Geographic Routing in \(d\)-dimensional Spaces with Guaranteed Delivery and Low Stretch

Simon S. Lam, Fellow, IEEE, and Chen Qian, Student Member, IEEE

Abstract—Almost all geographic routing protocols have been designed for 2D. We present a novel geographic routing protocol, named Multi-hop Delaunay Triangulation (MDT), for \(2D\), \(3D\), and higher dimensions with these properties: (i) guaranteed delivery for any connected graph of nodes and physical links, and (ii) low routing stretch from efficient forwarding of packets out of local minima. The guaranteed delivery property holds for node locations specified by accurate, inaccurate, or arbitrary coordinates. The MDT protocol suite includes a packet forwarding protocol together with protocols for nodes to construct and maintain a distributed MDT for routing. We present the performance of MDT protocols in \(2D\) and \(4D\) as well as performance comparisons of MDT routing versus representative geographic routing protocols for nodes in \(2D\) and \(3D\). Experimental results show that MDT provides the lowest routing stretch in the comparisons. Furthermore, MDT protocols are specially designed to handle churn, i.e., dynamic topology changes due to addition and deletion of nodes and links. Experimental results show that MDT’s routing success rate is close to 100\% during churn and node states converge quickly to a correct MDT after churn.

Index Terms—Geographic routing, greedy forwarding, distributed Delaunay triangulation, resilience to churn, protocol design.

I. INTRODUCTION

GEOGRAPHIC routing (also known as location-based or geometric routing) is attractive because the routing state needed for greedy forwarding at each node is independent of network size. Almost all geographic routing protocols have been designed for nodes in \(2D\). In reality, many wireless applications run on nodes located in \(3D\) [22], [1], [7], [8]. Furthermore, node location information may be highly inaccurate or simply unavailable.

Consider a network represented by a connected graph of nodes and physical links (to be referred to as the connectivity graph). Greedy forwarding of a packet may be stuck at a local minimum, i.e., the packet is at a node closer to the packet’s destination than any of the node’s directly-connected neighbors. Geographic routing protocols differ mainly in their recovery methods designed to move packets out of local minima. For general connectivity graphs in \(3D\), face routing methods designed for \(2D\) [4], [13], [14] are not applicable. Furthermore, Durocher et al. [7] proved that there is no “local” routing protocol that provides guaranteed delivery, even under the strong assumptions of a “unit ball graph” and accurate location information. Thus, designing a geographic routing protocol that provides guaranteed delivery in \(3D\) is a challenging problem.

We present in this paper a novel geographic routing protocol, MDT, that provides guaranteed delivery for a network of nodes in a \(d\)-dimensional space, for \(d \geq 2\). (Only Euclidean spaces are considered in this paper.) The guaranteed delivery property is proved for node locations specified by arbitrary coordinates; thus the property also holds for node locations specified by inaccurate coordinates or accurate coordinates. We show experimentally that MDT routing provides a routing (distance) stretch close to 1 for nodes in \(2D\) and \(3D\) when coordinates specifying node locations are accurate.\(^1\) When coordinates specifying node locations are highly inaccurate, we show that MDT routing provides a low routing (distance) stretch relative to other geographic routing protocols. Nodes may also be arbitrarily located in a virtual space with packets routed by MDT using the coordinates of nodes in the virtual space (instead of their coordinates in physical space). In this case, MDT routing still provides guaranteed delivery but the distance stretch in physical space may be high.

Geographic routing in a virtual space is useful for networks without location information or networks in which the routing cost between two directly-connected neighbors is neither a constant nor proportional to the physical distance between them (such as, ETT [6]). For example, a \(4D\) virtual space can be used for geographic routing of nodes physically located in a \(3D\) space. The extra dimension makes it possible to assign nodes to locations in the virtual space such that the Euclidean distance between each pair of nodes in the virtual space is a good estimate of the routing cost between them. The design of a positioning system to embed routing costs in a virtual space is a challenging problem for wireless networks without any-to-any routing support and beyond the scope of this paper. The problem is solved in a companion paper [23] where we show how to (i) make use of MDT protocols to embed routing costs in virtual spaces (such as \(4D\)), and (ii) extend MDT routing to optimize end-to-end path costs for any additive routing metric.

MDT was designed to leverage the guaranteed delivery property of Delaunay triangulation (DT) graphs. For nodes in \(2D\), Bose and Morin proved that greedy routing in a DT always finds a given destination node [3]. Lee and Lam [16] generalized their result and proved that in a \(d\)-dimensional Euclidean space (\(d \geq 2\)), given a destination location \(\ell\), greedy routing in a DT always finds a node that is closest to \(\ell\).

Figure 1(a) shows a \(2D\) space with three large obstacles and an arbitrary connectivity graph. Figure 1(b) shows the DT graph [9] of the nodes in Figure 1(a). In the DT graph,\(^1\)Routing and distance stretch are defined later.

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the dashed lines denote DT edges between nodes that are not connected by physical links. The MDT graph of the connectivity graph in Figure 1(a) is illustrated in Figure 1(c). By definition, the MDT graph includes every physical link in the connectivity graph and every edge in the DT graph. In MDT routing, when a packet is stuck at a local minimum of the connectivity graph, the packet is next forwarded, via a “virtual link,” to the DT neighbor that is closest to the destination. In short, the recovery method of MDT is to forward greedily in the DT graph which is guaranteed to succeed.

In this paper, we present MDT protocols for a set of nodes to construct and maintain a correct multi-hop DT (formal definition in Section II). In a multi-hop DT, two nodes that are neighbors in the DT graph communicate directly if there is a physical link between them; otherwise, they communicate via a virtual link, i.e., a multi-hop path provided by soft-state forwarding tables in nodes along the path.

MDT protocols are also designed specially for networks where node churn and link churn are nontrivial concerns. For example, in a wireless community network, nodes join and leave whenever computers in the community are powered on and off. Furthermore, the quality of wireless links may vary widely over time for many reasons (e.g., fading effects, external interference, and weather conditions). Link quality fluctuations cause dynamic addition and deletion of physical links in the connectivity graph used for MDT routing.

The MDT protocol suite consists of protocols for forwarding, join, leave, failure, maintenance, and system initialization. The MDT join protocol was proved correct for a single join. Thus it constructs a correct multi-hop DT when nodes join serially. The maintenance protocol enables concurrent joins at system initialization. Experimental results show that MDT constructs a correct multi-hop DT very quickly using concurrent joins. The join and maintenance protocols are sufficient for a system under churn to provide a routing success rate close to 100% and for node states to converge to a correct multi-hop DT after churn. The leave and failure protocols are used to improve accuracy and reduce communication cost.

MDT is communication efficient because MDT does not use flooding to discover multi-hop DT neighbors. MDT’s search technique is also not limited by a maximum hop count (needed in scoped flooding used by many wireless routing protocols) and is guaranteed to succeed when the existing multi-hop DT is correct.

Lastly, since MDT routing is designed to run correctly in any connected graph of nodes and physical links, it is possible to use MDT for layer-2 routing in wireline networks. For example, ROME [24], an architecture for metropolitan-scale layer-2 networks, makes use of MDT routing in virtual spaces.

For a given set of nodes, under the restrictive assumption that every node can directly communicate with every other node, Lee and Lam [16], [17] presented protocols for the construction and maintenance of a correct distributed DT. These protocols cannot be used for routing in an arbitrary connectivity graph because their assumption is not satisfied. Major contributions of this paper include the definition of a correct distributed multi-hop DT, a new two-step greedy forwarding protocol, proofs of guaranteed delivery by the new forwarding protocol and correctness of the join protocol, and as well as designing each protocol in the MDT suite to correctly construct/repair forwarding tables in paths between multi-hop DT neighbors to provide a correct distributed multi-hop DT.

A. Related work

There were several prior proposals to apply DT to geographic routing. None of them addressed the underlying technical issue that the DT graph of a wireless network is, in general, not a subgraph of its connectivity graph. In [30], requirements are imposed on the placement of nodes and links in 2D such the DT graph is a subgraph of the connectivity graph. In other approaches, the restricted DT graph [11] and the k-localized DT [20] are both approximations of the DT graph. These graphs were shown to be good spanners with constant stretch factors. However, being DT approximations, they do not provide guaranteed delivery. Furthermore, they were designed for nodes in 2D with connectivity graphs restricted to unit disk graphs. (A unit disk graph requires that a physical link exists between two nodes if and only if the distance between them is within a given radio transmission range.)

