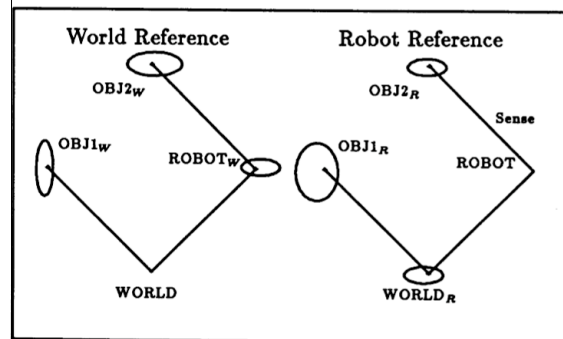
Example

- The robot senses object #1.
 - The robot moves.
 - The robot senses a *different* object #2.
 - Now the robot senses object #1 again.
- After each step, what does the robot know (in its landmark map) about each object, including itself?

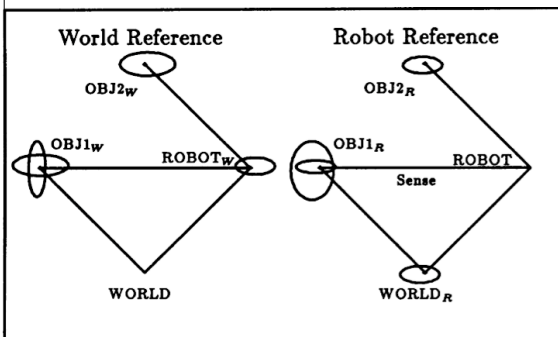
Robot Senses Object #1 and Moves



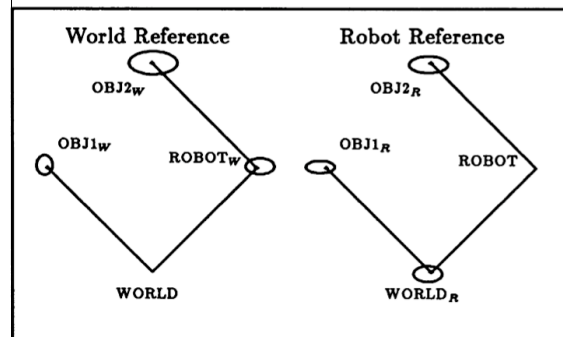
Robot Senses Object #2

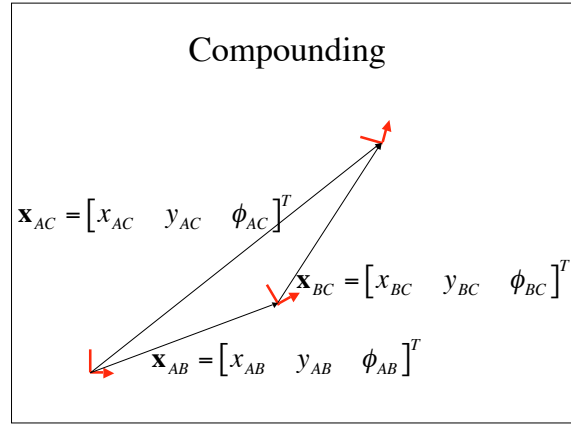
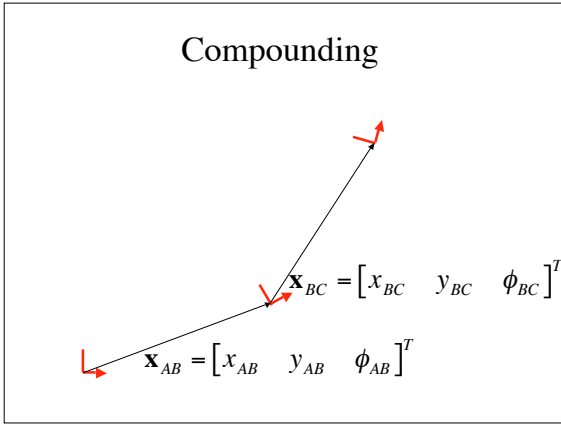


