Lecture 20:
The Spatial Semantic Hierarchy

CS 344R/393R: Robotics

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What is a Map?

- A map is a model of an environment that helps an agent plan and take action.
- A topological map is useful for travel planning.
- A metrical map is useful for inferring directions and distances.
- Both must be learned from observations.

Scale of Space

- **Small-scale space** is within the agent's perceptual surround.
  - "visual space" or "perceptual space"
- **Large-scale space** has structure that must be integrated from the agent's observations gathered over time and travel.
  - the "cognitive map"

Local Metrical Mapping Works

- In small-scale space, modern SLAM methods work extremely well with lasers.
  - Great progress with visual SLAM.

	Metrical Mapping	Topological Mapping
Small-scale space	Local SLAM	
Large-scale space		

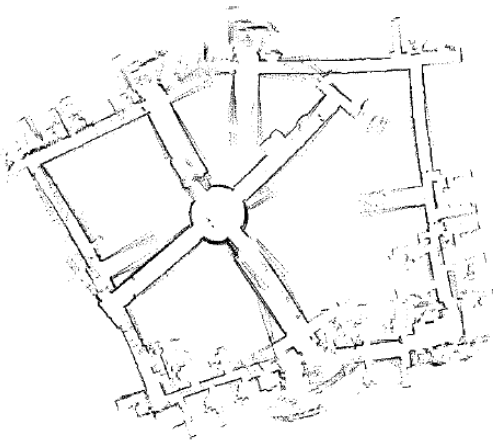
Global Metrical Mapping Is Hard

- Within a single global frame of reference over large-scale space, errors accumulate.
  - Sufficiently large loops are always a problem.

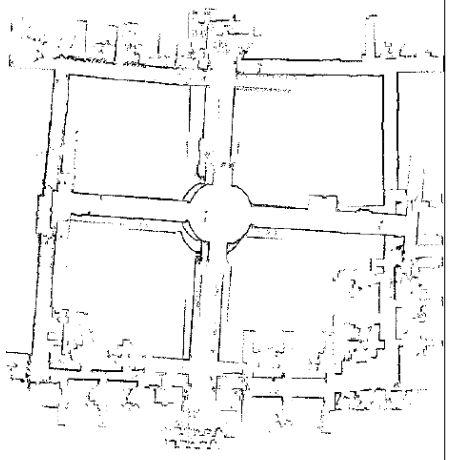
	Metrical Mapping	Topological Mapping
Small-scale space	Local SLAM	
Large-scale space	Cumulative errors Scalability	

Problem: Closing Large Loops

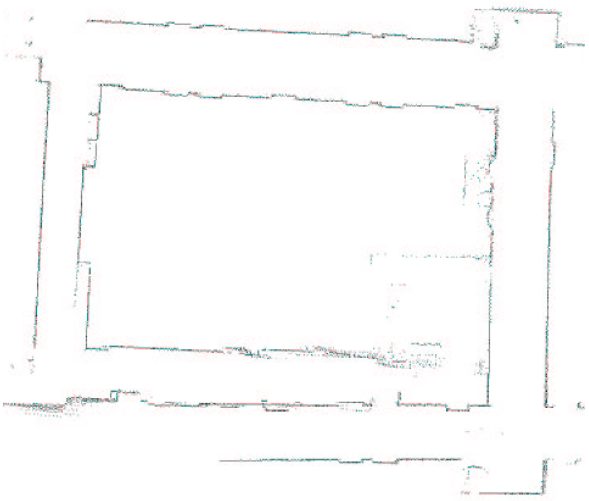
Raw Odometry



SLAM Corrected Odometry



Local matching can find false, but locally optimal, loop closures



Topological Mapping

- Describe large-scale space in terms of
  - Places (with local frames of reference)
  - Paths (with ordered sequences of places)
  - Regions (with sets of places and paths)
    - Paths can serve as boundaries
- Handles many practical planning problems, even without a metrical map

The Spatial Semantic Hierarchy

A hierarchy of *ontologies*.

- **Control**: select *control laws* to move reliably among *distinctive states*.
- **Causal**: *actions* such as turn and travel link *states*, which have sensory *views*.
- **Topological**: *places*, *paths*, and *regions* linked by connectivity, order, containment.
- **Metrical**: *frames of reference*, distance, direction, shape.