Many geographic routing protocols have been designed for nodes in 2D based upon greedy forwarding. Two of the earliest protocols, GFG [4] and GPSR [13], use face routing to move packets out of local minima. Bose et al. [2] proposed other memoryless routing algorithms for special planar graphs. These protocols provide guaranteed delivery for a planar graph. If the connectivity graph is not planar, a planarization algorithm (such as GG [10] or RNG [26]) is used to construct a connected planar subgraph. Successful construction requires
graphs. Each node in the graph belongs to a distributed DT. For recovery, the packet is routed upwards in the spanning tree until it reaches a point where greedy routing can again make progress.

Kim et al. [14] proposed CLDP which, given any connectivity graph, produces a subgraph in which each induced subgraph among the nodes faces a cycle of length \( n \). CLDP was designed to provide guaranteed delivery for nodes in 3D under the assumption that the graph is a tree. CLDP uses a distributed algorithm that requires each node to know the location of its neighbors.

Leong et al. proposed GDSTR [18] which provides guaranteed delivery for any connectivity graph. Initially, nodes exchange messages to compute and store a distributed spanning tree. Each node also computes and stores a convex hull of the locations of all of its descendants in the subtree rooted at the node; the resulting tree is called a hull tree. Subsequently, a packet is routed greedily until it is stuck at a local minimum. For recovery, the packet is routed upwards in the spanning tree until it reaches a point where greedy routing can again make progress.

GHG [21] and GRG [8] are geographic protocols designed for 3D. GHG assumes a unit-ball graph and accurate location information, which are unrealistic assumptions. GRG uses random recovery which is inefficient and does not provide guaranteed delivery. For Poisson distributed nodes and multi-hop graphs in 3D [28] (disk graphs in 2D [27]), the use of a "sufficiently" large transmission radius can provide guaranteed delivery.

Aside from MDT, there is one other geographic routing protocol that provides guaranteed delivery for general connectivity graphs in 3D, namely, GDSTR-3D [31]. For recovery, GDSTR-3D uses two distributed hull trees while MDT uses a distributed DT graph. GDSTR-3D, designed for sensor networks, assumes a static network topology; the protocol has no provision for any dynamic topology change.

Embedding nodes into a virtual space and assigning them virtual coordinates can provide guaranteed delivery for geographic routing. The VPD protocol [23] can be used by nodes to assign themselves coordinates in a Euclidean virtual space such that the distance between any two nodes in the space is a good estimate of the routing cost (in any additive metric) between them. For quasi-unit disk graph models, Sarkar et al. [25] proposed to use conformal mapping computed with Ricci flow to obtain an embedding such that greedy forwarding guarantees delivery. Xia et al. [29] proposed a unit tetrahedron cell mesh structure to obtain virtual coordinates to achieve deterministic greedy routing in 3D. PIE [12] produces a greedy embedding by isometrically embedding trees in non-Euclidean spaces of dimension \( \log^2(n) \). Its design targets Internet-like graphs. Each node in the graph belongs to \( O(\log(n)) \) trees. PIE achieves a success ratio of 100% for any connected graph.

B. Outline

The balance of this paper is organized as follows. In Section II, we present concepts, definitions, and model assumptions. In Section III, we present the MDT forwarding protocol. In Section IV, we present join, maintenance, and initialization protocols. In Section V, we present an experimental performance evaluation of MDT in 3D and 4D. We also present experimental results to demonstrate MDT’s resilience to node churn and link churn. In Section VI, we present performance comparisons of MDT with geographic routing protocols designed for 2D and 3D. We conclude in Section VII.

II. Concepts and Definitions

A triangulation of a set \( S \) of nodes (points) in 2D is a maximal planar subdivision of the convex hull of nodes in \( S \) into non-overlapping triangles such that the vertices of each triangle are nodes in \( S \). A DT in 2D is a triangulation such that the circumcircle of each triangle does not contain any other node inside \( S \). The definition of DT can be generalized to a higher dimensional space using simplexes and circum-hyperspheres. In each case, the DT of \( S \) is a graph to be denoted by \( DT(S) \).

Consider a set \( S \) of nodes in a \( d \)-dimensional space, for \( d \geq 2 \). Each node in \( S \) is identified by its location specified by coordinates. There is at most one node at each location. When we say node \( u \) knows node \( v \), node \( u \) knows node \( v \)'s coordinates. A node’s coordinates may be accurate, inaccurate, or arbitrary (that is, its known location may differ from its actual location). In Section II-A, we present the definition of a distributed DT and a key result from Lee and Lam [16], [17] that we need later.

A. Distributed DT

A distributed DT of a set \( S \) of nodes is specified by \( \{ u, N_u = \{ u \in S \} \} \), where \( N_u \) represents the set of \( u \)'s neighbor nodes, which is locally determined by \( u \).

**Definition 1.** A distributed DT is correct if and only if for every node \( u \in S \), \( N_u \) is the same as the neighbor set of \( u \) in \( DT(S) \).

\( ^2 \) A maximal planar subdivision is a subdivision such that no more edge connecting two nodes can be added to the subdivision without destroying its planarity.
To construct a correct distributed DT, each node, \( u \in S \), discovers a set \( C_u \) of nodes (\( C_u \) includes \( u \)). Then \( u \) computes \( DT(C_u) \) locally to determine its set \( N_u \) of neighbor nodes. Note that \( C_u \) is information discovered by \( u \) while \( S \) is global knowledge. For the extreme case of \( C_u = S \), \( u \) is guaranteed to know its neighbors in \( DT(S) \). However, the communication cost for each node to discover \( S \) (using, for example, a broadcast protocol) would be very high and not scalable. A necessary and sufficient condition \([16], [17]\) for a distributed DT to be correct is that for all \( u \in S \), \( C_u \) includes all neighbor nodes of \( u \) in \( DT(S) \). The condition’s necessity is obvious. Its sufficiency requires a nontrivial proof (see \([17]\)). This result enabled the design of efficient protocols for distributed DT construction.

### B. Model assumptions

Two nodes directly connected by a physical link are said to be physical neighbors. Each link is bidirectional. In our protocol descriptions, each link is assumed to provide reliable message delivery.\(^3\)

The graph of nodes and physical links may be arbitrary so long as it is a connected graph. We provide protocols to handle dynamic topology changes. In particular, new nodes may join and existing nodes may leave or fail.\(^4\) Furthermore, new physical links may be added and existing physical links that have become error-prone are deleted.

Each node runs the same protocols. After a node boots up, it knows all of its physical neighbors. Subsequently, it discovers other nodes, including its multi-hop DT neighbors, from sending and receiving protocol messages.

### C. Multi-hop DT

A multi-hop DT is specified by \( \{ < u, N_u, F_u > | u \in S \} \), where \( F_u \) is a soft-state forwarding table, and \( N_u \) is \( u \)'s neighbor set which is derived from information in \( F_u \). The multi-hop DT model generalizes the distributed DT model by relaxing the requirement that every node in \( S \) be able to communicate directly with each of its neighbors. (We use the term “neighbor” to refer to a DT neighbor.) In a multi-hop DT, the neighbor of a node may not be a physical neighbor; see, for example, nodes \( i \) and \( g \) in Figure 2.

For a node \( u \), each entry in its forwarding table \( F_u \) is a 4-tuple \( < source, pred, succ, dest > \), which is a sequence of nodes with \( source \) and \( dest \) being the source and destination nodes of a path, and \( pred \) and \( succ \) being node \( u \)'s predecessor and successor nodes in the path. In a tuple, \( source \) and \( pred \) may be the same node; also, \( succ \) and \( dest \) may be the same node. A tuple in \( F_u \) is used by \( u \) for message forwarding from \( source \) to \( dest \) or from \( dest \) to \( source \). For a specific tuple \( t \), we use \( t.source \), \( t.pred \), \( t.succ \), and \( t.dest \) to denote the corresponding nodes in \( t \).

