Unreliable Failure Detectors for Reliable Distributed Systems

A different approach

- Augment the asynchronous model with an unreliable failure detector for crash failures
- Define failure detectors in terms of abstract properties, not specific implementations
- Identify classes of failure detectors that allow to solve Consensus

The Model

General

- asynchronous system
- processes fail by crashing
- a failed process does not recover

Failure Detectors

- outputs set of processes that it currently suspects to have crashed
- the set may be different for different processes

Completeness

Strong Completeness Eventually every process that crashes is permanently suspected by every correct process

Weak Completeness Eventually every process that crashes is permanently suspected by some correct process

Accuracy

Strong Accuracy
No correct process is ever suspected

Weak Accuracy
Some correct process is never suspected

Accuracy

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No correct process is ever suspected

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Some correct process is never suspected

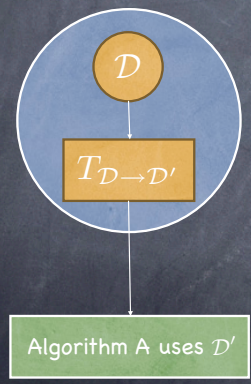
Eventual Strong Accuracy
There is a time after which no correct process is ever suspected

Eventual Weak Accuracy
There is a time after which some correct process is never suspected

Failure detectors

Completeness	Accuracy			
	Strong	Weak	Eventual strong	Eventual weak
Strong	Perfect P	Strong S	$\Diamond P$	$\Diamond S$
Weak	Quasi Q	Weak W	$\Diamond Q$	$\Diamond W$

Reducibility



$T_{\mathcal{D}\rightarrow\mathcal{D}'}$ transforms failure detector $\mathcal{D}$ into failure detector $\mathcal{D}'$

If we can transform $\mathcal{D}$ into $\mathcal{D}'$ then we say that $\mathcal{D}$ is stronger than $\mathcal{D}'$ ($\mathcal{D} \geq \mathcal{D}'$) and that $\mathcal{D}'$ is reducible to $\mathcal{D}$

If $\mathcal{D} \geq \mathcal{D}'$ and $\mathcal{D}' \geq \mathcal{D}$ then we say that $\mathcal{D}$ and $\mathcal{D}'$ are equivalent:

$$\mathcal{D} \equiv \mathcal{D}'$$

Simplify, Simplify!

- All weakly complete failure detectors are reducible to strongly complete failure detectors

$$P \geq Q, \quad S \geq W, \quad \Diamond P \geq \Diamond Q, \quad \Diamond S \geq \Diamond W$$

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$$P \geq Q, \quad S \geq W, \quad \Diamond P \geq \Diamond Q, \quad \Diamond S \geq \Diamond W$$

- All strongly complete failure detectors are reducible to weakly complete failure detectors (!)

$$Q \geq P, \quad W \geq S, \quad \Diamond Q \geq \Diamond P, \quad \Diamond W \geq \Diamond S$$

Weakly and strongly complete failure detectors are equivalent!

From Weak Completeness to Strong Completeness

Every process p executes the following:

$output_p$:= 0

cobegin

|| Task 1: repeat forever

{ p queries its local failure detector module $\mathcal{D}_p$}

$suspects_p := \mathcal{D}_p$

send $(p, suspects_p)$ to all

|| Task 2: when receive($q, suspects_q$) from some q

$output_p := (output_p \cup suspects_p) - \{q\}$

coend

The Theorems

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Theorem 3 In asynchronous systems in which processes can use $\Diamond W$, consensus can be solved as long as $f < n/2$

Theorem 4 A failure detector can solve consensus only if it satisfies weak completeness and eventual weak accuracy–i.e. $\Diamond W$ is the weakest failure detector that can solve consensus.

