

Open book and notes.

Max points = 50

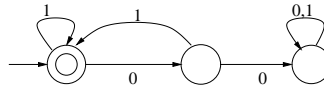
Time = 50 min

Do all questions.

## 1. (Finite State Machine; 28 points)

- (a) (Verification; 11 points) It is claimed that the following machine accepts a binary string in which every 0 is followed by a 1. Annotate the diagram and write the predicates which must be proven to verify this claim.

Let  $p(x)$  denote that in string  $x$  every 0 is followed by a 1. Let  $q(x)$  denote that in string  $x$  the last bit is a 0, and every 0, except the very last one, is followed by a 1.



- (b) (Finite State Transducer; 11 points) Draw a finite state transducer whose input is a binary string ending with a special symbol  $\#$ . It outputs a binary string ending with  $\#$  according to the following rule: a contiguous sequence of 0s is replaced by a single 0 and a contiguous sequence of  $n$  1s is replaced by a sequence of  $n + 1$  1s. Thus, 0010011 $\#$  becomes 0110111 $\#$ .
- (c) (Regular Expression; 6 points) Write 6 of the shortest strings of  $(000 | 1)^*(0 | 11)^*$ .

## 2. (Recursion and Induction; 22 points)

- (a) (4 points) Define a function, **between**, of three arguments,  $x$ ,  $y$  and  $z$ , each a number. The output is **True** iff  $x$  falls strictly between  $y$  and  $z$ .  $y$  may be less than, equal, or greater than  $z$ . Thus  
**between 3 5 7** is **False**,  
**between 3 5 2** is **True**,  
and **between 3 2 3** is **False**.
- (b) (6 points) Define a function which has a non-empty list of numbers as argument. Its output is **False** iff all numbers are identical.
- (c) (6 points) Define function **zip** which takes a pair of lists of equal lengths as argument and returns a list of pairs of corresponding elements. So,

$$\text{zip} ([1,2,3], ['a','b','c']) = [(1,'a'), (2,'b'), (3,'c')]$$

- (d) (6 points) Define function **unzip** which is the inverse of **zip**:

$$\text{unzip} (\text{zip} (xs,ys)) = (xs,ys)$$