For ease of exposition, we assume that a tuple and its “reverse” are inserted in and deleted from \( F_u \) as a pair. For example, \( < a, b, c, d > \) is in \( F_u \) if and only if \( < d, c, b, a > \) is in \( F_u \). (In fact, only one tuple is stored with each of its two endpoints being both source and destination.) A tuple in \( F_u \) with \( u \) itself as the source is represented as \( < -, -, succ, dest > \), which does not have a reverse in \( F_u \).

For an example of a forwarding path, consider the MDT graph in Figure 2. The DT edge between nodes \( g \) and \( i \) is a virtual link; messages are routed along the paths, \( g \rightarrow e \rightarrow h \rightarrow i \) and \( i \rightarrow h \rightarrow e \rightarrow g \), using the following tuples: \( < -, -, e, i > \) in node \( g \), \( < g, g, h, i > \) in node \( e \), \( < g, e, i, i > \) in node \( h \), and \( < -, -, h, g > \) in node \( i \).

Tuples in \( F_u \) are maintained as soft states. Each tuple is refreshed whenever there is packet traffic (e.g., application data or keep-alive messages) between its endpoints. A tuple that is not refreshed will be deleted when its timeout occurs.

**Definition 2.** A multi-hop DT of \( S \), \( \{ < u, N_u, F_u > | u \in S \} \), is correct if and only if the following conditions hold: i) the distributed DT of \( S \), \( \{ < u, N_u > | u \in S \} \), is correct; and ii) for every neighbor pair \( (u, v) \), there exists a unique \( k \)-hop path between \( u \) and \( v \) in the forwarding tables of nodes in \( S \), where \( k \) is finite.

For a dynamic network in which nodes and physical links may be added and deleted, we define a metric for quantifying the accuracy of a multi-hop DT. We consider a node to be in-system from when it has finished joining until when it starts leaving or has failed. Let \( MDT(S) \) denote a multi-hop DT of a set \( S \) of in-system nodes. Let \( N_{n}(MDT(S)) \) be the total number of correct neighbor entries and \( N_{wp}(MDT(S)) \) be the total number of wrong neighbor entries in the forwarding tables of all nodes. A neighbor \( v \) in \( N_{n} \) is correct when \( u \) and \( v \) are neighbors in \( DT(S) \) and wrong when \( u \) and \( v \) are not neighbors in \( DT(S) \). Let \( N_{edges}(DT(S)) \) be the number of edges in \( DT(S) \). Let \( N_{np}(MDT(S)) \) be the number of edges in \( DT(S) \) that do not have forwarding paths in the multi-hop DT of \( S \). The accuracy of \( MDT(S) \) is defined to be:

\[
\frac{N_{n}(MDT(S)) - N_{wp}(MDT(S)) - 2 \times N_{np}(MDT(S))}{2 \times N_{edges}(DT(S))}
\]

It is straightforward to prove that the accuracy of \( MDT(S) \) is 1 (or 100%) if and only if the multi-hop DT of \( S \) is correct.

**Terminology.** For a node \( u \), a physical neighbor \( v \) that has just booted up is represented in \( F_u \) by the tuple \( < -, -, -, v > \). A physical neighbor \( v \) that has sent a join request and received a join reply from a DT node is said to be a physical neighbor attached to the DT. It is represented in \( F_u \) by \( < -, -, v, v > \). We use \( P_u \) to denote \( u \)'s set of physical neighbors attached to
the DT. A node in \( P_u \) will become a DT node when it finishes executing the join protocol.

### III. MDT Forwarding Protocol

The key idea of MDT forwarding at a node, say \( u \), is conceptually simple: For a packet with destination \( d \), if \( u \) is not a local minimum, the packet is forwarded to a physical neighbor closest to \( d \); else, the packet is forwarded, via a virtual link, to a multi-hop DT neighbor closest to \( d \).

For a more detailed specification, consider a node \( u \) that has received a data message \( m \) to forward. Node \( u \) stores it with the format: \( m = \langle \text{dest}, \text{source}, \text{relay}, \text{data} \rangle \) in a local data structure, where \( \text{dest} \) is the destination location, \( \text{source} \) is the source node, \( \text{relay} \) is the relay node, and \( \text{data} \) is the payload of the message. Note that if \( \text{relay} \neq \text{null} \), message \( m \) is traversing a virtual link.

The MDT forwarding protocol at a node, say \( u \), is specified by the conditions and actions in Table II. To forward message \( m \) to a node closest to location \( \text{dest} \), the conditions in Table II are checked sequentially. The first condition found to be true determines the forwarding action. In particular, line 3 is for handling messages traversing a virtual link. Line 4 is greedy forwarding to physical neighbors. Line 5 is greedy forwarding to multi-hop DT neighbors.

The following theorem, which states that MDT forwarding in a correct multi-hop DT provides guaranteed delivery, is proved in the Appendix.

**Theorem 1:** Consider a correct multi-hop DT of a finite set \( S \) of nodes in a \( d \)-dimensional Euclidean space \((d \geq 2)\). Given a location \( \ell \) in the space, the MDT forwarding protocol succeeds to find a node in \( S \) closest to \( \ell \) in a finite number of hops.

### IV. MDT Protocol Suite

In addition to the forwarding protocol, MDT includes join, maintenance, leave, failure, and initialization protocols. The join protocol is designed to have the following correctness property: Given a system of nodes maintaining a correct multi-hop DT, after a new node has finished joining the system, the resulting multi-hop DT is correct. This property ensures that a correct multi-hop DT can be constructed for any system of nodes by starting with one node, say \( u \) with \( F_u = \emptyset \) initially, which is a correct multi-hop DT by definition, and letting the other nodes join the existing multi-hop DT serially.

Two nodes are said to join a system concurrently if their join protocol executions overlap in time. When two nodes join concurrently, the joins are independent if the sets of nodes whose states are changed by the join protocol executions do not overlap. For a large network, two nodes joining different parts of the network are likely to be independent. If nodes join a correct multi-hop DT concurrently and independently using the MDT join protocol, the resulting multi-hop DT is also guaranteed to be correct.

The maintenance protocol is designed to repair errors in node states after concurrent joins that are dependent, after nodes leave or fail, after the addition of physical links, and after the deletion of existing physical links (due to, for example, degraded link quality). Experimental results show that join and maintenance protocols are sufficient for a system of nodes to recover from dynamic topology changes and their multi-hop DT to converge to 100% accuracy.

MDT includes leave and failure protocols designed for a single leave and failure, respectively, for two reasons: (i) A departed node has almost all recovery information in its state to inform its neighbors how to repair their states. Such recovery information is not available to the maintenance protocol and would be lost if not provided by a leave or failure protocol before the node leaves or fails. (For failure recovery, each node \( u \) pre-stores the recovery information in a selected neighbor which serves as \( u \)’s monitor node.) Thus having leave and failure protocols allows the maintenance protocol, which has a higher communication cost, to run less frequently than otherwise. (ii) Concurrent join, leave, and failure occurrences in different parts of a large network are often independent of each other. After a leave or failure, node states can be quickly and effectively repaired by leave and failure protocols without waiting for the maintenance protocol to run. The leave and failure protocols are presented in the Appendix.

For a multi-hop DT, in addition to constructing and maintaining a distributed DT, join and maintenance protocols insert tuples into forwarding tables and update some existing tuples to correctly construct paths between multi-hop neighbors. Leave, failure, and maintenance protocols construct a new path between two multi-hop neighbors whenever the previous path between them has been broken due to a node leave/failure or a link deletion.

#### A. Join protocol

Consider a new node, say \( w \). It boots up and discovers its physical neighbors. If one of the physical neighbors is a DT node (say \( v \)) then \( w \) sends a join request to \( v \) to join the existing DT.\(^5\) In the MDT join protocol, a node uses the basic

\(^5\)If node \( w \) discovers only physical neighbors, it will not start the join protocol until it hears from a physical neighbor that is attached to the DT, e.g., it receives a token from such a node at system initialization.
search technique of Lee and Lam [16] to find its DT neighbors. First, greedy forwarding of w's join request finds w's closest DT neighbor. Subsequently, w sends a neighbor-set request to every new neighbor it has found; each new neighbor replies with a set of w's neighbors in the new neighbor's local view. If more new neighbors are found in the replies, w sends a neighbor-set request to each of them. This search process is iteratively repeated until w finds no more new neighbor in the replies. The MDT join protocol also constructs a forwarding path between w and every one of its multi-hop DT neighbors. A more detailed protocol description follows.

