Analysis of Binomial Congestion Control *

Y. Richard Yang, Simon S. Lam
Department of Computer Sciences
The University of Texas at Austin
Austin, TX 78712-1188
{*yangyang, lam*}@*cs.utexas.edu*

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1 Introduction

Binomial congestion control was proposed by Bansal and Balakrishnan in [2]. However, the sending rate derivation in [2] is greatly simplified and does not consider the effect of timeouts. Further, even though the authors use $\alpha = 1$ and $\beta = 0.6$ for TCP-friendliness in their experiments; this selection is not justified by their analysis. On the contrary, according to the authors, for $\alpha = 1$, they should select β such that $\frac{\alpha}{\beta} = \frac{1}{0.5}$, therefore, $\beta = 0.5$.

The motivation of this paper is to analyze the sending rate of binomial congestion window adjustment policy, considering both tripli-duplicate loss indications and timeout loss indications. We also consider the selection of α and β for IIAD and SQRT congestion control strategies [2] to be TCP-friendly. This paper suggests that the authors of Binomial should test their protocol under higher loss scenarios.

The balance of this paper is as follows. In Section 2, we describe the Binomial congestion control and state the analysis assumptions. The detail of the derivations is put in the Appendix. In Section 3, we use the sending rate formula to derive conditions under which a Binomial flow is TCP-friendly.

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2 Model and Analysis Assumptions

Formally, the Binomial window adjustment policy is

$$\begin{cases} w_{t+R} & \leftarrow & w_t + \alpha/w_t^k & \text{if no loss} \\ w_{t+\delta t} & \leftarrow & w_t - \beta w_t^l & \text{when loss} \end{cases} \tag{1}$$

We can see that TCP is a special case when $k = 0$, $l = 1$. In this paper, we analysis the two cases considered by the authors: when $k = 1$, $l = 0$, which is called IIAD (inverse-increase/additive decrease) and $k = l = 0.5$, which is called SQRT (square-root).

Window adjustment policy, however, is only one component of a complete congestion control protocol. Other mechanisms such as congestion detection and round-trip time estimation are needed to make a complete protocol. Since TCP congestion control has been studied extensively for many years, Binomial adopts these other mechanisms from TCP Reno [5, 6, 8, 1]. In the next subsection, we give a brief description of the Binomial congestion window adjustment algorithm. All other algorithms are the same as those of TCP Reno.

2.1 Congestion window adjustment

A Binomial session begins in the *slowstart* state. In this state, the congestion window size is doubled for every window of packets acknowledged. Upon the first congestion indication, the congestion window size is cut in half and the session enters the *congestion avoidance* state. In this state, the congestion window size is increased by α/W^k in each round-trip time, where W is the current congestion window size. Notice that in this analysis we assume that the receiver returns one new ACK for each received data packet. It is straightforward to extend the analysis to consider delayed ACK. Binomial reduces the window size when congestion is detected. Same as TCP Reno, Binomial detects congestion by two events: *triple-duplicate ACK* and *timeout*. If congestion is detected by a triple-duplicate ACK, Binomial changes the window size to $W - \beta W^l$. If the congestion indication is a timeout, the window size is set to 1.

2.2 Modeling assumptions

The assumptions and simplifications made in this analysis are summarized below.

- We assume that the sender always has data to send (i.e., a saturated sender). The receiver always advertises a large enough receiver window size such that the send window size is determined by the Binomial congestion window size.

- The sending rate is a random process. We have limited our efforts to modeling the mean value of the sending rate. An interesting future topic will be to study the variance of the sending rate which is beyond the scope of this paper.
- We focus on Binomial's congestion avoidance mechanisms. The impact of slowstart has been ignored.
- We model Binomial's congestion avoidance behavior in terms of rounds. A round starts with the back-to-back transmission of W packets, where W is the current window size. Once all packets falling within the congestion window have been sent in this back-to-back manner, no more packet is sent until the first ACK is received for one of the W packets. This ACK reception marks the end of the current round and the beginning of the next round. In this model, the duration of a round is equal to the round-trip time and is assumed to be independent of the window size. Also, it is assumed that the time needed to send all of the packets in a window is smaller than the round-trip time.
- We assume that losses in different rounds are independent. When a packet in a round is lost, however, we assume all packets following it in the same round are also lost. Therefore, p is defined to be the probability that a packet is lost, given that it is either the first packet in its round or the preceding packet in its round is not lost [7].
- To void having too many parameters, we assume that the receiver returns one new ACK for each received data packet, i.e., no delayed ACK. To model the effect of delayed ACK, we can simply replace all α with α/b, where α is the increasing parameter, and b is the number of data packets before an ACK is sent.
- To derive an analytic result, sometimes in the analysis we assume $E[W^t] \approx E[W]^t$, where W is the window size and $t \in (0, \infty)$.

