Traditional Uses of Compilers

Last lecture
- Field analysis

Today
- Register allocation

Register Allocation

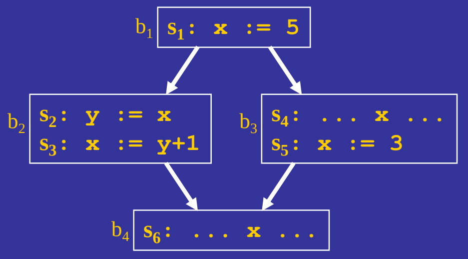
Problem
- Assume a load/store architecture
- Assign an unbounded number of **symbolic** registers to a fixed number of **architectural** registers (which might get renamed by the hardware to some number of **physical** registers)
- Simultaneously live data must be assigned to different architectural registers

Goal
- Minimize overhead of accessing data
  - Memory operations (loads & stores)
  - Register moves

Granularity of Allocation

What is allocated to registers?

- Variables **Can we do better?**
- Live ranges (*i.e.*, set of basic blocks in which a variable is live)
- Values (*i.e.*, definitions; same as variables with SSA & copy propagation)
- Webs (*i.e.*, du-chains with common uses)



Variables: 2 (**x** & **y**)
Live ranges: 2 ({b_1,b_2,b_3,b_4}, {b_2})
Values: 4 (s_1, s_2, s_3, s_5, $\phi(s_3,s_5)$)
Web: 3 ($s_1 \rightarrow s_2,s_4$;
$s_2 \rightarrow s_3$;
$s_3,s_5 \rightarrow s_6$)

What are the tradeoffs?

Each allocation unit is given a symbolic register name (*e.g.*, `s1`, `s2`, *etc.*)

Scope of Register Allocation

Expression

Local

➡ **Loop**

➡ **Global**

Interprocedural

Local Register Allocation for Loops

Idea

- Estimate the benefit of allocating variables in basic blocks or loops
- Allocate variables with greatest benefit to registers
- Estimates are a function of execution frequency (from profiles, heuristics)

Surprisingly effective!

- IBM 360/370 Fortran H compiler

Local Register Allocation for Loops (cont)

Definitions

- *ldcost*: Cost (time) of load instruction
- *stcost*: Cost of store instruction
- *mvcost*: Cost of register-to-register transfer instruction
- *usesave*: Savings (time) for each use of variable in a register vs. memory
- *defsave*: Savings for each assignment of variable in a register vs. memory
- Static counts for variable v: l_i, s_i, u_i, d_i (l_i and s_i are 0 or 1)

Benefit of allocating variable v to a register in block b_i is

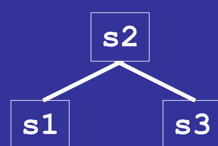
$$netsave(v,i) = u_i \cdot usesave + d_i \cdot defsave - l_i \cdot ldcost - s_i \cdot stcost$$

$$Benefit(v,L) = 10^{depth(L)} \sum_{i \in blocks(L)} netsave(v,i)$$

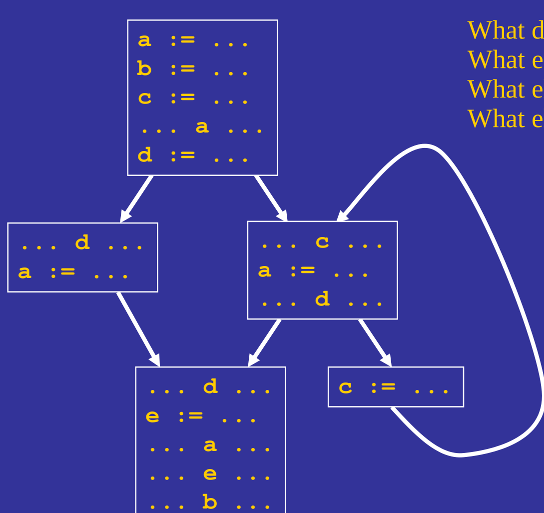
Global Register Allocation by Graph Coloring

Idea

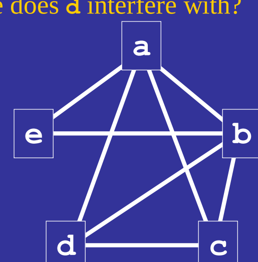
1. Construct **interference graph** $G=(N,E)$
   - Represents notion of "simultaneously live"
   - Nodes are units of allocation (*e.g.*, variables, live ranges, webs)
   - $\exists$ edge $(n_1,n_2) \in E$ if n_1 and n_2 are simultaneously live
   - Symmetric (not reflexive nor transitive)
2. Find ***k*-coloring** of G (for k registers)
   - Adjacent nodes can't have same color
3. **Allocate** the same register to all allocation units of the same color
   - Adjacent nodes must be allocated to distinct registers