**Finding the closest node and path construction.** Node w joins by sending a join request to node v with w's own location as the destination location. MDT forwarding is used to forward the join request to a DT node z that is closest to w (success is guaranteed by Theorem 1). A forwarding path between w and z is constructed as follows. When w sends the join request to v, it stores the tuple <−, v, w> in its forwarding table. Subsequently, suppose an intermediate node (say u) receives the join request from a physical neighbor (say v) and forwards it to a physical neighbor (say e), the tuple <w, v, e, e> is stored in F_u.

When node z finally receives the join request of w from a physical neighbor (say d), it stores the tuple <−, v, w> in its forwarding table for the reverse path. The join reply is forwarded along the reverse path from z to w using tuples stored when the join request traveled from w to z earlier. Additionally, each such tuple is updated with z as an endpoint. For example, suppose node x receives a join reply from z to w from its physical neighbor e. Node x changes the existing tuple <e, e, *, w> in F_x to <z, e, *, w>, where * denotes any node already in the tuple.

After node w has received the join reply, it notifies each of its physical neighbors that w is now attached to the DT and they should change their tuple for w from <−, −, −, w> to <−, −, −, w>.

**Physical-link shortcuts.** The join reply message, at any node along the path from z to w (including node z), can be transmitted directly to w if node z is a physical neighbor (i.e., for message m, there is a tuple t in the forwarding table such that t.dest = m.dest). If such a physical-link shortcut is taken, the path previously set up between z and w is changed. Tuples with z and w as endpoints stored by nodes in the abandoned portion of the previous path will be deleted because they will not be refreshed by the endpoints.

A physical-link shortcut can also be taken when other messages in the MDT join, maintenance, leave, and failure protocols are forwarded, but they require the stronger condition: there is a tuple t in the forwarding table such that t.succ = t.dest = m.dest, that is, the shortcut can be taken only if m.dest is a physical neighbor attached to the DT.

**Finding DT neighbors.** Node w, after receiving the join reply from node z, sends a neighbor-set request to z for neighbor information. At this time, C_z, the set of nodes known to z includes both w and z. Node z computes DT(C_z), finds nodes that are neighbors of w in DT(C_z), and sends them to w in a neighbor-set reply message.

When w receives the neighbor-set reply from z, w adds neighbors in the reply (if any) to its candidate set, C_w, and updates its neighbor set, N_w, from computing DT(C_w). If w finds new neighbors in N_w, w sends neighbor-set requests to them for more neighbor information. The joining node w iteratively repeats the above search process until it cannot find any more new neighbor in N_w. At this time w has successfully joined and become a DT node.

Nodes in C_w, the set of nodes known to a node u, are maintained as hard states in distributed DT protocols [16], [17]. In MDT protocols, nodes in C_w are maintained as soft states. More specifically, tuples in F_u are maintained as soft states. By definition, C_u consists of nodes in \( \{ u \} \cup \{ v \mid v = t.dest, t \in F_u \} \) as well as new nodes that may become DT neighbors. A new node in C_u is deleted if it does not become the destination of a tuple in F_u within a timeout period. Furthermore, whenever a tuple t is deleted from F_u upon timeout, each endpoint of t is deleted from C_u unless it is also an endpoint of another tuple.

**Path construction to multi-hop DT neighbors.** The MDT join protocol also constructs a forwarding path between the joining node w and each of its multi-hop neighbors. Whenever w learns a new node y from the join reply or a neighbor-set reply sent by some node, say x, node w sends a neighbor-set request to x, with x as the relay and y as the destination (that is, in neighbor-set request m, m relay = x and m dest = y). Note that a forwarding path has already been established between w and x. Also, since x and y are DT neighbors, a forwarding path exists between x and y (given that w is joining a correct multi-hop DT). As the neighbor-set request is forwarded and relayed from w to y, tuples with w and y as endpoints are stored in forwarding tables of nodes along the path from w to y. The forwarding path that has been set up between w and y is then used by y to return a neighbor-set reply to w.

Note that m.relty serves two different functions in different types of MDT protocol messages [15]. In a data message (also a join request message), m.relty is used to indicate a multi-hop DT neighbor that can route the message out of a local minimum. In a neighbor-set request message sent by a joining node (say u), m.relty is the node that previously informed u of the new node to which the neighbor-set request is destined.

**Example.** Let node a in Figure 2 be a joining node. Suppose a has found b, c, and d to be DT neighbors and it has just learned from b that j is a new neighbor. Node a sends a neighbor-set request to j with b indicated in the message as the relay. Because the existing multi-hop DT (of 9 nodes) is correct, a unique forwarding path exists between node b and node j, which is b − e − h − j. After receiving the message, b forwards it to e on the b − e − h − j path. At b and every node along the way to j, a tuple with endpoints a and j is stored in the node’s forwarding table. When the neighbor-set reply from j travels back via h, node b searches F_h and finds that node a is a physical neighbor attached to the DT (see Figure 2). Node h then transmits j’s reply directly to node a. (This is an example of a physical-link shortcut.) Subsequently, nodes a and j will select and refresh only the path a − h − j between them. Tuples previously stored in nodes b, e, and h for endpoints a and j will be deleted upon timeout. Lastly, from j’s reply, a learns no new neighbor other than b, c, and
d. Without any more new neighbor to query, a’s join protocol execution terminates and it becomes a DT node.

Theorem 2: Let S be a set of nodes and w be a joining node that is a physical neighbor of at least one node in S. Suppose the existing multi-hop DT of S is correct, w joins using the MDT join protocol, and no other node joins, leaves, or fails. Then the MDT join protocol finishes and the updated multi-hop DT of $S \cup \{w\}$ is correct.

A proof of Theorem 2 is presented in the Appendix. A pseudocode specification of the MDT join protocol can be found in our technical report [15].

B. Maintenance protocol

The MDT maintenance protocol for repairing node states is designed for systems with frequent addition and deletion of nodes and physical links. For a distributed DT to be correct, each node must know all of its neighbors in the global DT. Towards this goal, each node (say u) runs the maintenance protocol by first querying a subset of its neighbors, one for each simplex including u in $DT(C_u)$. More specifically, node u selects the smallest subset $V$ of neighbors such that every simplex including $u$ in $DT(C_u)$ includes one node in $V$. Node u then sends a neighbor-set request to each node in $V$. A node $z$ that has received the neighbor-set request adds $u$ to $C_z$ and computes $DT(C_z)$. Node $z$ then sends a neighbor-set reply containing neighbors of $u$ in $DT(C_z)$ to $u$.

Node $u$ adds new nodes found in each neighbor-set reply to $C_u$; it then computes $DT(C_u)$ to get $N_u$. If $u$ finds a new neighbor, say $v$, in $N_u$, node $u$ sends a neighbor-set request to $x$ if $x$ satisfies the following condition:

$\text{C1. The simplex in } DT(C_u) \text{ that includes both } u \text{ and neighbor } x \text{ does not include any node to which } u \text{ has sent a neighbor-set request.}$

Node $u$ keeps sending neighbor-set requests until it cannot find any more new neighbor in $N_u$ that satisfies C1. Node $u$ then sends neighbor-set notifications to neighbors in $N_u$ that have not been sent neighbor-set requests (these notifications announce $u$’s presence and do not require replies). The protocol code for constructing forwarding paths between node $u$ and each new neighbor is the same as in the MDT join protocol.

If after sending a neighbor-set request to a node, say $v$, and a neighbor-set reply is not received from $v$ within a timeout period, node $v$ is deemed to have failed. Node $u$ sends a failure notification about $v$ to inform each node in $u$’s updated neighbor set. These notifications are unnecessary since MDT uses soft states; they are performed to speed up convergence to correct node states.

Each node runs the maintenance protocol independently, controlled by a timeout value $T_m$. After a node has finished running the maintenance protocol, it waits for time $T_m$ before starting the maintenance protocol again. The value of $T_m$ should be set adaptively. When a system has a low churn rate, a large value should be used for $T_m$ to reduce communication cost. We found that if each node runs the maintenance protocol repeatedly, the node states converge to a correct multi-hop DT very quickly. (See results from our system initialization experiments in Section V-C and churn experiments in Section V-F.)

