

## Causal Delivery in Synchronous Systems

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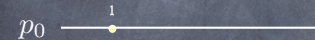
DR1: At time  $t$ ,  $p_0$  delivers all messages  
it received with timestamp up to  $t - \Delta$   
in increasing timestamp order

## Causal Delivery with Lamport Clocks

DR1.1: Deliver all received messages in  
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Problem: Lamport Clocks don't provide **gap detection**

Given two events  $e$  and  $e'$  and their clock values  $LC(e)$  and  $LC(e')$ —where  $LC(e) < LC(e')$ —determine whether some event  $e''$  exists s.t.  
 $LC(e) < LC(e'') < LC(e')$

## Stability

DR2: Deliver all received **stable** messages in increasing (logical clock) timestamp order.

A message  $m$  received by  $p$  is stable at  $p$  if  $p$  will never receive a future message  $m'$  s.t.

$$TS(m') < TS(m)$$

## Implementing Stability

- Real-time clocks
  - wait for  $\Delta$  time units

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- Real-time clocks
  - wait for  $\Delta$  time units
- Lamport clocks
  - wait on each channel for  $m$  s.t.  $TS(m) > LC(e)$
- Design better clocks!

# Clocks and STRONG Clocks

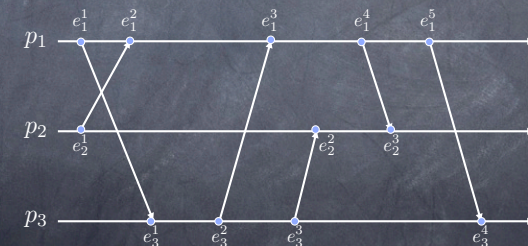
- Lamport clocks implement the **clock condition**:  
 $e \rightarrow e' \Rightarrow LC(e) < LC(e')$
- We want new clocks that implement the **strong clock condition**:  
 $e \rightarrow e' \equiv SC(e) < SC(e')$

# Causal Histories

- The **causal history** of an event  $e$  in  $(H, \rightarrow)$  is the set  
 $\theta(e) = \{e' \in H \mid e' \rightarrow e\} \cup \{e\}$

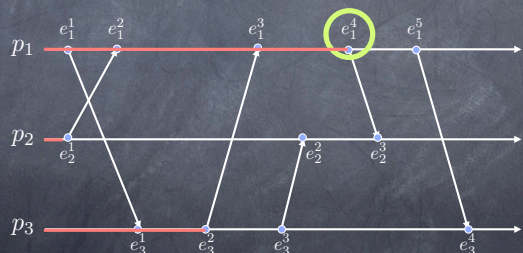
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$$e \rightarrow e' \equiv \theta(e) \subset \theta(e')$$

## How to build $\theta(e)$

Each process  $p_i$ :

- initializes  $\theta$ :  $\theta := \emptyset$
- if  $e_i^k$  is an **internal** or **send** event, then
 
$$\theta(e_i^k) := \{e_i^k\} \cup \theta(e_i^{k-1})$$
- if  $e_i^k$  is a **receive** event for message  $m$ , then
 
$$\theta(e_i^k) := \{e_i^k\} \cup \theta(e_i^{k-1}) \cup \theta(\text{send}(m))$$

## Pruning causal histories

- Prune segments of history that are known to all processes (Peterson, Bucholz and Schlichting)
- Use a more clever way to encode  $\theta(e)$

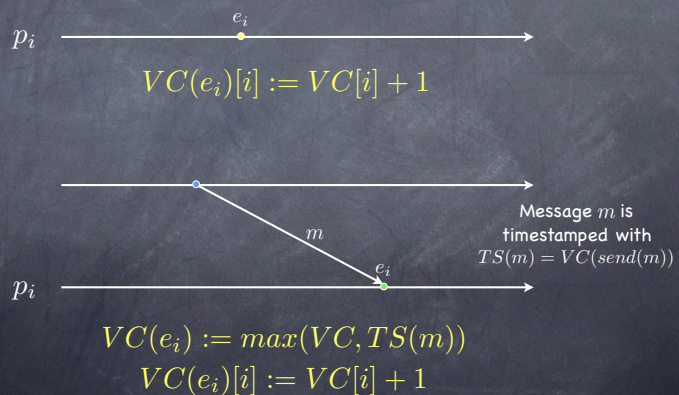
## Vector Clocks

- Consider  $\theta_i(e)$ , the projection of  $\theta(e)$  on  $p_i$
- $\theta_i(e)$  is a prefix of  $h^i$ :  $\theta_i(e) = h_i^{k_i}$  - it can be encoded using  $k_i$
- $\theta(e) = \theta_1(e) \cup \theta_2(e) \cup \dots \cup \theta_n(e)$  can be encoded using  $k_1, k_2, \dots, k_n$

