

Sudoku examples

The “Hardest” Sudoku

Sudoku

• “hardest Sudoku puzzle”

8	5				2	4		
7	2							9
		4						
			1		7			2
3		5				9		
	4							
				8			7	
	1	7						
				3	6		4	

Sudoku

• Propagate 9

8	5				2	4		
7	2							9
		4						
			1		7			2
3		5				9		
	4							
				8			7	
	1	7						
				3	6		4	

Sudoku

• Notice full column.

8	5				2	4		
7	2							9
		4						
			1		7			2
3		5				9		
	4							
				8			7	
	1	7						9
				3	6		4	

Sudoku

• Propagate 2.

8	5				2	4		
7	2							9
		4						
			1		7			2
3		5				9		
	4							
				8			7	
	1	7					9	
				3	6		4	

Sudoku

• Propagate 2 again.

8	5				2	4		
7	2							9
		4						
			1		7			2
3		5				9		
	4							
				8			7	
	1	7					9	
				3	6		4	

Sudoku

- Result.

8	5			2	4		
7	2						9
		4					2
			1		7		2
3		5				9	
	4						
			8			7	
	1	7				9	
			3	6		4	

Sudoku

- Propagate 7.

8	5			2	4		
7	2						9
		4					
			1		7		2
3		5				9	
	4						
				8		7	
	1	7				9	
			3	6		4	

Sudoku

- Notice 1,2 together

8	5			2	4		
7	2						9
		4					2
			1		7		2
3	7	5				9	
	4						
			8			7	
	1	7				9	
			3	6		4	

Sudoku

- Represent a pair of cells.
– They are “filled”.

8	5			2	4		
7	2						9
		4					2
			1		7		2
3	7	5				9	
¹ / ₂	4	¹ / ₂					
			8			7	
	1	7				9	
			3	6		4	

Sudoku

- Propagate 4, 7, 1, and 9.

8	5			2	4		
7	2						9
		4					2
			1	4	7		2
3	7	5				9	1 4
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7				9	
			7	3	6		4

Sudoku

- Eliminate all but one.

8	5			2	4		
7	2						9
		4					2
			1	4	7		2
3	7	5				9	1 4
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7				9	
			7	3	6		4

Sudoku

- Assign remaining value.

8	5			2	4		
7	2						9
		4					2
			1	4	7		2
3	7	5			8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7					9
			7	3	6		4

Sudoku

- D.row1 = {6,8,9} => {F.A,F.B} = {3,5}

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ / ₅	³ / ₅ 2
3	7	5			8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7					9
			7	3	6		4

Sudoku

- Another pair in box E.
– Next step is quite tricky.

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ / ₅	³ / ₅ 2
3	7	5	² / ₆	² / ₆	8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7					9
			7	3	6		4

Sudoku

- Notice 3 and 8.
– Propagate 3 (next 8).

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ / ₅	³ / ₅ 2
3	7	5	² / ₆	² / ₆	8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7					9
			7	3	6		4

Sudoku

- Now propagate: 8
– Same pair in Box I.

A	B	C
D	E	F
G	H	I

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ / ₅	³ / ₅ 2
3	7	5	² / ₆	² / ₆	8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7					9
			7	3	6		4

Sudoku

- Gives another cell-pair.

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ / ₅	³ / ₅ 2
3	7	5	² / ₆	² / ₆	8	9	1
¹ / ₂	4	¹ / ₂					
			9	8	1		7
	1	7				³ / ₈	³ / ₈ 9
			7	3	6		4

Sudoku

- Propagate 6.
– Because I.row2 is full.

8	5			2	4		
7	2						9
		4					2
			1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
¹ ₂	4	¹ ₂					
			9	8	1	7	7
6	1	7				³ ₈	9 ³ ₈
			7	3	6	4	

Sudoku

- Notice 6,8 and Box D.
– Only one cell left!

8	5			2	4		
7	2						9
		4					2
	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
¹ ₂	4	¹ ₂					
			9	8	1		7
6	1	7				³ ₈	9 ³ ₈
			7	3	6	4	

Sudoku

- Lots of progress!
– This is “hardest”?

8	5			2	4		
7	2						9
		4					2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
¹ ₂	4	¹ ₂					
			9	8	1		7
6	1	7				³ ₈	9 ³ ₈
			7	3	6	4	

Sudoku

- Notice 8,9 in Box G.