C. Initialization protocols

Serial joins by token passing. Starting from one node, other nodes join serially using the join protocol. The ordering of joins is controlled by the passing of a single token from one node to another.

Concurrent joins by token broadcast. Starting from one node, other nodes join concurrently using the join and maintenance protocols. The ordering of joins is controlled by a token broadcast protocol. Initially, a token is installed in a selected node. When a node has a token, it runs the join protocol once (except the selected node) and then the maintenance protocol repeatedly, controlled by the timeout value $T_m$. It also sends a token to each physical neighbor that is not known to have joined the multi-hop DT. Each token is sent after a random delay uniformly distributed over time interval $[1, \tau]$, where $\tau$ is in seconds. If a node receives more than one token, any duplicate token is discarded.

V. PERFORMANCE EVALUATION

A. Methodology

We evaluate MDT protocols using a packet-level discrete-event simulator in which every protocol message created is routed and processed hop by hop from its source to destination. We will not evaluate metrics that depend on congestion, e.g., end-to-end throughput and latency. Hence, queuing delays at a node are not simulated. Instead, message delivery times from one node to the next are sampled from a uniform distribution over a specified time interval. Time-varying wireless link characteristics and interference problems are modeled by allowing physical links to be added and deleted dynamically.

Creating general connectivity graphs. To create general connectivity graphs for simulation experiments, a physical space in 3D (2D) is first specified. Obstacles are then placed in the physical space. The number, location, shape, and size of the obstacles are constrained by the requirement that the unoccupied physical space is not disconnected by the obstacles. (Any real network environment can be modeled accurately if computational cost is not a limiting factor.) Nodes are then placed randomly in the unoccupied physical space. Let $R$ denote the radio transmission range. Physical links are then placed using the following algorithm: For each pair of nodes, if the distance between them is larger than $R$ or the line between them intersects an obstacle, there is no physical link; else a physical link is placed between the nodes with probability $p$. We refer to $p$ as the connection probability and $1 - p$ as the missing link probability. If a graph created using the above procedure is disconnected, it is not used. Alternatively, to replicate the connectivity graph of a real network, missing links between neighbors can be specified deterministically rather than with probability $1 - p$. 

6The maintenance protocol can use the same iterative search technique used in the join protocol. However, experimental results show that condition C1 can be used to reduce the number of neighbor-set messages sent by the maintenance protocol without any impact on its effectiveness to find DT neighbors.
Inaccurate coordinates. The known coordinates of a node may be highly inaccurate [19] because some localization methods have large location errors. In our experiments, after placing nodes in the physical space, their “known” coordinates are then generated with randomized location errors. The location errors are generated to satisfy a location error ratio, \( e \), which is defined to be the ratio of the average location error to the average distance between nodes that are physical neighbors. We experimented with location error ratios from 0 to 2.

Definitions. The routing stretch value of a pair of nodes, \( s \) and \( d \), in a multi-hop DT of \( S \) is defined to be the ratio of the number of physical links in the MDT route to the number of physical links in the shortest route in the connectivity graph between \( s \) and \( d \). The routing stretch of the multi-hop DT is defined to be the average of the routing stretch values of all source-destination pairs in \( S \). The distance stretch of the multi-hop DT is defined similarly with distance replacing number of physical links as metric.

B. Design of experiments

Our simulation experiments were designed to evaluate geographic routing in the most challenging environments. In general, everything else being equal, the challenge is bigger for a higher dimensional space, larger obstacles, a higher missing link probability, a lower node density, a larger network size, or larger node location errors. Furthermore, we performed experiments to evaluate MDT’s resilience to dynamic topology changes at very high churn rates. In the geographic routing literature, no other protocol has been shown to meet all of these challenges.

Our simulator enables evaluation of geographic routing protocols in the most challenging environments. In the simulator, any connectivity graph can be created to represent any real network environment with obstacles of different shapes and sizes. The connectivity graphs created as described above have properties of real wireless networks, unlike unit-disk and unit-ball graphs used in prior work on geographic routing.\(^7\) We experimented with obstacles of different shapes and sizes, and nodes with large location errors or arbitrary coordinates in 2D, 3D, and 4D. In this paper, we present experimental results for large obstacles, such as those shown in Figure 3(a), because large obstacles are more challenging to geographic routing than small ones; these very large obstacles may represent tall buildings in an outdoor space or large machinery in a factory. Between neighbors that are in line of sight and within radio transmission range, we experimented with a missing link probability as high as 0.5.

Node density (average node degree) is an important parameter that impacts geographic routing performance. We present experiments for node densities of 13.5 for 3D and 9.7 for 2D. These node densities are relatively low compared with node densities used in prior work on geographic routing. We found that node densities lower than 13.5 for 3D and 9.7 for 2D would result in many disconnected graphs for spaces with large obstacles and a high missing link probability. We also conducted experiments for higher node densities which resulted in better MDT performance, thus allowing us to conclude that MDT works well for a wide range of node densities. When we scale up the network size in a set of experiments, we increase the space and obstacle sizes to keep node density approximately the same. For experiments with different missing link probabilities, we vary the radio transmission range to keep node density approximately the same.

C. System initialization experiments

We have performed numerous experiments using our initialization protocols. In every experiment, a correct multi-hop DT is constructed. Concurrent joins can do so much faster than serial joins but with a higher message cost (see Figure 10 for message cost comparison).

Figures 3(b)-(c) show results from two sets of experiments using concurrent-join initialization. In each experiment, the physical space is a 1000 × 1000 × 1000 3D space, with three large obstacles, placed as shown in Figure 3(a). The size of one obstacle is 200 × 300 × 1000. Each of the other two is 200 × 350 × 1000 in size. The obstacles occupy 20% of the physical space. Connectivity graphs are then created for 300 nodes using the procedure described in Section V-A for radio transmission range \( R = 305 \) and link connection probability \( p = 0.5 \). The average node degree, i.e., number of physical neighbors per node, is 13.5. The (known) coordinates of the nodes are inaccurate with location error ratio \( e = 1 \).

The first set of experiments is for low-speed networks with one-hop message delays sampled from 100 ms to 200 ms (average \( = 150 \) ms) and a maintenance protocol timeout duration of 60 seconds. The second set of experiments is for

\(^7\)In a recent paper on 3D routing, unit-ball graphs were still used for simulation experiments [31].
faster networks with one-hop message delays sampled from 10 ms to 20 ms (average = 15 ms) and a maintenance protocol timeout duration of 10 seconds.

In the legend of Figures 3(b)-(c), “max. token delay” is maximum token delay $\tau$. In each experiment, note that accuracy of the multi-hop DT is low initially when many nodes are joining at the same time. With a smaller $\tau$, more nodes initiate their join process earlier at about the same time, resulting in a lower MDT accuracy at the beginning. However, accuracy improves and converges to 100% quickly for all $\tau$ values. In every experiment, after each node’s initial join, the node had run the maintenance protocol only once or twice by the time 100% accuracy was achieved.

D. MDT performance in 3D

We evaluated the performance of MDT routing for 100 to 1300 nodes in 3D. We present results from four different sets of experiments using connectivity graphs created in a 3D space with and without obstacles, for node locations specified by accurate and inaccurate coordinates. There are four cases:

- accurate coordinates ($e = 0$), few missing links ($p = 0.9$), no obstacle
- inaccurate coordinates ($e = 1$), few missing links ($p = 0.9$), no obstacle
- accurate coordinates ($e = 0$), many missing links ($p = 0.5$), large obstacles (obs)
- inaccurate coordinates ($e = 1$), many missing links ($p = 0.5$), large obstacles (obs)

For 300 nodes, dimensions of the physical space and obstacles are the same as in Figure 3(a). For a smaller (or larger) number of nodes, dimensions of the physical space and obstacles are scaled down (or up) proportionally. For each obs experiment, the three obstacles are randomly placed in the horizontal plane. $R = 305$ is used for $p = 0.5$ and $R = 250$ is used for $p = 0.9$ such that the average node degree is approximately 13.5. At the beginning of each experiment, a correct multi-hop DT was first constructed. Routing success rate was 100% in every experiment and is not plotted.