3 TCP-friendly Binomial Congestion Control

As derived in Appendix, the sending rate of both IIAD and SQRT can be expressed as

$$T_{Binomial}(\alpha, \beta, p, R, T_0) \approx \frac{1}{R\sqrt{\frac{\beta}{\alpha}p} + T_0 \min\left(1, 3\sqrt{\frac{\beta}{\alpha}p}\right)p(1+32p^2)} \qquad (2)$$

where p is the loss rate, R the mean round-trip time, and T_0 the timeout. We should emphasize that to derive (2), in some cases we have assumed p is small. For detail, refer to the Appendix.

To be TCP-friendly, we need to match $T_{Binomial}(\alpha, \beta, p, R, T_0)$ to that of TCP sending rate formula, which is

$$T_{TCP}(p, R, T_0) \approx \frac{1}{R\sqrt{\frac{2}{3}p} + T_0 \min\left(1, 3\sqrt{\frac{3}{8}p}\right)p(1+32p^2)} \tag{3}$$

Under low loss scenario, the first terms in the denominators of (2) and (3) dominates, and we have the expression:

$$\frac{\beta}{\alpha} = \frac{2}{3} \tag{4}$$

For example, when the Binomial congestion control uses $\alpha = 1$, we select $\beta = 0.66$ so that the control is TCP-friendly.

To consider the sensitivity of the TCP-friendliness on the β parameters, we define

$$F(\alpha, \beta) = \frac{\frac{1}{R\sqrt{\frac{2}{3}p}}}{\frac{1}{R\sqrt{\frac{\beta}{\alpha}p}}} \tag{5}$$

$$= \sqrt{\frac{3\beta}{2\alpha}} \tag{6}$$

Under small loss rate p, F is the relative throughput of a IIAD/SQRT flow and a TCP flow. Figure 1 plots F as a function of β when $\alpha = 1$. Compare Figure 1 with the experimental results in Figure 16 of [2], we find that the two figures are very similar. This can be considered a validation of (2).

However, it is important to point out that F is valid only when loss rate p is small. When loss rate is high, we should use the complete sending rate formula to derive the TCP-friendly α and β, using the methods as in [**?**]. It also suggests that the authors of Binomial should evaluate Binomial under high loss scenarios.

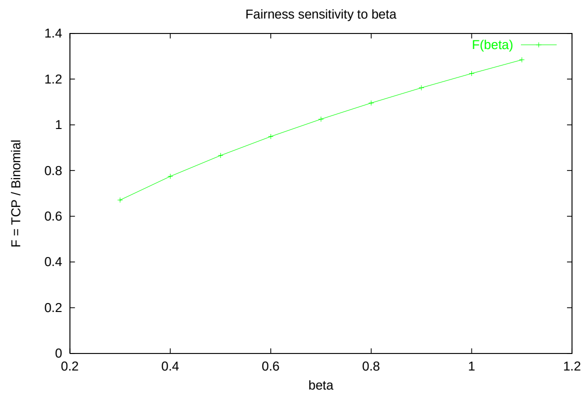

Figure 1: F as a function of β when $\alpha = 1$

A Sending Rate Derivation

We carry the derivation in two steps. In the first step, we only consider the case when congestion indications are exclusively of type "triple duplicate" ACK (TD). In the next step, we consider both TD and timeout loss indications.