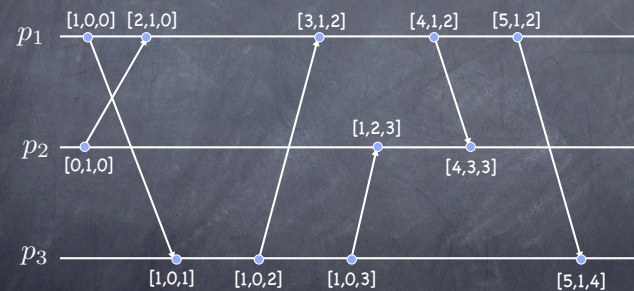
Represent  $\theta$  using an  $n$ -vector  $VC$  such that

$$VC(e)[i] = k \Leftrightarrow \theta_i(e) = h_i^{k_i}$$

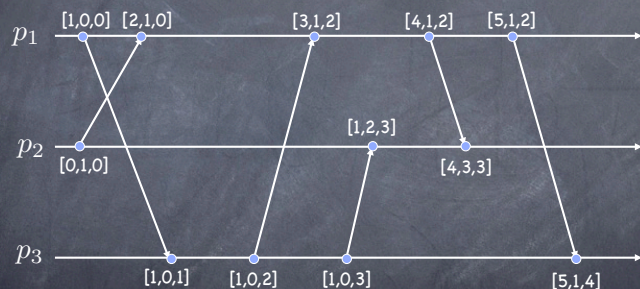
## Update rules



## Example



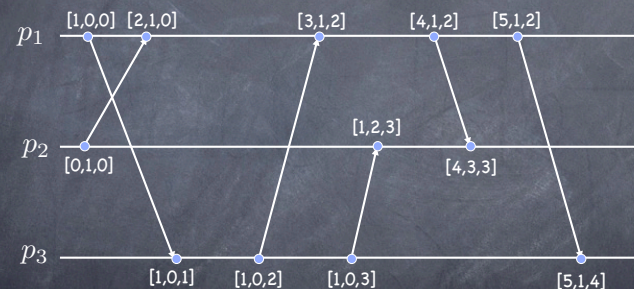
## Operational interpretation



$$VC(e_i)[i] =$$

$$VC(e_i)[j] =$$

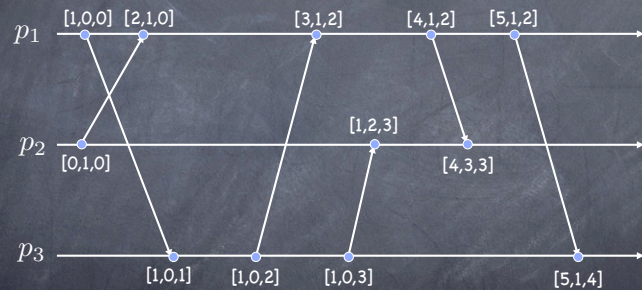
## Operational interpretation



$$VC(e_i)[i] = \text{no. of events executed by } p_i \text{ up to and including } e_i$$

$$VC(e_i)[j] =$$

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$VC(e_i)[i]$  = no. of events executed by  $p_i$  up to and including  $e_i$

$VC(e_i)[j]$  = no. of events executed by  $p_j$  that happen before  $e_i$  of  $p_i$

## VC properties: event ordering

Given two vectors  $V$  and  $V'$ , **less than** is defined as:

$$V < V' \equiv (V \neq V') \wedge (\forall k : 1 \leq k \leq n : V[k] \leq V'[k])$$

🕒 **Strong Clock Condition:**  $e \rightarrow e' \equiv VC(e) < VC(e')$

🕒 **Simple Strong Clock Condition:**

Given  $e_i$  of  $p_i$  and  $e_j$  of  $p_j$ , where  $i \neq j$

$$e_i \rightarrow e_j \equiv VC(e_i)[i] \leq VC(e_j)[i]$$

🕒 **Concurrency**

Given  $e_i$  of  $p_i$  and  $e_j$  of  $p_j$ , where  $i \neq j$

$$e_i \parallel e_j \equiv (VC(e_i)[i] > VC(e_j)[i]) \wedge (VC(e_j)[j] > VC(e_i)[j])$$

## VC properties: consistency

🕒 **Pairwise inconsistency**

Events  $e_i$  of  $p_i$  and  $e_j$  of  $p_j$  ( $i \neq j$ ) are pairwise inconsistent (i.e. can't be on the frontier of the same consistent cut) if and only if

$$(VC(e_i)[i] < VC(e_j)[i]) \vee (VC(e_j)[j] < VC(e_i)[j])$$

🕒 **Consistent Cut**

A cut defined by  $(c_1, \dots, c_n)$  is consistent if and only if

$$\forall i, j : 1 \leq i \leq n, 1 \leq j \leq n : (VC(e_i^{c_i})[i] \geq VC(e_j^{c_j})[i])$$