The Basic SSH

- **Strengths**
  - The robustness of commonsense knowledge comes from having multiple, different, coordinated representations for knowledge.
  - Makes few assumptions about sensors, effectors, or the environment.
- **Weaknesses**
  - Hill-climbing to distinctive states is awkward, and seems like unnecessary physical motion.
  - *What if we really want a global metrical map?*
  - *What if we really know about our sensors?*

Solution: The Hybrid SSH

- **Local metrical maps**
  - Metrical SLAM methods work well locally.
  - Localization substitutes for hill-climbing
- **Global topological maps**
  - Represent structural hypotheses explicitly.
- **Global metrical map**
  - Build on the skeleton of the topological map

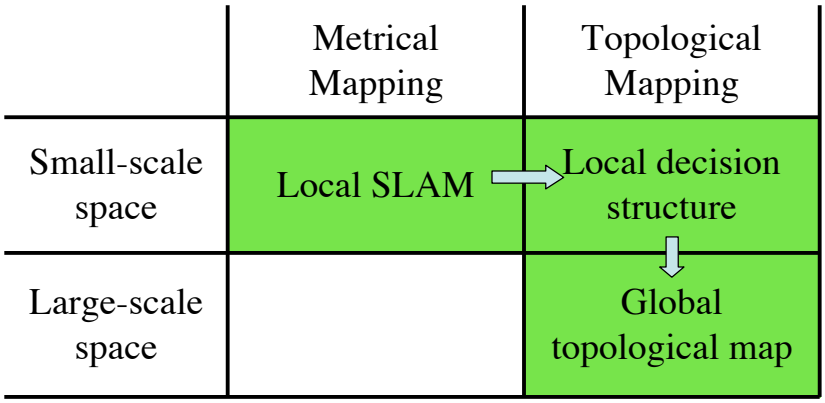
Identify the Local Topology

- Identify the local decision structure of each place neighborhood.
  - Travel experience as graph exploration

	Metrical Mapping	Topological Mapping
Small-scale space	Local SLAM →	Local decision structure
Large-scale space		

Build the Global Topological Map

- Decide when and how loops are closed
  - When does the next place match a previous place?
- Build a tree of all possible topologies



Searching the Tree of All Possible Maps

- The tree is **guaranteed** to contain the true map
  - All consistent maps are created.
  - Only inconsistent ones are deleted.
- Select the best consistent map for planning.
  - Remember the tree.
  - The current best map could be refuted.

Axioms for Map Structure

- These axioms can rule out possible maps.
  - Logically inconsistent, hence impossible
  - Outside the set of permissible maps

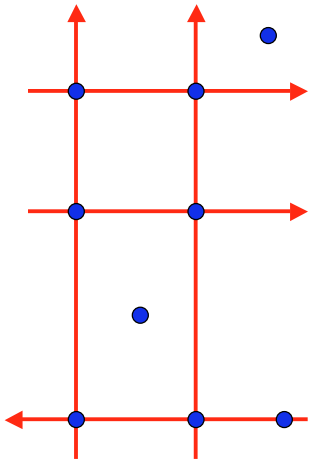
- **Causal**: predict results of actions
- **Topological**: order relations on paths
- **Boundary**: paths divide the world
- **Metrical**: triangle inequality

The Topological Map is a Graph of Places and Paths

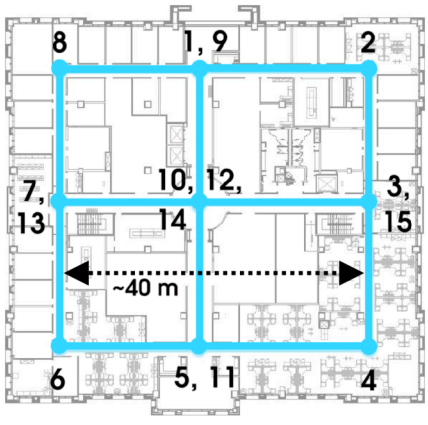
- The topological map is a bipartite graph:
  - Nodes = Places ∪ Paths
  - Edges = relations: *on*(*place*,*path*)
- Each path has a 1-D direction *dir* ∈ {+,–}
- An order relation, *order*(*path*,*a*,*b*,*dir*), for the places on each path.
- Each directed path is a *boundary*, describing places as on its right and its left.