Solving consensus using S

S: Strong Completeness, Weak Accuracy

- at least some correct process c is never suspected

- Each process p has its own failure detector
- Input values are chosen from the set {0,1}

Notation

We introduce the operators $\oplus, \star, \otimes$

They operate element-wise on vectors whose entries have values from the set {0, 1, $\perp$}

$v \star \perp = v \quad \perp \star v = v$

$v \star v = \perp \quad \perp \star \perp = \perp$

$v \oplus \perp = v \quad \perp \oplus v = v$

$v \oplus v = v \quad \perp \oplus \perp = \perp$

$v \otimes \perp = \perp \quad \perp \otimes v = \perp$

$v \otimes v = v \quad \perp \otimes \perp = \perp$

Given two vectors A and B, we write $A \leq B$ if $A[i] \neq \perp$ implies $B[i] \neq \perp$

Solving Consensus using any $\mathcal{D} \in S$

1: V_p := $(\perp, \ldots, \perp, v_p, \perp, \ldots, \perp)$ {p's estimate of the proposed values}
2: Δ_p:= $(\perp, \ldots, \perp, v_p, \perp, \ldots, \perp)$
3: {phase 1} {asynchronous rounds r_p, $1 \leq r_p \leq n-1$}
4: for r_p := 1 to $n-1$
5: send (r_p, Δ_p, p) to all
6: wait until [$\forall q$: received (r_p, Δ_q, q) or $q \in \mathcal{D}_p$] {query the failure detector}
7: O_p := V_p
8: V_p := $V_p \oplus (\oplus_{q\ received} \Delta_q)$
9: Δ_p := $V_p \star O_p$ {value is only echoed the first time it is seen}
10: {phase 2}
11: send (r_p, V_p, p) to all
12: wait until [$\forall q$: received (r_p, V_q, q) or $q \in \mathcal{D}_p$]
13: V_p := $\otimes_{q\ received} V_q$ {computes the "intersection", including V_p}
14: {phase 3}
15: decide on leftmost non-$\perp$ coordinate of V_p

A useful Lemma

1: V_p := ($\perp$, ..., $\perp$, v_p, $\perp$, ..., $\perp$) {p's estimate of the proposed values}
2: Δ_p := ($\perp$, ..., $\perp$, v_p, $\perp$, ..., $\perp$)
3: {phase 1} {asynchronous rounds r_p, $1 \leq r_p \leq n-1$}
4: for r_p := 1 to n-1
5: send (r_p, Δ_p ,p) to all
6: wait until [$\forall$q: received (r_p, Δ_q ,q) or $q \in \mathcal{D}_p$]
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Lemma 1 After phase 1 is complete, $V_c \leq V_p$ for all processes p that complete phase 1

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Lemma 1 After phase 1 is complete, $V_c \leq V_p$ for all processes p that complete phase 1

Proof We show that

$$V_c[i] = v_i \wedge v_i \neq \perp \Rightarrow \forall p : V_p[i] = v_i$$

Let r be the first round when c sees v_i

1. $r \leq n-2$
   - c will send to all Δ_c with v_i in round r
   - By weak accuracy, all correct processes receive v_i by next round
2. $r = n-1$
   - v_i has been forwarded $n-1$ times: every other process has seen v_i

Two additional cool lemmas

1: $V_p := (\perp, \ldots, \perp, v_p, \perp, \ldots, \perp)$ {p's estimate of the proposed values}
2: $\Delta_p := (\perp, \ldots, \perp, v_p, \perp, \ldots, \perp)$
3: {Phase 1} {asynchronous rounds r_p, $1 \le r_p \le n-1$}
4: **for** $r_p := 1$ **to** n-1
5: **send** (r_p, Δ_p, p) **to** all
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12: **wait until** [$\forall q$: received (r_p, V_q, q) **or** $q \in \mathcal{D}_p$]
13: $V_p := \otimes_q$ received V_q {computes the "intersection", including V_p}
14: {Phase 3}
15: decide on leftmost non-$\perp$ coordinate of V_p

Lemma 2 After Phase 2 is complete, $V_c = V_p$ for each p that completes phase 2

Proof

- All processes that completed phase 2 have received V_c.
  Since V_c is the smallest V vector,
  $$V_c[i] \neq \perp \Rightarrow V_p[i] \neq \perp \quad \forall p$$
- By the definition of $\otimes$
  $$V_c[i] = \perp \Rightarrow V_p[i] = \perp \quad \forall p$$
  after phase 2

Lemma 3 $V_c \neq (\perp, \perp, \perp, \ldots, \perp)$

Solving consensus

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Theorem The protocol to the left satisfies Validity, Agreement, and Termination

Proof

Left as an exercise

A lower bound - I

Theorem Consensus with $\Diamond P$ requires $f < n/2$

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Theorem Consensus with $\Diamond P$ requires $f < n/2$

Proof

- Suppose n is even, and a protocol exists that solves consensus when $f = n/2$
- Divide the set of processes in two sets of size $n/2$, P_1 and P_2