8	5			2	4		
7	2						9
		4					2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
¹ ₂	4	¹ ₂					
			9	8	1		7
6	1	7				³ ₈	9 ³ ₈
	⁸ ₉	⁸ ₉	7	3	6	4	

Sudoku

- Eliminate all but A.G := 1
– This resolves {1,2} pair in D.

8	5			2	4		
7	2						9
1		4					2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
¹ ₂	4	¹ ₂					
			9	8	1		7
6	1	7				³ ₈	9 ³ ₈
	⁸ ₉	⁸ ₉	7	3	6	4	

Sudoku

- Box G gets 4 and 5.
• Box I gets 1 and 2 (+ a pair)

8	5			2	4		
7	2						9
1		4					2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅ 2
3	7	5	² ₆	² ₆	8	9	1 4
2	4	1					
			9	8	1		7
6	1	7				³ ₈	9 ³ ₈
	⁸ ₉	⁸ ₉	7	3	6	4	

Sudoku

- Finish Box G
- Next propagate 1, then 7

8	5				2	4		
7	2							9
1		4						2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1						
4	3	2	9	8	1	⁵ ₆	7	⁵ ₆
6	1	7				³ ₈	9	³ ₈
5	⁸ ₉	⁸ ₉	7	3	6	2	4	1

Sudoku

- Next . . .

8	5			1	2	4		7
7	2					1		9
1		4		7				2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1				7		
4	3	2	9	8	1	⁵ ₆	7	⁵ ₆
6	1	7				³ ₈	9	³ ₈
5	⁸ ₉	⁸ ₉	7	3	6	2	4	1

Sudoku

- In the top row, where is 9?
- Then propagate 9 & left col.

8	5	9		1	2	4		7
7	2					1		9
1		4		7	9			2
9	⁶ ₈	⁶ ₈	1	4	7	³ ₅	³ ₅	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1		9		7	⁶ ₈	⁶ ₈
4	3	2	9	8	1	⁵ ₆	7	⁵ ₆
6	1	7				³ ₈	9	³ ₈
5	⁸ ₉	⁸ ₉	7	3	6	2	4	1

Sudoku

- Getting optimistic!
- Propagate 3

8	5	9		1	2	4		7
7	2	3				1		9
1	6	4		7	9			2
9	8	6	1	4	7	³ ₅	³ ₅	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1	⁵ ₃	9	³ ₅	7	⁶ ₈	⁶ ₈
4	3	2	9	8	1	⁵ ₆	7	⁵ ₆
6	1	7				³ ₈	9	³ ₈
5	9	8	7	3	6	2	4	1

Sudoku

- Now 6.
- Resolves two pairs.

8	5	9		1	2	4		7
7	2	3				1		9
1	6	4		7	9			2
9	8	6	1	4	7	³ ₅	³ ₅	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1	5	9	3	7	8	⁶ ₈
4	3	2	9	8	1	⁶ ₅	7	⁵ ₆
6	1	7				³ ₈	9	³ ₈
5	9	8	7	3	6	2	4	1

Sudoku

- Propagate 5, then 3 and 8.
- From here, it's easy.

8	5	9		1	2	4	3	7
7	2	3	8			1		9
1	6	4	3	7	9	5	2	8
9	8	6	1	4	7	³ ₅	⁵ ₃	2
3	7	5	² ₆	² ₆	8	9	1	4
2	4	1	5	9	3	7	8	6
4	3	2	9	8	1	6	7	5
6	1	7				⁸ ₃	9	³ ₈
5	9	8	7	3	6	2	4	1

Sudoku

- Finish it off!
– “Hardest”? Nah!

8	5	9	6	1	2	4	3	7
7	2	3	8	5	4	1	6	9
1	6	4	3	7	9	5	2	8
9	8	6	1	4	7	3	5	2
3	7	5	2	6	8	9	1	4
2	4	1	5	9	3	7	8	6
4	3	2	9	8	1	6	7	5
6	1	7	4	2	5	8	9	3
5	9	8	7	3	6	2	4	1

Sudoku Strategies

- For each number . . . For each box . . .
 - Does the box already contain the number?
 - Move on.
 - Is there only one possible cell?
 - Fill it in.
 - Are there two possible cells?
 - Create a pair constraint.
 - Are all possible cells in a row or column?
 - Propagate in the direction they point.
 - Otherwise
 - Move on.

Sudoku Strategies (2)

- When there are exactly two possible cells...
 - Create a constraint: when one cell becomes impossible, the other must get that value.
- For an empty cell,
 - Eliminate all values in its row, column, and box
 - If exactly one value is left, assign that value