A.1 Congestion indications are exclusively triple-duplicate ACKs

We first consider the case when congestion indications are exclusively of type "triple duplicate" ACK (TD). Consider a Binomial flow starting at time $t = 0$. For any given time $t > 0$, define N_t as the number of packets transmitted in the interval $[0, t]$, and $T_t = N_t/t$, the sending rate on that interval. Note that T_t is the number of packets sent per unit of time regardless of their eventual fate (i.e. whether they are received or not). Thus, T_t represents the sending rate of the connection. We define the long-term steady-state rate T to be

$$T = \lim_{t \to \infty} T_t = \lim_{t \to \infty} \frac{N_t}{t} \tag{7}$$

Define a TD period (TDP) to be the interval of time between two TD congestion indications. For the ith TD period we define random variable Y_i as the number of packets send in the period, A_i the duration of the period, and W_i the window size at the end of the period. Consider $\{W_i\}$ to be a Markov regenerative process with rewards $\{Y_i\}$. From renewal theory [3, 4], we know that

$$T = \frac{E[Y]}{E[A]} \tag{8}$$

In order to derive an expression for T, the long-term steady-state Binomial sending rate, we next derive expressions for the means of Y and A.

Consider a TD period as in Figure 2.

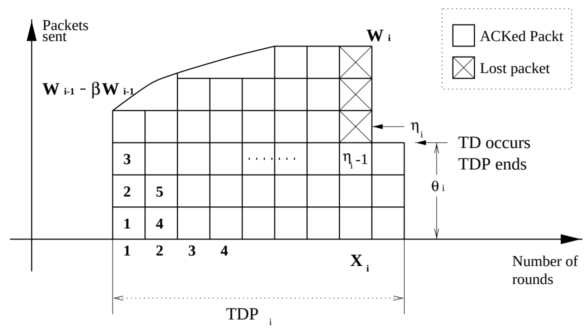

Figure 2: A triple-duplicate period (TDP)

A TD period starts immediately after a TD congestion indication, and thus the congestion window size at the start of the ith TD period is equal to $W_{i-1} - \beta W_{i-1}^{l}$. At the end of each round, the window is incremented by α / W^k, where W is the window size at the beginning of the round. We denote by η_i the first packet lost in TDP_i, and X_i the round where this loss occurs. After packet η_i, $W_i - 1$ more packets are sent in an additional round before a TD congestion indication occurs (and the current TD period ends). Thus a total of $Y_i = \eta_i + W_i - 1$ packets are sent in $X_i + 1$ rounds. It follows that:

$$E[Y] = E[\eta] + E[W] - 1 \tag{9}$$

To derive $E[\eta]$, consider a random process $\{\eta_i\}$, where η_i is the number of packets sent in a TD period up to and including the first packet that is lost. Based on the assumption that packets are lost in a round independently of any packets lost in *other* rounds, $\{\eta_i\}$ is a sequence of independent and identically distributed (i.i.d.) random variables. Given the loss model, the probability of $\eta_i = k$ is equal to the probability that exactly $k - 1$ packets are successfully acknowledged before a loss occurs

$$P[\eta = k] = (1 - p)^{k-1} p, \quad k = 1, 2, \ldots \tag{10}$$

The mean of η is thus

$$E[\eta] = \sum_{k=1}^{\infty}(1-p)^{k-1}pk = \frac{1}{p} \tag{11}$$

Plugging (11) into (9), we have

$$E[Y] = \frac{1-p}{p} + E[W] \tag{12}$$

To derive $E[W]$ and $E[A]$, consider again TDP_i. Define r_{ij} to be the duration of the jth round of TDP_i. Then, the duration of TDP_i is $A_i = \sum_{j=1}^{X_i+1} r_{ij}$. Consider the round-trip time r_{ij} to be random variables that are assumed to be independent of congestion window size, and thus independent of the round number, j. It follows that

$$E[A] = (E[X]+1)E[r] \tag{13}$$

Henceforth, let $R = E[r]$ denote the average value of the round-trip time.

Finally, to derive an expression for $E[X]$, consider the evolution of W_i as a function of the number of rounds. First we observe that during the ith TD period, the window size increases between $W_{i-1} - \beta W_{i-1}^l$ and W_i (see Figure 2).

Consider the differential equation:

$$\frac{dW}{dt} = \frac{\alpha}{RW^k} \tag{14}$$

Solve the differential equation, we have that for $t \in [0, RX_i]$

$$W(t) = \left(\frac{\alpha(k+1)}{R}t + W_{i-1}^{k+1}(1-\beta W_{i-1}^{l-1})^{k+1}\right)^{\frac{1}{k+1}} \tag{15}$$

From (15), and plug in $W(RX_i) = W_i$, we solve the expression for X_i as

$$X_i = \frac{1}{(k+1)\alpha}\left(W_i^{k+1} - W_{i-1}^{k+1}(1-\beta W_{i-1}^{l-1})^{k+1}\right) \tag{16}$$