Interference Graph Exercise (Variables)

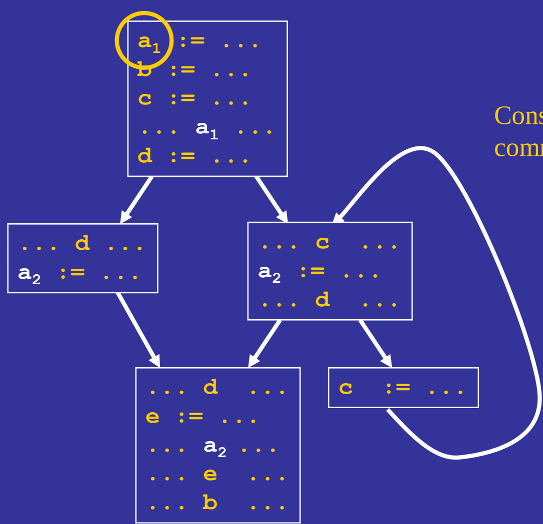


What does **a** interfere with?
What else does **b** interfere with?
What else does **c** interfere with?
What else does **d** interfere with?

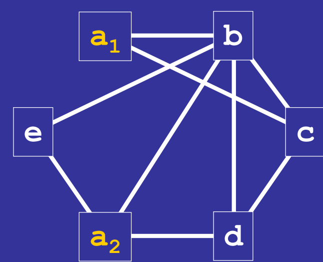


How many colors do we need?

Interference Graph Example (Webs)



Consider webs (du-chains w/ common uses) instead of variables



Building the Interference Graph

Use results of live variable analysis

for each symbolic-register s_i **do**
  for each symbolic-register s_j ($j < i$) **do**
    for each def $\in$ {definitions of s_i} **do**
      if (s_j is live at def) **then**
        $E \leftarrow E \cup (s_i, s_j)$

Allocating Registers Using the Interference Graph

***K*-coloring**
- Color nodes using up to *k* colors
- Adjacent nodes must have different colors

Allocating to *k* registers ≡ finding a *k*-coloring of the interference graph
- Adjacent nodes must be allocated to distinct registers

But. . .
- Optimal graph coloring is NP-complete
  - Register allocation is NP-complete, too (must approximate)
- What if we can't *k*-color a graph?

Spilling

If we can't find a k-coloring of the interference graph
- Spill variables (nodes) to stack until the graph is colorable

How does spilling help?
- It reduces the live range of the spilled variable

Which variables should we spill?
- The least frequently accessed variables
- Break ties by choosing nodes with the most conflicts in the interference graph
- Yes, these are heuristics!

Weighted Interference Graph

Goal

– Weight(s) = $\sum_{\forall \text{references } r \text{ of } s} f(r)$ $f(r)$ is execution frequency of r

Static approximation

– Use some reasonable scheme to rank variables

– One possibility

  – Weight(s) = 1

  – Nodes after branch: ½ weight of branch

  – Nodes in loop: 10 × weight of nodes outside loop

Simple Greedy Algorithm for Register Allocation

for each $n \in N$ **do** { select n in decreasing order of weight }

  if n can be colored **then**

    do it { reserve a register for n }

  else

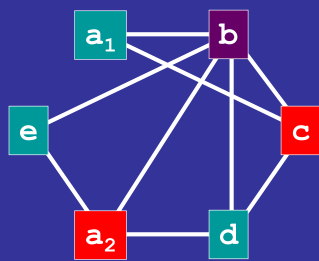
    Remove n (and its edges) from graph { allocate n to stack (spill) }

Note

– Reserve 2-3 temp registers for manipulating data on stack

Example

Attempt to 3-color this graph (, ,)



Weighted order:
a$_1$
b
c
d
a$_2$
e

What if you use a different weighting?

Problems with this approach?

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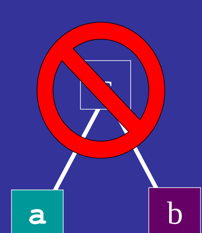
Example

Attempt to 2-color this graph (,)



Weighted order:
a
b
c

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Improvement #1: Simplification Phase

Idea

- Nodes with < *k* neighbors are guaranteed colorable

Remove them from the graph first

- Reduces the degree of the remaining nodes

Must spill only when all remaining nodes have degree ≥ *k*

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Algorithm [Chaitin82]

```
while interference graph not empty do
    while ∃ a node n with < k neighbors do        }
        Remove n from the graph                   } simplify
        Push n on a stack                         }
    if any nodes remain in the graph then { blocked with ≥ k edges }  }
        Pick a node n to spill          { lowest spill-cost or }      }
        Add n to spill set              {    highest degree }         } spill
        Remove n from the graph                                       }
if spill set not empty then
    Insert spill code for all spilled nodes  { store after def; load before use }
    Reconstruct interference graph & start over
while stack not empty do          }
    Pop node n from stack         } color
    Allocate n to a register      }
```

