| **CS 337** | **Quiz 3** | **12/10/03** |
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Open book and notes.

| Max points = 50 | Time = 50 min | Do all questions. |
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1. (String Matching; 32 points)

   (a) (Rabin-Karp algorithm; 8 points) In the Rabin-Karp algorithm, suppose we are searching the text for 2 digit strings. Suppose the hash function for a 2 digit number $n$ is $val(n) = n \bmod 7$. So, we have

   | | | | | | |
   | --- | --- | --- | --- | --- | --- |
   | *text* | 2 | 4 | 1 | 5 | 7 |
   | *val(n)* | | 3 | 6 | 1 | 1 |

   Construct a string of 8 digits, with as many different digits as possible, in which $val(n) = 5$ for every 2-digit contiguous substring. Avoid 3 3 3 3 3 3 3 3.

   (b) (8 points) Let $(ab)^n a$, for $n \geq 1$, denote the string obtained by $n$-fold repetition of $ab$ followed by $a$. Thus, $(ab)^2 a$ is the string $ababa$. Show that $c^i((ab)^n a)$, where $1 \leq i \leq n$, is $(ab)^{n-i} a$.

   (c) (6 points) The algorithm for core computation includes the following code fragment; see notes on "String Matching", Page 10.

   **if** $p[\overline{u}] = p[\overline{v}]$
     **then** $c(v') := u'$
     **else** $c(v') := \epsilon$
   **endif** ;

   Is it possible that $u = \epsilon$ and $c(v') \neq \epsilon$ after execution of this portion of the program? Show a small example to support your claim.

   (d) (KMP Algorithm; 10 points) Apply the KMP algorithm on the following pattern and text. Show only the different values of $l$ (see page 6 of the notes on "String Matching").

   | *index* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
   | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
   | *text* | a | b | a | b | a | a | b | b | a | b | a | b |
   | *pattern* | a | b | a | b | a | b | | | | | | |

2. (Data Parallel Programming; 18 points)

   (a) (8 points) Consider the function $f$ on integers that returns a powerlist, defined as follows.

   $$\begin{aligned} f\ 0 &= \langle 0\ 1 \rangle \\ f(n+1) &= (f\ n) \mid rev(f\ n) \end{aligned}$$

Prove that for $n \geq 1$, $f(n+1) = (f\ n) \mid (f\ n)$. Use the following definition of $rev$.

$$\begin{aligned} rev\langle x\rangle &= \langle x\rangle \\ rev(p \mid q) &= (rev\ q) \mid (rev\ p) \end{aligned}$$

(b) (10 points) Let $u$ and $v$ be powerlists of equal length whose elements are $n$-bit binary strings ($n$ is the same for both $u$ and $v$). Suppose $u \sim v$ denotes that the corresponding elements of $u$ and $v$ are apart by Hamming distance 1. For example, with $n = 2$, let

$$\begin{aligned} x &= \ 00\ \ 01\ \ 11\ \ 10 \\ y &= \ 10\ \ 11\ \ 01\ \ 00 \\ z &= \ 11\ \ 10\ \ 00\ \ 01 \end{aligned}$$

Then, $x \sim y$ , $y \sim z$, but $x \not\sim z$.

i. (2 points) Simplify $(u \bowtie v) \sim (r \bowtie s)$. No proof needed.

ii. (4 points) Define a boolean function $gr$ where $(gr\ u)$ holds iff each pair of adjacent elements of $u$ (counting wrap-around) are apart by Hamming distance 1. So, $(gr\ x)$, $(gr\ y)$ and $(gr\ z)$ hold. You may use any permutation function defined in the notes.

iii. (4 points) Using your definition of $gr$, express $gr(u \bowtie v)$ in simpler terms.