Figures 4(a)-(b) show that both routing stretch and distance stretch versus network size are close to 1 for the easy case of accurate coordinates ($e = 0$), few missing links ($p = 0.9$), and no obstacle. Either inaccurate coordinates ($e = 1$) or many missing links ($p = 0.5$) and large obstacles (obs) increase both the routing stretch and distance stretch of MDT routing. Note that both the routing and distance stretch of MDT remain low as network size becomes large.\(^8\)

Storage cost. The most important routing information stored in a node is the set of nodes it uses for forwarding; the known coordinates of each node in the set are stored in a location table. We use 4 bytes per dimension for storing each node’s coordinates (e.g., 12 bytes for a node in 3D); this design choice is intended for very large networks. The coordinates of a node are used as its global identifier. Each node is also represented by a 1-byte local identifier in our current implementation. The location table stores pairs of global and local identifiers (e.g., 13 bytes per node for nodes in 3D). In the forwarding table, local identifiers are used to represent nodes in tuples. To illustrate MDT’s storage cost in bytes, consider the case of 1300 nodes, $e = 1$, and $p = 0.5$ with obstacles. The average location table size is 540.2 bytes. The average forwarding table size is 88.8 bytes. The average location table size is 86% of the combined storage cost. We found that this percentage is unchanged for all network sizes (100 - 1300) in each set of experiments, indicating that the forwarding table size is also proportional to the number of distinct nodes stored.

In this paper, the storage cost is measured by the average number of distinct nodes a node needs to know (and store) to perform forwarding. This represents the storage cost of a node’s minimum required knowledge of other nodes. This metric, unlike counting bytes, requires no implementation assumptions which may cause bias when different routing protocols are compared. Figure 4(c) shows the storage cost per node versus network size. As expected, either inaccurate coordinates ($e = 1$) or many missing links ($p = 0.5$) and large obstacles require more storage per node due to the need for more multi-hop DT neighbors. For comparison, the bottom curve is the average number of physical neighbors per node.

Varying obstacle locations. Each data point plotted in Figures 4(a)-(c) is the average value of 50 simulation runs for 50 different connectivity graphs each of which was created from a different placement of the obstacles. Also shown as bars are the 10th and 90th percentile values. Observe that the intervals between 10th and 90th percentile values are small for all data points. (These intervals are also small in experimental results to be presented in Figures 5 and 8-11.

\(^8\)Distance stretch is almost the same as routing stretch (except in 4D experiments for which physical distance is not meaningful) and will not be shown again.
and will be omitted from those figures for clarity.) The small intervals between 10th and 90th percentile values demonstrate that varying obstacle locations has negligible impact on MDT routing performance.

**Varying number and size of obstacles.** Aside from varying the locations of obstacles, we also experimented with varying the number and size of obstacles. In particular, we repeated the experiments in Figure 4 for 6 obstacles and also for 9 obstacles. In each such experiment, the fraction of physical space occupied by obstacles was kept at 20%. We found the resulting changes in MDT’s routing stretch, distance stretch, and storage cost to be too small to be visible when plotted in Figures 4.9 However, when we increased the fraction of physical space occupied by obstacles from 20% to 30%, the resulting increases in MDT’s routing and distance stretch were significant (about 6%).

### E. MDT performance in 4D

To illustrate how MDT can be used in 4D, consider the connectivity graphs created for the set of experiments in Figure 4 with many missing links ($p = 0.5$) and large obstacles. Suppose the nodes have no location information. We experimented with two cases: (i) Each node assigns itself an arbitrary location in a 4D space and sends its (arbitrary) coordinates to its physical neighbors. These coordinates are used by MDT protocols to construct and maintain a multi-hop DT as well as for routing. (ii) After a multi-hop DT has been constructed by the nodes using the initial (arbitrary) coordinates, each node then runs VPoD [23] which is a virtual positioning protocol that does not require any node location information, any special nodes (such as, landmarks), nor any use of flooding. Nodes use VPoD to change their coordinates by comparing distances with routing costs to their physical and DT neighbors. A new multi-hop DT is then constructed using the new coordinates. After several iterations, the node coordinates will converge to achieve the following property [23]: The distance between any two nodes in the virtual space is a good estimate of the routing cost (in any additive metric) between them.

For the results presented in Figure 5, we used 1 (hop) as the routing metric between two physical neighbors. Each data point plotted in Figure 5 is the average value from 50 experiments.

9Performance measures from experiments for 9 obstacles are smaller than those from experiments for 3 obstacles by less than 0.5%.

For comparison, we have also plotted the results for MDT routing using inaccurate 3D coordinates, that is, the case of ($e = 1$, $p = 0.5$, obs) in Figure 4. Figure 5(a) on routing stretch, plotted in logarithmic scale, shows that MDT routing using 4D virtual coordinates is better than using inaccurate coordinates in 3D. Figure 5(b) on storage cost shows that MDT routing using inaccurate coordinates in 3D is better than using 4D virtual coordinates. In both figures, MDT routing using arbitrary coordinates has the worst performance. Routing success rate was 100% in every experiment and is not shown.

### F. Resilience to churn

We performed a large number of experiments to evaluate the performance of MDT protocols for systems under churn, with 300 nodes in a $1000 \times 1000 \times 1000$ 3D physical space. Like the experiments used to evaluate MDT routing stretch in Figure 4, four sets of experiments were performed using connectivity graphs created with and without three large obstacles, for node locations specified by accurate and inaccurate coordinates. The average node degree is kept at approximately 13.5 for every experiment.

In a node churn experiment, the rate at which new nodes join is equal to the churn rate; the rate of nodes leaving and the rate of nodes failing are each equal to half the churn rate. In a link churn experiment, the churn rate is equal to the rate at which new physical links are added and the rate at which existing physical links are deleted. In each experiment, the 300 nodes initially maintain a correct multi-hop DT. Churn begins at time=0 and ends at time=60 seconds.

Figure 6 presents results from node churn experiments for low-speed networks where one-hop message delays are sampled from $[100 \text{ ms}, 200 \text{ ms}]$. The maintenance timeout value is 60 seconds. The churn rate is 100 nodes/minute in Figures 6(a)-(b) and varies in Figure 6(c). Figure 6(a) shows the accuracy of the multi-hop DT versus time. The accuracy returns to 100% quickly after churn. Figure 6(b) shows the routing success rate versus time. The success rate is close to 100% during churn and returns to 100% quickly after churn. Figure 6(c) shows the communication cost (per node per second) versus churn rate.

By Little’s Law, for 300 nodes and a churn rate of 100 nodes per minute, the average lifetime of a node is $300/100 = 3$ minutes, which represents a very high churn rate for most practical systems.

Figure 7 presents results from link churn experiments for low-speed networks with a maintenance timeout value of 60
communication costs to construct a multi-hop DT initially large. There are two types of extra communication costs: (i) storage cost converges to an asymptotic constant as \(N\) physical neighbors; see Figure 4(c). Observe that the extra difference between the MDT storage cost and the number of routing protocol. The extra storage cost of MDT is the and communication costs when compared to a simple greedy and few missing links (\(p = 0.9\)). The radio transmission range is \(R = 150\). The average node degree is 16.5. The performance results are plotted versus location error ratio, from \(e = 0\) (no error) to \(e = 2\) (very large location errors).

VI. PERFORMANCE COMPARISON

A. Comparison of 2D protocols

The geographic routing protocols, GPSR running on GG, RNG, and CLDP graphs [13], [14], and GDSTR [18] were designed for routing in 2D. We implemented these protocols in our simulator. The experiments in Figure 8 were carried out for 300 nodes in a 1000 \(\times\) 1000 2D space with no obstacle and few missing links (\(p = 0.9\)). The radio transmission range is \(R = 150\). The average node degree is 16.5. The performance results are plotted versus location error ratio, from \(e = 0\) (no error) to \(e = 2\) (very large location errors).

The experiments of Figure 9 were carried out for 300 nodes in a 1000 × 1000 2D space with three randomly placed obstacles (a 200 × 300 rectangle and two 200 × 350 rectangles) and many missing links ($p = 0.5$). The radio transmission range is $R = 150$. The average node degree is 9.7. The performance results are plotted versus location error ratio, from $e = 0$ to $e = 1$.