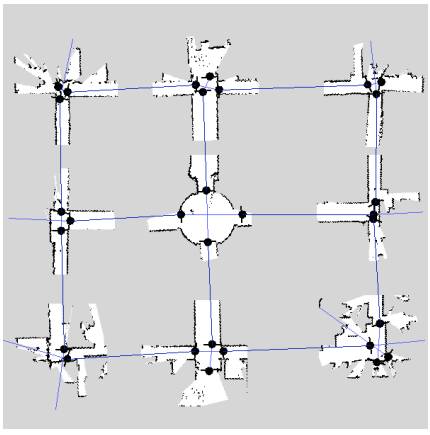
Deeper Topological Inference

- Each map has richer topological concepts and relations:
  - A *place* has a circular order of directed paths
  - *Boundary relations* hold between path & places
  - Useful for route planning
- Refute maps that violate the topological axioms



The Topological Map Links Local Place Maps



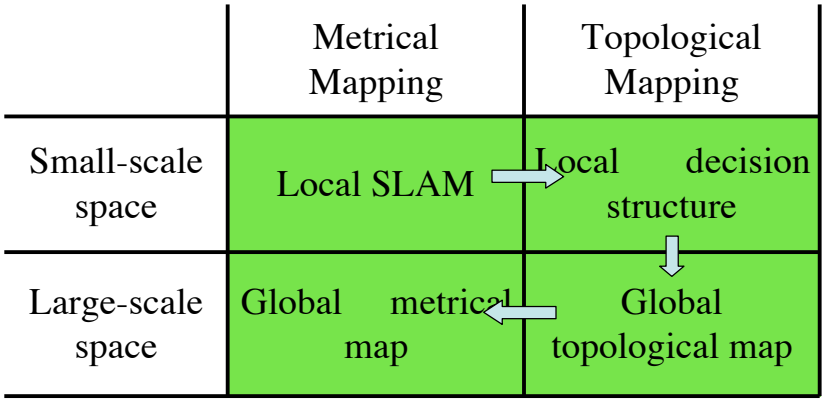


Roadmap

- Local metrical maps
  - *Given local maps of each place…*
- Global topological maps
  - *Given a single best structural hypothesis …*
- **Global metrical map**
  - **Displacement along each travel segment**
  - **Global layout of places**
  - **All robot poses in the global frame of reference**

Global Metrical Map

- Use the topological map as a skeleton.
  - Lay out places in a single global frame of reference.
  - Fill in the details from local places and segments.



Given the Topological Map …

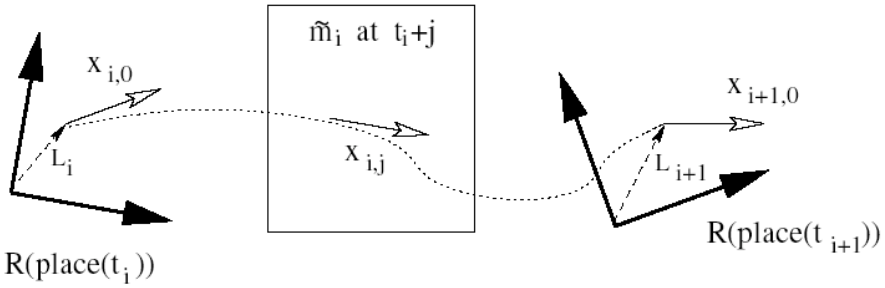
- The loop-closing problem is solved.
  - The topological map specifies which loops close, and where.
- Each place has an accurate local metrical map in its own local frame of reference.
- Continuous behavior divides into segments at distinctive place neighborhoods
- The global metrical map combines information from separate local maps.

The Global Metrical Map: Factoring the Problem

- **Displacements**: the pose of each place in the frame of reference of its predecessor.
- **Layout**: the pose of each place in the global frame of reference.
- **Robot poses**: the robot pose at each timestep in the global frame of reference.
- **Global map**: range sensor endpoints starting from known robot poses.

