

Open book and notes.

Max points = 50

Time = 50 min

Do all questions.

1. (Rabin-Karp algorithm for string search; 8 points)
 - (a) (4 points) In the Rabin-Karp algorithm is q required to be prime? Suppose it is not, what is the advantage of having both d (the number of symbols in the alphabet) and q as powers of 2?
 - (b) (4 points) We have one very long text string (like the collected works of Shakespeare). We expect to receive many requests to search it for different patterns. How can we apply the Rabin-Karp algorithm effectively? Assume that each pattern length will be at least 20. Explain your algorithm in no more than 5 sentences. There is no unique correct answer.
2. (KMP algorithm for string search; 17 points)
 - (a) (6 points) For strings u and v , write $u \sqsubseteq v$ to denote that u is a prefix of v and $u \leq v$ to denote that u is a suffix of v . Assume that both these relations are partial orders. Prove that \preceq is a partial order.
 - (b) (3 points) Let $v = \text{"ababab"}$. Show a string u such that $c(v) \preceq u \prec v$.
 - (c) (8 points) Consider the following fragment from the program given in Page 166 of your notes.

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while  $p[\bar{u}] \neq p[\bar{v}] \wedge u \neq \epsilon$  do
   $\{u = c^i(v), \text{ for some } i, i > 0\}$ 
   $u := c(u)$ 
endwhile ;
 $\{u = c^i(v), \text{ for some } i, i > 0 \text{ and } (p[\bar{u}] = p[\bar{v}] \vee u = \epsilon)\}$ 

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Is it ever possible that *both* $p[\bar{u}] = p[\bar{v}]$ and $u = \epsilon$ hold on termination of the loop? If no, say why; if yes, show an example.

3. (Relational Algebra; 10 points)
 - (a) (5 points) Consider the relations in Tables 6.1 (page 144), 6.5 (page 147) and 6.4 (page 146) of your notes. Call these relations R , S and T , respectively. Write a query to find the names of theatres which are showing G-rated movies in which Audrey Hepburn is acting. You don't have to simplify or compute the value of the query.
 - (b) (5 points) Show that for relations R , S , and attribute a , $\pi_a(R \cap S) = \pi_a(R) \cap \pi_a(S)$ does not necessarily hold.
4. (Powerlist; 15 points)
 - (a) (6 points) Define function *swap* on powerlists which transposes adjacent elements of the list, as shown below.

$$\begin{aligned} \text{swap}\langle x \rangle &= \langle x \rangle \\ \text{swap}\langle 1\ 2\ 3\ 4 \rangle &= \langle 2\ 1\ 4\ 3 \rangle \end{aligned}$$

- (b) (9 points) Use the following definitions of left rotate (lr), right rotate (rr) and reverse (rev).

$$\begin{aligned} lr\langle x \rangle &= \langle x \rangle & lr(p \bowtie q) &= lr(q) \bowtie p \\ rr\langle x \rangle &= \langle x \rangle & rr(p \bowtie q) &= q \bowtie rr(p) \\ rev\langle x \rangle &= \langle x \rangle & rev(p \bowtie q) &= rev(q) \bowtie rev(p) \end{aligned}$$

Prove that for any powerlist u , $rev(rr(u)) = lr(rev(u))$.