The fact that Y_i packets are transmitted in TDP_i is expressed by

$$\begin{aligned} Y_i &= \int_0^{RX_i} W(t)dt + \theta_i & (17)\\ &= \frac{W_i^{k+2} - W_{i-1}^{k+2}(1-\beta W_{i-1}^{l-1})^{k+2}}{\alpha(k+2)} + \theta_i & (18) \end{aligned}$$

where θ_i is the number of packets sent in the last round. Consider that θ_i, the number of packets in the last round, is uniformly distributed between 1 and W_i, and thus

$$E[\theta] = E[W]/2 \tag{19}$$

$\{W_i\}$ is a Markov process for which a stationary distribution can be obtained numerically. However, a simpler approximate solution can be obtained.

Next, we consider two special cases. The first case is called IIAD (*inverse-increase/additive decrease*); the second, SQRT (because the increase and decrease are proportional to the square-root of the current window).

A.1.1 IIAD (k = 1, l= 0)

First, plug in $k = 1$, $l = 0$ into (16), we have

$$X_i = \frac{1}{2\alpha}\left(W_i^2 - (W_{i-1} - \beta)^2\right) \tag{20}$$

Take expectation on (20), and we have

$$E[X] = \frac{2\beta E[W] - \beta^2}{2\alpha} \tag{21}$$

Plug in $k = 1$, $l = 0$ into (18), take expectations on both sides, compare to (12), we have

$$\begin{aligned} \frac{1-p}{p} + \frac{E[W]}{2} &= E\left[\frac{W^3 - (W-\beta)^3)}{3\alpha}\right] && (22) \\ &= \frac{3\beta E[W^2] - 3\beta^2 E[W] + \beta^3}{3\alpha} && (23) \end{aligned}$$

Since $Var[W] = E[W^2] - E[W]^2$, and we assume the variance of W is small, therefore, we can approximate $E[W^2]$ by $E[W]^2$. We solve the Equation (23) and derive the expression for $E[W]$ as

$$E[W] \approx \frac{\alpha + 2\beta^2}{4\beta} + \sqrt{\frac{\alpha}{\beta p} + \frac{3\alpha^2 - 48\alpha\beta + 12\alpha\beta^2 - 4\beta^4}{144\beta^2}} \tag{24}$$

Simplify, and we have

$$E[W] = \sqrt{\frac{\alpha}{\beta p}} + o(1/\sqrt{p}) \tag{25}$$

Therefore, for small value of p, we have

$$E[W] \approx \sqrt{\frac{\alpha}{\beta p}} \tag{26}$$

According to (21), and plug in the expression for $E[W]$, we can derive the expression for $E[X]$, simplify, and we have

$$E[X] = \sqrt{\frac{\beta}{\alpha p}} + o(1/\sqrt{p}) \tag{27}$$

Next, consider the derivation for $E[A]$. Plugging the expression for $E[X]$ into (13), we have

$$\begin{aligned} E[A] &= R(E[X]+1) && (28) \\ &= R\sqrt{\frac{\beta}{\alpha p}} + o(1/\sqrt{p}) && (29) \end{aligned}$$

Then, according to (8) for T, (12) for $E[Y]$, (24) for $E[W]$, (29) for $E[A]$, we have

$$\begin{aligned} T &= \frac{\frac{1-p}{p} + E[W]}{E[A]} && (30) \\ &\approx \frac{\frac{1-p}{p} + \sqrt{\frac{\alpha}{\beta p}}}{R\sqrt{\frac{\beta}{\alpha p}}} && (31) \end{aligned}$$

Simplify, and we have

$$T \approx \frac{1}{R}\sqrt{\frac{\alpha}{\beta p}} + o(1/\sqrt{p}) \tag{32}$$

A.1.2 SQRT (k = l = 0.5)

First, plug in $k = l = 0.5$ into (16), we have

$$X_i = \frac{1}{1.5\alpha}\left(W_i^{1.5} - (W_{i-1} - \beta W_{i-1}^{0.5})^{1.5}\right) \tag{33}$$

Assume $E[W^t] \approx E[W]^t$, take expectations on (33), we have

$$E[X] = \frac{E[W]^{1.5} - E[W]^{1.5}(1 - \frac{\beta}{\sqrt{E[W]}})^{1.5}}{1.5\alpha} \tag{34}$$