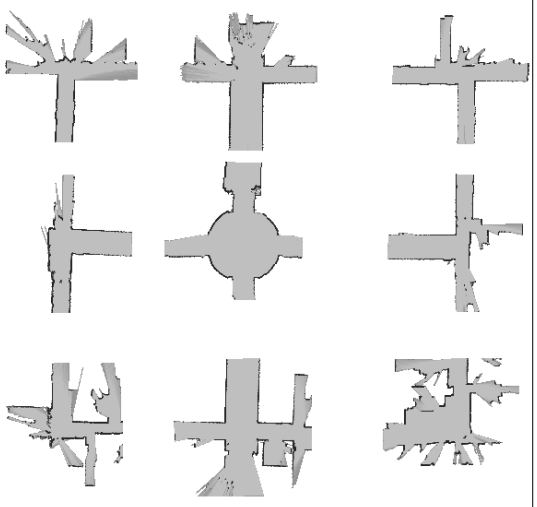
Estimating Displacements

- Use incremental SLAM to estimate pose $x_{i+1,0}$ in the frame of reference of m_i.
- Localize to get $x_{i+1,0}$ in frame m_{i+1}.
- Derive displacement λ_i between the two place poses.



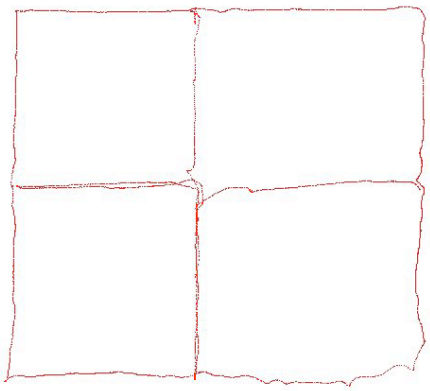
Estimating Place Layout

- Local displacements propagate to global place layout.
  - Loop-closings are especially helpful.
- Relaxation search converges quickly to a maximum likelihood layout.



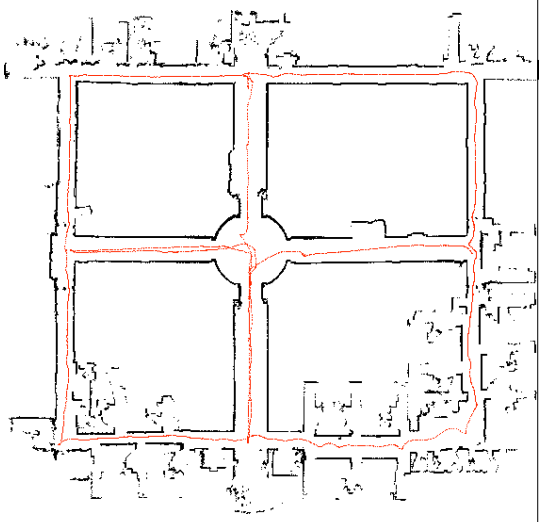
Estimating Robot Poses

- **Given** a max likelihood place layout
- and the trajectory of robot poses
- **define** a fixed anchor pose each time the trajectory passes through a place neighborhood
- **interpolate** poses in each segment, using corrected odometry.



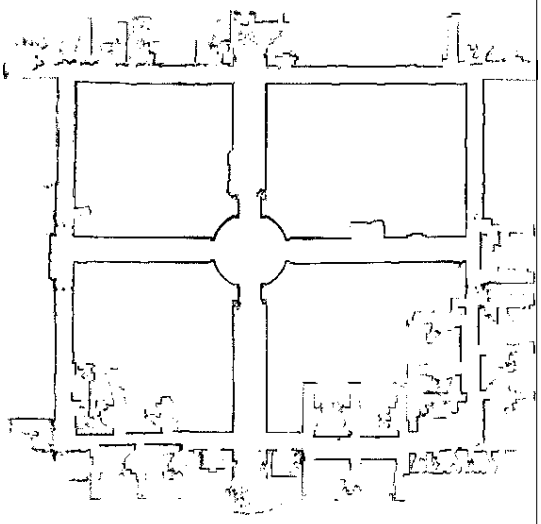
Global SLAM with new poses

- The pose distribution is a highly accurate proposal distribution.
- Treat it as providing corrected odometry.
- Now do SLAM in the global frame of reference.
- Or just mapping given localization.