In Figure 8(a) and Figure 9(a) the routing success rates of MDT and GDSTR are both 100% for all $e$ values (it was 100% in every experiment). As the location error ratio ($e$) increases from 0, the routing success rates of RNG, GG, and CLDP drop off gradually from 100%. For $e > 0.6$ in Figure 8(a) and $e > 0.3$ in Figure 9(a), their routing success rates drop significantly.

Figure 8(b) and Figure 9(b), in logarithmic scale, show that MDT has the lowest routing stretch for all $e$ values, with GDSTR a close second, followed by CLDP, GG, and RNG in that order. Note that routing stretch increases as $e$ increases for all protocols.

Figure 8(c) and Figure 9(c) show storage costs comparisons. The GPSR protocols (CLDP, GG, and RNG) have the lowest storage cost, with the storage costs of GDSTR and MDT about the same.

**Comparison of graph construction costs.** We compare MDT’s message cost to construct a correct multi-hop DT with message costs of CLDP graph construction using serial probes [14] and GDSTR hull tree construction [18]. The physical space is a 2D square with three large rectangular obstacles, occupying 20% of the physical space. There are many missing links ($p = 0.5$). Nodes have inaccurate coordinates ($e = 1$). The number $N$ of nodes is varied from 100 to 1300. For the radio transmission range $R = 150$, the sizes of the physical space and obstacles are determined for each value of $N$ such that the average node degree is approximately 12.

In Figure 10, the vertical axis is in logarithmic scale. The message cost of a protocol is the average number of messages sent per node (we did not account for message size differences among the protocols). Note that each GDSTR message is a broadcast message sent by a node to all of its physical neighbors and is counted only as one message sent. Messages sent by CLDP and MDT are unicast messages.

Figure 10 shows that with the average number of messages sent per node as metric, GDSTR has the best message cost performance for up to 900 nodes. For more than 900 nodes MDT (serial joins) has the lowest cost. CLDP has a very high cost. Note that the CLDP and GDSTR curves increase gradually with $N$. The MDT curves are flat.

**B. Comparison of 3D protocols**

We compare the routing performance of MDT with GRG [8] and GDSTR-3D [31]. We implemented the basic version of GRG in our simulator. Several techniques to improve the performance of GRG are presented for unit ball graphs [8]. Since arbitrary connectivity graphs are used in our experiments, these techniques are not applicable and not implemented.

GDSTR-3D uses two hull trees for recovery. For each tree, each node stores two 2D convex hulls to aggregate the locations of all descendants in the subtree rooted at the node; the two 2D convex hulls approximate a 3D convex hull at each node. We implemented GDSTR-3D using its authors’ TinyOS 2.x source code available at Google Sites.

Unlike other geographic protocols, each node in GDSTR-3D stores 2-hop neighbors and uses 2-hop greedy forwarding to reduce routing stretch at the expense of a much larger storage cost per node. This performance tradeoff may not be appropriate for networks with limited nodal storage.

A non-geographic routing protocol, VRR [5], is included in the comparison. We implemented VRR for static networks without joins and failures.11 For each pair of virtual neighbors, we used the shortest path (in hops) between them as the forwarding path (the routing stretch value is 1 between virtual neighbors). Thus, the routing stretch and storage cost results shown in Figure 11(b)-(c) for VRR are slightly optimistic. In VRR, each node also stores 2-hop neighbors for forwarding.

MDT can be easily modified to use 2-hop greedy forwarding. We present results for both MDT (which uses 1-hop

greedy forwarding) and MDTv2 (which uses 2-hop greedy forwarding).

In our experiments, the number $N$ of nodes is varied from 100 to 1300. The physical space and large obstacles are the same as the ones used in Figure 4. The average node degree was kept at approximately 13.5. Experiments were performed using connectivity graphs created for the following case: inaccurate coordinates ($e = 1$), many missing links ($p = 0.5$), and three large obstacles that occupy 20% of the physical space.

Figure 11(a) shows that MDT (also MDTv2), GDSTR-3D, and VRR all achieve 100% routing success rate while the routing success rate of GRG is about 86%. Figure 11(b), in logarithmic scale, shows that the routing stretch of GRG is very high, the routing stretch of VRR is high for $N > 300$, and both increase with $N$. The routing stretch of MDTv2 is the lowest and slightly lower than that of GDSTR-3D for every network size (the differences are, however, too small to be seen in Figure 11(b)). MDT, which uses 1-hop greedy forwarding, ranks a close third.

In Figure 11(c), GDSTR-3D, VRR, MDTv2 have large per-node storage costs, because each node stores 2-hop neighbors as well as physical neighbors. The storage cost of MDTv2 is smaller than those of GDSTR-3D and VRR. Both GRG and MDT have much lower storage costs because they use 1-hop greedy forwarding. The per-node storage cost of GRG, equal to the average number of physical neighbors, is the lowest of the five protocols.

**MDT versus GDSTR-3D.** MDT, MDTv2, and GDSTR-3D all provide guaranteed delivery in 3D and achieve routing stretch close to 1. GDSTR-3D has a higher storage cost than MDTv2 and a much higher storage cost than MDT. One clear advantage MDT (or MDTv2) has over GDSTR-3D is that MDT is highly resilient to dynamic topology changes (both node churn and link churn) while GDSTR-3D is designed for a static topology without provision to handle any dynamic topology change. Another advantage of MDT is that it provides guaranteed delivery for nodes with arbitrary coordinates in higher dimensions ($d > 3$).

**VII. Conclusions**

MDT is the only geographic routing protocol that provides guaranteed delivery in 2D, 3D, and higher dimensions. The graph of nodes and physical links is required to be connected, but may otherwise be arbitrary. MDT’s guaranteed delivery property holds for nodes with accurate, inaccurate, or arbitrary coordinates.

Experimental results show that MDT constructs a correct multi-hop DT very quickly at system initialization. MDT is also highly resilient to both node churn and link churn. Furthermore, MDT achieves a routing stretch (also distance stretch) close to 1.

The performance of MDT scales well to a large network size ($N$). We observed that, as $N$ becomes large, MDT’s routing (distance) stretch and per-node storage cost converge to horizontal asymptotes. MDT does not use special nodes (such as, beacons and landmarks) that are required in many wireless routing protocols; every MDT node runs the same protocols. Each node computes its own local DT with computation cost dependent upon its storage cost, rather than $N$. Lastly, MDT’s per-node communication costs for constructing and maintaining a correct multi-hop DT are fairly low and independent of $N$.

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Given a location $\ell$ in the space, the MDT forwarding protocol succeeds to find a node in $S$ closest to $\ell$ in a finite number of hops.

Proof:

1) By definition, a correct multi-hop DT of $S$ is a correct distributed DT of $S$. The distributed DT maintained by nodes in $S$ is the same as $DT(S)$.

2) Given a correct multi-hop DT, each DT neighbor of a node $u$ in $S$ is either a physical neighbor or connected to $u$ by a forwarding path of finite length (in hops) that exists in $\{P(v) : v \in S\}$.

3) When a message, say $m$, arrives at a node, say $u$, if the condition in line 1, 2, or 6 in Table II is true, then a node closest to $\ell$ is found. If the conditions in lines 1-3 are all false, node $u$ performs greedy forwarding in lines 4-5. If it succeeds to find in $P_v$ a physical neighbor $v$ that is closer to $\ell$ than node $u$, message $m$ is transmitted directly to $v$ (lines 4 in Table II); else, greedy forwarding is performed over the set of DT neighbors (line 5 in Table II). The proof of Theorem 1 in [16] for a distributed DT guarantees that either node $u$ is closest to $\ell$ or there exists in $N_u$ a node $v$ that is closer to $\ell$ than $u$. Therefore, if node $u$ is not a closest node to $\ell$, executing the greedy forwarding code (lines 4-5 in Table II) finds a node $v$ that is closer to $\ell$ than node $u$.

4) Any other node in $S$ that is closer to $\ell$ than $u$ will not use the actions in lines 4-5 in Table II to send message $m$ back to node $u$. It is, however, possible for message $m$ to visit node $u$ again in the forwarding path between two DT neighbors that are closer to $\ell$ than $u$. In this case, the condition of line 3 in Table II must be true for $m$ at node $u$. Thus, node $u$ executes the greedy forwarding code (lines 4-5 in Table II) for message $m$ at most once. This property holds for every node. By 2), 3), and the assumption that $S$ has a finite number of nodes, MDT forwarding finds a closest node in $S$ to $\ell$ in a finite number of hops.