Plug in $k = l = 0.5$ into (18), take expectations on both sides, assume $E[W^t] \approx E[W]^t$, and compare to (12), we have

$$\frac{1-p}{p} + \frac{E[W]}{2} = \frac{E[W]^{2.5}(1-(1-\frac{\beta}{\sqrt{E[W]}})^{2.5})}{2.5\alpha} \tag{35}$$

To get an analytical expression for $E[W]$, approximate $(1-\frac{\beta}{\sqrt{E[W]}})^{2.5}$ as $1 - 2.5\frac{\beta}{\sqrt{E[W]}}$, we solve the equation to get

$$E[W] = \frac{\alpha}{4\beta} + \frac{1}{2}\sqrt{\frac{\alpha^2}{4\beta^2} + \frac{4\alpha}{\beta}\frac{1-p}{p}} \tag{36}$$

Simplify, and we have

$$E[W] = \sqrt{\frac{\alpha}{\beta p}} + o(1/\sqrt{p}) \tag{37}$$

Therefore, for small value of p, we have

$$E[W] \approx \sqrt{\frac{\alpha}{\beta p}} \tag{38}$$

Plug in (36) into (34), simplify, and we have

$$E[X] = \sqrt{\frac{\beta}{\alpha p}} + o(1/\sqrt{p}) \tag{39}$$

Next, consider the derivation for $E[A]$. Plug in the expression for $E[X]$ into (13), we have have

$$E[A] = R(E[X]+1) \tag{40}$$

$$= R\sqrt{\frac{\beta}{\alpha p}} + o(1/\sqrt{p}) \tag{41}$$

Then, according to (8) for T, (12) for $E[Y]$, (36) for $E[W]$, (41) for $E[A]$, we have

$$T = \frac{\frac{1-p}{p} + E[W]}{E[A]} \tag{42}$$

$$\approx \frac{\frac{1-p}{p} + \sqrt{\frac{\alpha}{\beta p}}}{R\sqrt{\frac{\beta}{\alpha p}}} \tag{43}$$

Simplify, and we have

$$T \approx \frac{1}{R}\sqrt{\frac{\alpha}{\beta p}} + o(1/\sqrt{p}) \tag{44}$$

Summarize the result for IIAD and SQRT, we found that for both cases,

$$E[W] \approx \sqrt{\frac{\alpha}{\beta p}} \tag{45}$$

and

$$E[X] \approx \sqrt{\frac{\beta}{\alpha p}} \tag{46}$$

A.2 Congestion indications are triple-duplicate ACKs and timeouts

Next, we extend the analysis to include timeouts. The derivation in this section is the same as in [7] except for $\hat{Q}(E(W))$. However, we include it here for completeness.

In the previous section, we considered Binomial flows where all congestion indications are due to "triple-duplicate" ACKs. However, under certain circumstances the majority of window decreases can be due to timeouts. Therefore, a good model should also capture timeout congestion indications.

Timeout occurs when packets (or ACKs) are lost, and less than three duplicate ACKs are received. The sender waits for a period of time denoted by T_0, and then retransmits the first unacknowledged packet. Following a timeout, the congestion window is reduced to one, and one packet is resent in the first round after a timeout. If this retransmission is unsuccessful, the period of timeout doubles to $2T_0$; this doubling is repeated for each unsuccessful retransmission until $64T_0$ is reached, after which the timeout period remains constant at $64T_0$.

Figure 3 shows a trace with both TDP and timeouts.

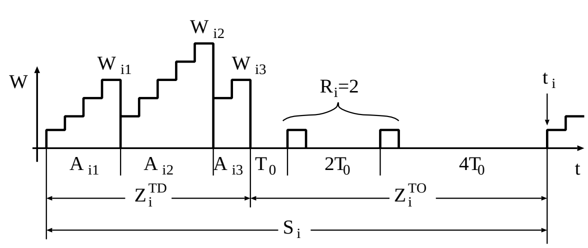

Figure 3: A trace with both TDP and timeouts

Let Z_i^{TO} denote the duration of a sequence of timeouts and Z_i^{TD} the time interval between two consecutive timeout sequences. Define S_i to be

$$S_i = Z_i^{TD} + Z_i^{TO} \tag{47}$$

Also, define M_i to be the number of packets sent during S_i. Then $\{(S_i, M_i)\}$ is an i.i.d sequence of random variables, and we have

