1. (Rabin-Karp algorithm for string search; 8 points)
   (a) (4 points) In the Rabin-Karp algorithm is \( q \) required to be prime? Suppose it is not, what is the advantage of having both \( d \) (the number of symbols in the alphabet) and \( q \) as powers of 2?
   (b) (4 points) We have one very long text string (like the collected works of Shakespeare). We expect to receive many requests to search it for different patterns. How can we apply the Rabin-Karp algorithm effectively? Assume that each pattern length will be at least 20. Explain your algorithm in no more than 5 sentences. There is no unique correct answer.

2. (KMP algorithm for string search; 17 points)
   (a) (6 points) For strings \( u \) and \( v \), write \( u \subseteq v \) to denote that \( u \) is a prefix of \( v \) and \( u \preceq v \) to denote that \( u \) is a suffix of \( v \). Assume that both these relations are partial orders. Prove that \( \preceq \) is a partial order.
   (b) (3 points) Let \( v = \text{“ababab”} \). Show a string \( u \) such that \( c(v) \preceq u < v \).
   (c) (8 points) Consider the following fragment from the program given in Page 166 of your notes.

     ```
     while \( p[u] \neq p[v] \land u \neq \epsilon \) do
     \{u = c^i(v), for some \( i, i > 0 \) \}
     u := c(u)
     endwhile ;
     \{u = c^i(v), for some \( i, i > 0 \) and \( (p[u] = p[v] \lor u = \epsilon) \) \}
     ```

     Is it ever possible that both \( p[u] = p[v] \) and \( u = \epsilon \) hold on termination of the loop? If no, say why; if yes, show an example.

3. (Relational Algebra; 10 points)
   (a) (5 points) Consider the relations in Tables 6.1 (page 144), 6.5 (page 147) and 6.4 (page 146) of your notes. Call these relations \( R \), \( S \) and \( T \), respectively. Write a query to find the names of theatres which are showing G-rated movies in which Audrey Hepburn is acting. You don’t have to simplify or compute the value of the query.
   (b) (5 points) Show that for relations \( R, S \), and attribute \( a \), \( \pi_a(R \cap S) = \pi_a(R) \cap \pi_a(S) \) does not necessarily hold.

4. (Powerlist; 15 points)
   (a) (6 points) Define function \( \text{swap} \) on powerlists which transposes adjacent elements of the list, as shown below.
\[ \text{swap}(x) = (x) \]
\[ \text{swap}(1 \ 2 \ 3 \ 4) = (2 \ 1 \ 4 \ 3) \]

(b) (9 points) Use the following definitions of left rotate (\( \text{lr} \)), right rotate (\( \text{rr} \)) and reverse (\( \text{rev} \)).

\[
\begin{align*}
\text{lr}(x) & = (x) \quad & \text{lr}(p \bowtie q) & = \text{lr}(q) \bowtie p \\
\text{rr}(x) & = (x) \quad & \text{rr}(p \bowtie q) & = q \bowtie \text{rr}(p) \\
\text{rev}(x) & = (x) \quad & \text{rev}(p \bowtie q) & = \text{rev}(q) \bowtie \text{rev}(p)
\end{align*}
\]

Prove that for any powerlist \( u \), \( \text{rev}(\text{rr}(u)) = \text{lr}(\text{rev}(u)) \).