Robot Senses Object #1 Again



Updated Estimates After Constraint





Compounding

- Let \mathbf{x}_{AB} be the pose of object B in the frame of reference of A. (Sometimes written B_A)
- Given \mathbf{x}_{AB} and \mathbf{x}_{BC} , calculate \mathbf{x}_{AC} .

$$\mathbf{x}_{AC} = \mathbf{x}_{AB} \oplus \mathbf{x}_{BC} = \begin{bmatrix} x_{BC} \cos \phi_{AB} - y_{BC} \sin \phi_{AB} + x_{AB} \\ x_{BC} \sin \phi_{AB} + y_{BC} \cos \phi_{AB} + y_{AB} \\ \phi_{AB} + \phi_{BC} \end{bmatrix}$$

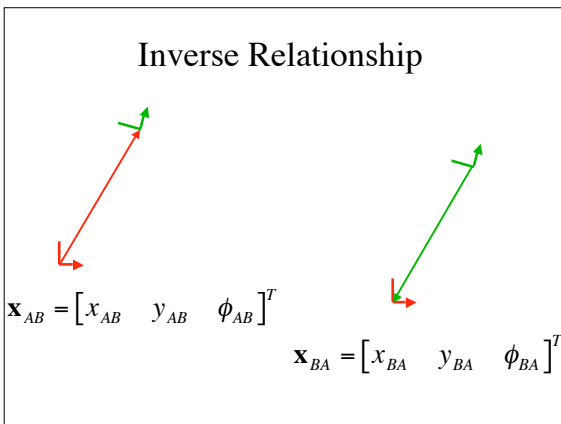
- Compute $\mathbf{C}(\mathbf{x}_{AC})$ from $\mathbf{C}(\mathbf{x}_{AB})$, $\mathbf{C}(\mathbf{x}_{BC})$, and $\mathbf{C}(\mathbf{x}_{AB}, \mathbf{x}_{BC})$.

Computing Covariance

- Consider the linear mapping $\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{b}$

$$\begin{aligned} \mathbf{y} &= \mathbf{M}\mathbf{x} + \mathbf{b} \\ \mathbf{C}(\mathbf{y}) &= \mathbf{C}(\mathbf{M}\mathbf{x} + \mathbf{b}) \\ &= E[(\mathbf{M}\mathbf{x} + \mathbf{b} - (\mathbf{M}\hat{\mathbf{x}} + \mathbf{b}))(\mathbf{M}\mathbf{x} + \mathbf{b} - (\mathbf{M}\hat{\mathbf{x}} + \mathbf{b}))^T] \\ &= E[\mathbf{M}(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{M}(\mathbf{x} - \hat{\mathbf{x}}))^T] \\ &= E[\mathbf{M}(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{M}^T] \\ &= \mathbf{M} E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] \mathbf{M}^T \\ &= \mathbf{M}\mathbf{C}(\mathbf{x})\mathbf{M}^T \end{aligned}$$

- Apply this to nonlinear functions by using Taylor Series.



The Inverse Relationship

- Let \mathbf{x}_{AB} be the pose of object B in the frame of reference of A.
- Given \mathbf{x}_{AB} , calculate \mathbf{x}_{BA} .

$$\mathbf{x}_{BA} = (-)\mathbf{x}_{AB} = \begin{bmatrix} -x_{AB} \cos \phi_{AB} - y_{AB} \sin \phi_{AB} \\ x_{AB} \sin \phi_{AB} - y_{AB} \cos \phi_{AB} \\ -\phi_{AB} \end{bmatrix}$$

- Compute $\mathbf{C}(\mathbf{x}_{BA})$ from $\mathbf{C}(\mathbf{x}_{AB})$

Composite Relationships

- Compounding combines relationships head-to-tail: $\mathbf{x}_{AC} = \mathbf{x}_{AB} \oplus \mathbf{x}_{BC}$
- Tail-to-tail combinations come from observing two things from the same point: $\mathbf{x}_{BC} = (\ominus \mathbf{x}_{AB}) \oplus \mathbf{x}_{AC}$
- Head-to-head combinations come from two observations of the same thing: $\mathbf{x}_{AC} = \mathbf{x}_{AB} \oplus (\ominus \mathbf{x}_{CB})$
- They provide new relationships between their endpoints.