$$T = \frac{E[M]}{E[S]} \tag{48}$$

Extend the definition of TD period defined previously to include periods starting after, or ending in, a TO congestion indication (besides periods between two TD congestion indications). Let n_i be the number of TD periods in interval Z_i^{TD}. For the jth TD period of interval Z_i^{TD} we define Y_{ij} to be the number of packets sent in the period, A_{ij} to be the duration of the period, X_{ij} to be the number of rounds in the period, and W_{ij} to be the window size at the end of the period. Also, R_i denotes the number of packets sent during timeout sequence Z_i^{TO}. We have

$$\begin{aligned} M_i &= \sum_{j=1}^{n_i} Y_{ij} + R_i \\ S_i &= \sum_{j=1}^{n_i} A_{ij} + Z_i^{TO} \end{aligned}$$

And thus,

$$E[M] = E[\sum_{j=1}^{n_i} Y_{ij}] + E[R] \tag{49}$$

$$E[S] = E[\sum_{j=1}^{n_i} A_{ij}] + E[Z^{TO}] \tag{50}$$

If n_i is an i.i.d. sequence of random variables, independent of $\{Y_{ij}\}$ and $\{A_{ij}\}$, then for any i we have

$$E[(\sum_{j=1}^{n_i} Y_{ij})_i] = E[n]E[Y] \tag{51}$$

$$E[(\sum_{j=1}^{n_i} A_{ij})_i] = E[n]E[A] \tag{52}$$

To derive $E[n]$, observe that, during Z_i^{TD}, the time between two consecutive timeout sequences, there are n_i TDPs, where each of the first $n_i - 1$ end in a TD, and the last TDP ends in a TO. It follows that in Z_i^{TD} there is one TO out of n_i loss indications. Therefore, if we denote by Q the probability that a congestion indication ending a TDP is a TO, we have $Q = 1/E[n]$. Consequently,

$$T = \frac{E[Y] + QE[R]}{E[A] + QE[Z^{TO}]} \tag{53}$$

Since Y_{ij} and A_{ij} do not depend on timeouts, their means are those derived before.

However, we still need to derive expressions for Q, $E[R]$, $E[Z^{TO}]$.

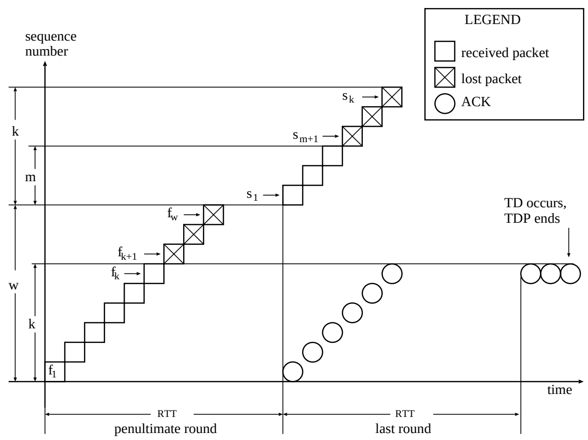

Figure 4: Packet and ACK transmissions preceding a loss indication

First consider Q. Consider the round of packets where a loss indication occurs; this round will be referred to as the “penultimate” round (see Figure 4) . We choose the ACK such that ACKs acknowledge individual packets (i.e. ACKs are not delayed). We will see that the analysis does not depend on whether ACKs are delayed or not. Let w be the current window size. Thus packet $f_1, \ldots, f_w$ are sent in the penultimate round. Packets $f_1, \ldots, f_k$ are acknowledged, and packets f_{k+1} is the first packet to be lost (or not ACKed). We again assume packet losses are correlated within a round: if a packet is lost, so too are all packets that follow, until the end of the round. Thus all packets following f_{k+1} are also lost. However, since packets $f_1, \ldots, f_k$ are ACKed, another k packets, $s_1, \ldots, s_k$ are sent in the next round, which we refer to the “last” round. This round of packet may have another loss, say packet s_{m+1}. Again, our assumption on packet loss correlation mandates that packets $s_{m+2}, \ldots, s_k$ are also lost in the last round. The m packets successfully sent in the last round are responded to by ACKs for packet f_k, which are counted as duplicate ACKs. These ACKs are not delayed, so the number of duplicate ACKs is equal to the number of successfully received packets in the last round. If the number of such ACKs is higher than three, then a TD indication occurs, otherwise a TO occurs. In both cases the current period ends. We denote by $A(w,k)$ the probability that first k packets are ACKed in a round of w packets, given there is a sequence of one or more losses in the round. Then