A lower bound - II

Consider three executions:

$P_1 \leftarrow 0; P_2 \leftarrow 0$

All processes in P_2 crash before they can propose

Detectors work perfectly

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P_1 decides 0

after t_1

$P_1 \leftarrow 1; P_2 \leftarrow 1$

All processes in P_1 crash before they can propose

Detectors work perfectly

P_2 decides 1

after t_2

A lower bound - II

Consider three executions:

$P_1 \leftarrow 0; P_2 \leftarrow 0$	$P_1 \leftarrow 0; P_2 \leftarrow 1$	$P_1 \leftarrow 1; P_2 \leftarrow 1$
All processes in P_2 crash before they can propose	No process crashes	All processes in P_1 crash before they can propose
Detectors work perfectly	Detectors make mistakes: until $max(t_1, t_2)$, P_1 believes P_2 crashed, and vice versa	Detectors work perfectly
P_1 decides 0		P_2 decides 1
after t_1		after t_2

A lower bound - II

Consider three executions:

$P_1 \leftarrow 0; P_2 \leftarrow 0$	$P_1 \leftarrow 0; P_2 \leftarrow 1$	$P_1 \leftarrow 1; P_2 \leftarrow 1$
All processes in P_2 crash before they can propose	No process crashes	All processes in P_1 crash before they can propose
Detectors work perfectly	Detectors make mistakes: until $max(t_1, t_2)$, P_1 believes P_2 crashed, and vice versa	Detectors work perfectly
P_1 decides 0	P_1 decides 0 P_2 decides 1	P_2 decides 1
after t_1		after t_2

The case of the Rotating Coordinator

Solving consensus with $\Diamond W$ (actually, $\Diamond S$)

- Asynchronous rounds
- Each round has a coordinator c
- $c_{id} = (r \bmod n) + 1$
- Each process p has an opinion $v_p \in \{0, 1\}$ (with a time of adoption t_p)
- Coordinator collects opinions to form a suggestion
- If they believe c to be correct, processes adopt its suggestion and make it their own opinion
- A suggestion adopted by a majority of processes is "locked"

One round, four phases

Phase 1

Each process, including c, sends its opinion timestamped r to c

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Phase 2

c waits for first $\lceil n/2+1 \rceil$ opinions with timestamp r
c selects v, one of the most recently adopted opinions
v becomes c's suggestion for round r
c sends its suggestion to all

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Phase 3

Each p waits for a suggestion, or for failure detector to signal c is faulty
If p receives a suggestion, p adopts it as its new opinion and ACKs to c
Otherwise, p NACKs to c

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Each p waits for a suggestion, or for failure detector to signal c is faulty
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Otherwise, p NACKs to c

Phase 4

c waits for first $\lceil n/2+1 \rceil$ responses
if all ACKs, then c decides on v and sends DECIDE to all
if p receives DECIDE, then p decides on v

Consensus using $\Diamond S$

v_p := input bit; r := 0; t_p := 0; $state_p$:= undecided
while p undecided do
  r := $r+1$
  c := $(r \bmod n)+1$
{phase 1: all processes send opinion to current coordinator}
  p sends (p, r, v_p, t_p) to c
{phase 2: current coordinator gather a majority of opinions}
  c waits for first $\lceil n/2+1 \rceil$ opinions (q, r, v_q, t_q)
  c selects among them the value v_q with the largest t_q
  c sends (c, r, v_q) to all
{phase 3: all processes wait for new suggestions from the current coordinator}
  p waits until suggestion (c, r, v) arrives or $c \in \Diamond S_p$
  if suggestion is received then {v_p := v; t_p := r; p sends (r, ACK) to c}
  else p sends (r, NACK) to c
{phase 4: coordinator waits for majority of replies. If majority adopted the coordinator's suggestion, then coordinator sends request to decide}
  c waits for first $\lceil n/2+1 \rceil$ (r, ACK) or (r, NACK)
  if c receives $\lceil n/2+1 \rceil$ ACKs, then c sends (r, DECIDE, v) to all
when p delivers (r, DECIDE, v) then {p decides v; $state_p$:= decided}

$\Diamond S$ Consensus as Paxos

- All processes are acceptors
- In round r, node $(r \bmod n)+1$ serves both as a distinguished proposer and as a distinguished learner
- The round structure guarantees a unique proposal number
- The value that a proposer proposes when no value is chosen is not determined