1. (String Matching; 32 points)

(a) (Rabin-Karp algorithm; 8 points) Suppose you are looking for the pattern 26 in the text 3141582653599793, where \( \text{val}(n) = n \mod 11 \). How many string matches do you have to attempt which ultimately fail?

(b) (KMP algorithm; 10 points) Show that you can determine if pattern \( p \) is in text \( t \) simply from the cores of the prefixes of \( pt \).

(c) (4 points) The algorithm for core computation includes the following code fragment; see notes on “String Matching”, Page 10.

\[
\text{if } p[u] = p[v] \\
\text{then } c(v') := u' \\
\text{else } c(v') := \epsilon \\
\text{endif ;}
\]

Is it possible that \( u = \epsilon \) and \( c(v') \neq \epsilon \) after execution of this portion of the program? Show a small example to support your claim.

(d) (KMP Algorithm; 10 points) Apply the KMP algorithm on the following pattern and text. Show only the different values of \( l \) (see page 6 of the notes on “String Matching”).

<table>
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<tbody>
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</table>

2. (Data Parallel Programming; 18 points)

(a) (Batcher Merge; 8 points) Suppose \( u \) is a sorted list. Show that

\[ u \text{ bm } u = u \uplus u \]

where \( \text{bm} \) is the Batcher Merge function (see Page 13 of your notes on Powerlist).

Hint: Use the following fact: if \( p \) and \( q \) partition a sorted list \( L \), then \( p \text{ bm } q = L \).

(b) (Prefix sum; 10 points) Let \( ps \) \( L \) be the prefix sum of \( L \). Suppose the corresponding operator \( \uplus \) is commutative as well as associative. Argue that \( ps(p \uplus q) = (ps p) \uplus (ps q) \). If \( \uplus \) is not commutative, show that \( ps(p \uplus q) = (ps p) \uplus (ps q) \) may not hold.

Hint: I don’t need a formal proof for the commutative result; let \( p = (p_0 \cdots p_n) \) and \( q = (q_0 \cdots q_n) \).