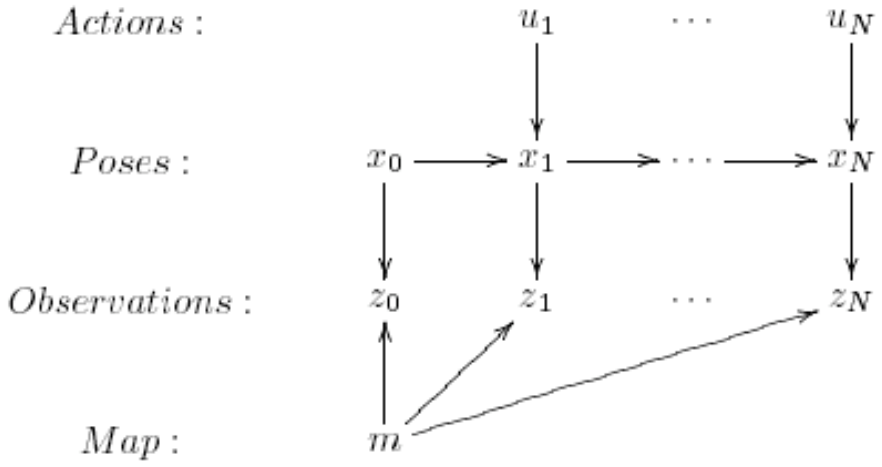
The Global Metrical Map

- The result is an accurate map in the global frame of reference.
- Cumulative error is eliminated by the topological map.
- More experience reduces any remaining errors.



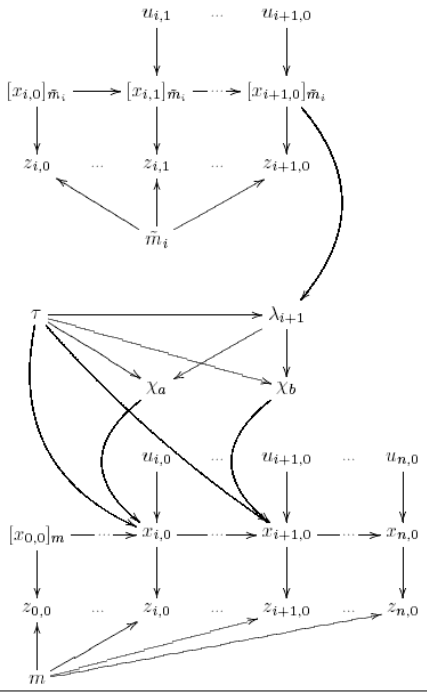
Dynamic Bayesian Network

- The well-known DBN for local SLAM.



Factored DBN

- For building the global metrical map on the topological skeleton τ.
  - Local maps m_i
  - Displacements λ
  - Place layout χ
  - Global poses x
  - Global map m



Three Levels of Map

- **Local perceptual map**
  - Use it for motion control and hazard avoidance
  - Scroll old map off the horizon
  - Identify places, gateways, distinctive states, views, and actions
- **Topological map**
  - Use it for route planning, global topological localization, and explanation
  - Learn through incremental, active exploration, branching on structural ambiguities
- **Global metrical map**
  - Use it for relative-position queries
  - Build it incrementally on the topological skeleton

The Spatial Semantic Hierarchy

- Robustness comes from multiple representations, with different strenths and weaknesses.
- The *Basic* SSH combines control, causal, topological and metrical representations.
- The *Hybrid* SSH combines topological representations for large-scale space with metrical representations for small-scale space.

References

- Beeson, Modayil & Kuipers, **Factoring the mapping problem: Mobile robot map-building in the Hybrid Spatial Semantic Hierarchy**. IJRR, 2009.
  - Kuipers, **An intellectual history of the Spatial Semantic Hierarchy**. In Jefferies & Yeap (edited volume), Springer, 2008
- Remolina & Kuipers, **Towards a general theory of topological maps**. AIJ, 2004.
- Kuipers, **The Spatial Semantic Hierarchy**. AIJ, 2000.

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Next

- What if we succeed?
  - Social and ethical implications of intelligent robotics, and/or …
  - AI and consciousness.