Theorem 2. Let $S$ be a set of nodes and $w$ be a joining node that is a physical neighbor of at least one node in $S$. Suppose the existing multi-hop DT of $S$ is correct, $w$ joins using the MDT join protocol, and no other node joins, leaves, or fails. Then the MDT join protocol finishes and the updated multi-hop DT of $S \cup \{w\}$ is correct.

Proof: By Theorem 1, the join request of $w$ succeeds to find a DT node (say $z$) closest to $w$, which sends back a joint reply. By a property of DT, node $z$, being closest to $w$, is guaranteed to be a neighbor of $w$ in $DT(S \cup \{w\})$. A forwarding path is constructed between $w$ and $z$. Subsequently, because the multi-hop DT of $S$ is correct, forwarding paths are constructed between $w$ and each neighbor it sends a neighbor-set request. After receiving a request from $w$, each neighbor of $w$ updates its own neighbor set to include $w$. They also send back replies to $w$. By Lemma 9 in [16], the join process finishes and $N_w$ consists of all neighbor nodes of $w$ in $DT(S \cup \{w\})$.

By construction, two DT neighbors select only one path to use between them by refreshing only tuples stored in nodes along the selected path. Therefore, the path between each pair of neighbors in $DT(S \cup \{w\})$ is unique after the join. Each path also has a finite number of hops because (i) the path from the joining node to its closest DT node ($z$) has a finite number of hops (by Theorem 1), and (ii) the path from the joining node to each of its other DT neighbors is either a one-hop path or the concatenation of two paths, each of which has a finite number of hops. By Definition 3, the updated multi-hop
DT is correct.

In the above proof, we make use of Lemma 9 in [16] for a distributed DT in which every node can directly communicate with every other node. Let $S’$ denote $S \cup \{w\}$. The main ideas used in the proof of this lemma are the following: i) The existing node $z \in S$ closest to the joining node $w$ is a neighbor of $w$ in $DT(S’)$. ii) For any two neighbors of $w$ in $DT(S’)$, say $u$ and $v$, if the facet shared by the Voronoi cells of $u$ and $w$ is adjacent to the facet shared by the Voronoi cells of $v$ and $w$ in $DT(S’)$, then $u$ and $v$ are neighbors in $DT(S)$.

Therefore, having found at least one neighbor in $DT(S’)$, $w$ can find any other neighbor in $DT(S’)$ by following a sequence of existing edges in $DT(S)$. A detailed proof is presented in [16].

We next describe the MDT leave and failure protocols designed for handling a single leave and failure, respectively. Concurrent join, leave, and failure occurrences in different parts of a large network are often independent of each other. In this case, after a leave or failure, node states can be quickly and effectively repaired by these leave and failure protocols without waiting for the maintenance protocol to run upon timeout.

**Leave protocol.** Consider a node $u$ that leaves gracefully. When node $u$’s neighbors update their states, it is not sufficient for a neighbor $v$ to delete $u$ from $C_v$ and $N_v$. This is because $v$ may have a new neighbor $z$ that was not a neighbor of $v$ before $u$’s departure and $v$ does not know $z$ after $u$’s departure. However, such a node $z$ is always a neighbor of $u$ prior to $u$’s departure (Lemma 10 in [16]). Therefore node $u$ can notify neighbor $v$ that $u$ is leaving and provide $v$ with the following information:

1. $v$’s neighbor set $N^u_v$ in $DT(N_u)$.
2. A graph $G=<V,E>$, where the set of vertices $V=N_0$, and the set of edges, $E = \{(x,y) \mid x, y \text{ are neighbors in } DT(N_u) \text{ and } F_u \text{ does not contain a tuple with } x \text{ and } y \text{ as endpoints}\}$.

We use $vertex$ to refer to a node in graph $G$ and $route$ to refer to a path in graph $G$ connecting two vertices. Note that all vertices in $G$ are DT nodes. Edges in $G$ connect neighbors in the multi-hop DT of $S$. By the definition of $G$, none of these edges uses $u$ as a node in its forwarding path.

After receiving a leave notification, $v$ computes a route in $G$ to every node $z$ in its updated neighbor set. Suppose such a route exists in $G$ between $v$ and $z$. Node $v$ sends to $z$ a path-recover message along the route as follows: The path-recover message is relayed by vertices along the route. Two adjacent vertices in the route, being neighbors in the multi-hop DT of $S$, are connected by a physical link or a forwarding path. At every hop along the route from $v$ to $z$, a tuple with $v$ and $z$ as endpoints is stored, thus establishing a forwarding path between $v$ and $z$. The leave protocol is highly efficient for repairing node states after a leave.

For some rare cases, the leave protocol may not be able to repair all node states after a leave for two reasons. First, the leaving node $u$ may be an articulation point of the connectivity graph. Second, even if $u$ is not an articulation point, it is possible that $v$ and some neighbor $z$ are disconnected in $G$ because the forwarding paths of all routes between them in the $DT(N_u)$ graph use node $u$ to forward messages. In this case, node $v$ exits the leave protocol and immediately runs the maintenance protocol to repair node states.

A pseudocode specification of the MDT leave protocol is presented in our technical report [15].

**Failure protocol.** The failure protocol is similar to the leave protocol and almost as efficient. The key idea is that every node $u$ prepares recovery information for its neighbors in case $u$ fails. The recovery information includes, for each neighbor $v$, its neighbor set $N^u_v$ in $DT(N_u)$ after $u$’s departure as well as the graph $G$ in the leave protocol. Node $u$ selects one of its neighbors (say $m$) as its monitor node and sends to $m$ the recovery information. (The recovery information is updated by $u$ whenever there is a change in $N_u$.) The monitor node $m$ periodically probes $u$ to check that $u$ is alive. When $m$ detects failure of $u$, $m$ sends to each of $u$’s former neighbors its recovery information prepared by $u$.

Simon S. Lam (F’85) received the B.S.E.E. degree with Distinction from Washington State University, Pullman, in 1969, and the M.S. and Ph.D. degrees in engineering from the University of California, Los Angeles, in 1970 and 1974, respectively. From 1971 to 1974, he was a Postgraduate Research Engineer with the ARPA Network Measurement Center at UCLA, where he worked on satellite and radio packet switching networks. From 1974 to 1977, he was a Research Staff Member with the IBM T. J. Watson Research Center, Yorktown Heights, NY. Since 1977, he has been on the faculty of the University of Texas at Austin, where he is Professor and Regents Chair in computer science, and served as Department Chair from 1992 to 1994. He served as Editor-in-Chief of IEEE/ACM Transactions on Networking from 1995 to 1999. He served on the editorial boards of IEEE/ACM Transactions on Networking, IEEE Transactions on Software Engineering, IEEE Transactions on Communications, Proceedings of the IEEE, Computer Networks, and Performance Evaluation. He co-founded the ACM SIGCOMM conference in 1983 and the IEEE International Conference on Network Protocols in 1993.

Professor Lam is a Member of the National Academy of Engineering and a Fellow of ACM. He received the 2004 ACM SIGCOMM Award for lifetime contribution to the field of communication networks, the 2004 ACM Software System Award for inventing secure sockets and prototyping the first secure sockets layer (named Secure Network Programming), the 2004 W. Wallace McDowell Award from the IEEE Computer Society, as well as the 1975 Leon G. Abraham Prize and the 2001 William R. Bennett Prize from the IEEE Communications Society.

Chen Qian (M’08) is a Ph.D. candidate at Department of Computer Science, the University of Texas at Austin, where he is the recipient of the James C. Browne Outstanding Graduate Fellowship. He received the B.Sc. degree from Nanjing University in 2006, and the M.Phil. degree from the Hong Kong University of Science and Technology in 2008. His research interests include computer networking, data-center networks and cloud computing, and scalable routing and multicast protocols. He has published research papers in a number of conferences and journals including ACM SIGMETRICS, IEEE ICNP, IEEE ICDCS, IEEE PerCom, IEEE/ACM Transactions on Networking, and IEEE Transactions on Parallel and Distributed Systems. He is a student member of IEEE and ACM.