$$A(w,k) = \frac{(1-p)^k p}{1-(1-p)^w} \tag{54}$$

Also, let $C(n,m)$ denote the probability that m packets are ACKed in sequence in the last round (where n packets are sent) and the rest of the packets in the round, if any are lost. Then

$$C(n,m) = \begin{cases} (1-p)^m p & \text{if m} \leq \text{n-1} \\ (1-p)^n & \text{otherwise} \end{cases} \tag{55}$$

Then, $\hat{Q}(m)$, the probability that a loss in a window of size w is a TO, is given by

$$\hat{Q}(w) = \begin{cases} 1 & \text{if w} \leq 3 \\ \sum_{k=0}^{2} A(w,k) + \sum_{k=3}^{w} A(w,k) \sum_{m=0}^{2} C(k,m) & \text{otherwise} \end{cases} \tag{56}$$

After some algebraic manipulation, we have

$$\hat{Q}(w) = \min\left(1, \frac{(1-(1-p)^3)(1+(1-p)^3(1-(1-p)^{w-3}))}{1-(1-p)^w}\right) \tag{57}$$

Observe that

$$\lim_{p \to 0} \hat{Q}(w) = \frac{3}{w} \tag{58}$$

A numerical approximation of $\hat{Q}(w)$ then is

$$\hat{Q}(w) \approx \min(1, \frac{3}{w}) \tag{59}$$

Q, the probability that a congestion indication is a TO, is

$$Q = \sum_{w=1}^{\infty} \hat{Q}(w) P[W = w] = E[\hat{Q}] \tag{60}$$

We approximate

$$Q \approx \hat{Q}(E[W]) \tag{61}$$

where $E[W]$ is from (45).

Next, consider the derivations of $E[R]$ and $E[Z^{TO}]$.

A sequence of k TOs occurs when there are $k-1$ consecutive losses (the first loss is given) followed by a successfully transmitted packet. Consequently, the number of TOs in a TO sequence has a geometric distribution, and thus

$$P[R=k] = p^{k-1}(1-p) \tag{62}$$

Then we calculate the mean of R as

$$E[R] = \sum_{k=1}^{\infty} kP[R=k] = \frac{1}{1-p} \tag{63}$$

Next, consider $E[Z^{TO}]$, the average duration of a timeout sequence excluding retransmissions, which can be calculated in a similar way. We know that the first six timeouts in one sequence have length $2^{i-1}T_0$, with all immediately following timeouts having length $64T_0$. Then the duration of a sequence with k timeout is

$$L_k = \begin{cases} (2^k-1)T_0 & \text{for k} \leq 6 \\ (63+64(k-6))T_0 & \text{for k} \geq 7 \end{cases} \tag{64}$$

And the mean of Z^{TO} is

$$\begin{aligned} E[Z^{TO}] &= \sum_{k=1}^{\infty} L_k P[R=k] && (65) \\ &= T_0 \frac{1+p+2p^2+4p^3+8p^4+16p^5+32p^6}{1-p} && (66) \end{aligned}$$

Now we can plug (12) for $E[Y]$, (63) for $E[R]$, (13) for $E[A]$, (66) for $E[Z^{TO}]$, and (61) for Q into (53), and have

$$T = \frac{\frac{1-p}{p} + E[W] + \hat{Q}(E[W])\frac{1}{1-p}}{R(E[X]+1) + \hat{Q}(E[W])T_0\frac{f(p)}{1-p}} \tag{67}$$

where

$$f(p) = 1+p+2p^2+4p^3+8p^4+16p^5+32p^6 \tag{68}$$

Now we can plug the common expression (45) of IIAD and SQRT for $E[W]$, the common expression (46) of IIAD and SQRT for $E[X]$, and (59) for $\hat{Q}$ into (67), simplify, and we have

$$T_{\alpha,\beta}(p, R, T_0, b) \approx \frac{1}{R\sqrt{\frac{\beta}{\alpha}p} + T_0 \min\left(1, 3\sqrt{\frac{\beta}{\alpha}p}\right)p(1+32p^2)} \tag{69}$$

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