Merging Information

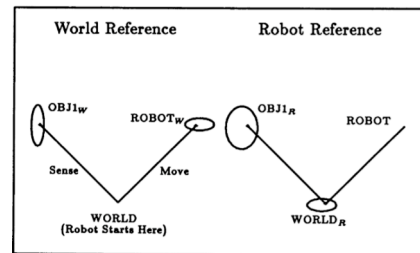
- An uncertain observation of a pose is combined with previous knowledge using the extended Kalman filter.
 - Previous knowledge: $\mathbf{x}_k^-, \mathbf{P}_k^-$
 - New observation: \mathbf{z}_k, \mathbf{R}
- Update: $\mathbf{x} = \mathbf{x}(\text{new}) \otimes \mathbf{x}(\text{old})$
- Can integrate dynamics as well.

EKF Update Equations

- Predictor step: $\hat{\mathbf{x}}_k^- = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_k)$
 $\mathbf{P}_k^- = \mathbf{A} \mathbf{P}_{k-1} \mathbf{A}^T + \mathbf{Q}$
- Kalman gain: $\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_k^- \mathbf{H}^T + \mathbf{R})^{-1}$
- Corrector step: $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - h(\hat{\mathbf{x}}_k^-))$
 $\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_k^-$

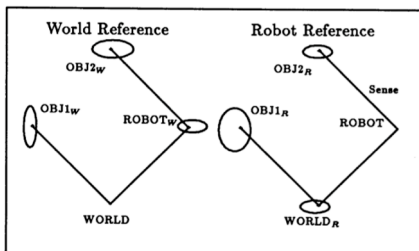
Reversing and Compounding

- $\text{OBJ1}_R = (\ominus \text{ROBOT}_W) \oplus \text{OBJ1}_W$
- $= \text{WORLD}_R \oplus \text{OBJ1}_W$



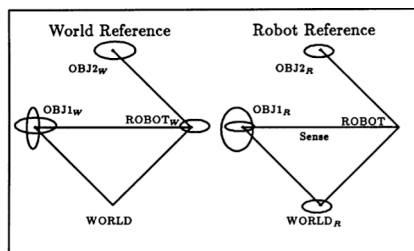
Sensing Object #2

- $\text{OBJ2}_W = \text{ROBOT}_W \oplus \text{OBJ2}_R$



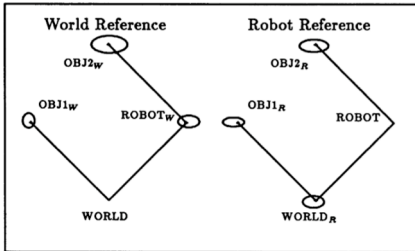
Observing Object #1 Again

- $\text{OBJ1}_W = \text{ROBOT}_W \oplus \text{OBJ1}_R$



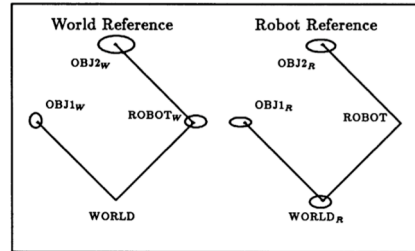
Combining Observations (1)

- $OJB1_W = OJB1_W(\text{new}) \otimes OJB1_W(\text{old})$
- $OJB1_R = OJB1_R(\text{new}) \otimes OJB1_R(\text{old})$



Combining Observations (2)

- $ROBOT_W(\text{new}) = OJB1_W \oplus (\ominus OJB1_R)$
- $ROBOT_W = ROBOT_W(\text{new}) \otimes ROBOT_W(\text{old})$



Useful for Feature-Based Maps

- We'll see this again when we study FastSLAM.