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More on Spilling

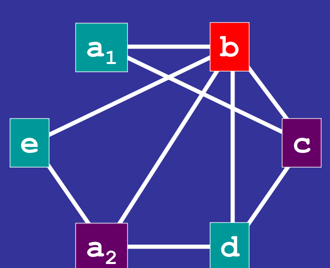
Chaitin's algorithm restarts the whole process on spill
- Necessary, because spill code (loads/stores) uses registers
- Okay, because restarts usually only happen a couple times

Alternative
- Reserve 2-3 registers for spilling
- Don't need to start over
- But have fewer registers to work with

Example

Attempt to 3-color this graph (, ,)

Stack:
d
c
b
a_2
a_1
e



Weighted order:
e
a_1
a_2
b
c
d

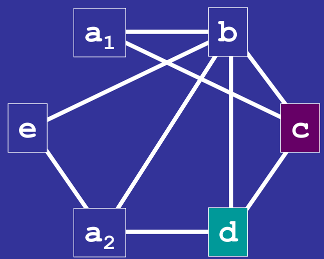
How are the nodes ordered here?

Example

Attempt to 2-color this graph (,)

Spill Set:
e
a_1
a_2
b

Stack:
d
c

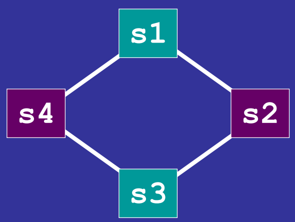


Weighted order:
e
a_1
a_2
b
c
d

Many nodes remain uncolored even though we could clearly do better

The Problem: Worst Case Assumptions

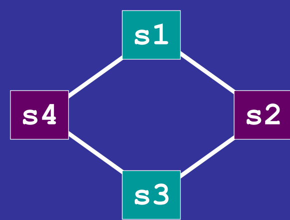
Is the following graph 2-colorable?



Clearly 2-colorable

- But Chaitin’s algorithm leads to an immediate block and spill.
- The algorithm assumes the worst case, namely, that all neighbors will be assigned a different color

Improvement #2: Optimistic Spilling



Idea

- Some neighbors might get the same color
- So nodes with *k* neighbors **might** be colorable
- Blocking does not imply that spilling is necessary
  - Push blocked nodes on stack (rather than place in spill set)
  - Check colorability upon popping the stack, when more information is available

} Defer decision

Algorithm [Briggs *et al*. 89]

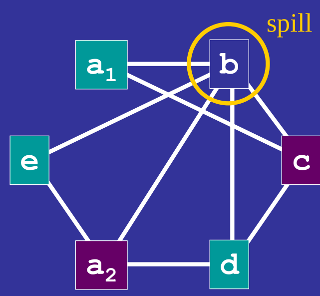
```
while interference graph not empty do
    while ∃ a node n with < k neighbors do      } simplify
        Remove n from the graph
        Push n on a stack
    if any nodes remain in the graph then       { blocked with ≥ k edges }
        Pick a node n to block                  { lowest spill-cost/highest degree }
        Push n on stack                         } defer decision
        Remove n from the graph
while stack not empty do
    Pop node n from stack                       } make decision
    if n is colorable then
        Allocate n to a register
    else
        Insert spill code for n                 { Store after def; load before use }
        Reconstruct interference graph & start over
```

Example

Attempt to 2-color this graph (■ , ■)

Stack:
d
c
b*
a_2*
a_1*
e*

*** blocked node**



Weighted order:
e
a_1
a_2
b
c
d

4 nodes were blocked
Only 1 node was spilled

Improvement #3: Live Range Splitting [Chow & Hennessy 84]

Idea

- Start with variables as our allocation unit
- When a variable can’t be allocated, split it into multiple subranges for separate allocation
- Selective spilling: put some subranges in registers, some in memory
- Insert memory operations at boundaries

Why is this a good idea?

Improvement #4: Rematerialization

Idea

- Selectively re-compute values rather than loading from memory
- "Reverse CSE"

Easy case

- Value that can be computed in single instruction, and
- All operands are available

Examples

- Constants
- Addresses of global variables
- Addresses of local variables (on stack)

Coalescing

Move instructions

- Code generation can produce unnecessary move instructions

```
mov t1, t2
```

- If we can assign `t1` and `t2` to the same register, we can eliminate the move

Idea

- If `t1` and `t2` are not connected in the interference graph, **coalesce** them into a single variable

Problem?

- Coalescing can increase the number of edges and make a graph uncolorable
- Limit coalescing to avoid uncolorable graphs

Next Time

Lecture

- More register allocation
  - Allocation across procedure calls