**CS 337** **Solution to Quiz 2** **11/3/04**

1. (Recursive Programming; 28 points)

   (a)
```
tie ([], [])        = []
tie (x:xs) (y: ys) = (x,y): (tie (xs,ys))

untie []        = ([], [])
untie (x,y): zs = ((x:xs), (y: ys))
                  where (xs,ys) = untie zs
```

   (b) Define function `gpos` of two arguments $k$ and $n$, both integers and $n \geq 0$, where `(gpos k n)` returns a list of `n` consecutive integers starting at `k`. Then,
```
pos n = gpos 1 n
```
   The definition of `gpos` is given by
```
gpos k 0 = []
gpos k (n+1) = k: (gpos (k+1) n)
```

   (c)
```
check f g zs = glt (f zs) (g zs)
glt xs [] = True
glt [] ys = False
glt (x: xs) (y: ys) = (x >= y) && (glt xs ys)
```

2. (Finite State Machines)

   (a) The following machine accepts strings with even number of zeroes.

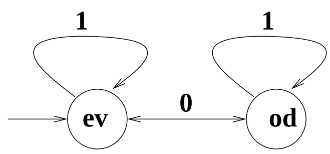


Figure 1: Machine that accepts strings with even number of zeroes

The following machine accepts strings ending in "11".

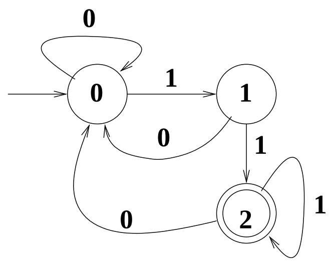


Figure 2: Machine that accepts strings ending in "11"

Combine the two machines.

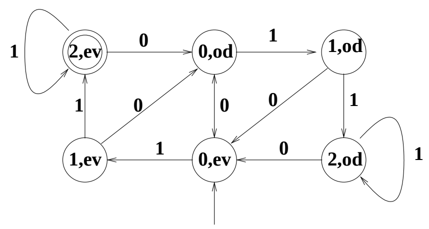


Figure 3: Combination of the two machines given above

(b) Define the following predicates.

$emp(x)$ : string $x$ is empty
$p0(x)$ : last symbol of string $x$ is 0
$p1(x)$ : last symbol of string $x$ is 1
$eq(x)$ : number of zero blocks in $x$ equals the number of one blocks
$lt(x)$ : number of zero blocks in $x$ is 1 less than the number of one blocks
$gt(x)$ : number of zero blocks in $x$ is 1 more than the number of one blocks

Annotate as follows:

A: $emp \wedge eq$
B: $p0 \wedge gt$
C: $p1 \wedge lt$
D: $p1 \wedge eq$
E: $p0 \wedge eq$

Proof for transition:

C to E: $p1(x) \wedge lt(x) \;\Rightarrow\; p0(x0) \wedge eq(x0)$
E to E: $p0(x) \wedge eq(x) \;\Rightarrow\; p0(x0) \wedge eq(x0)$
E to C: $p0(x) \wedge eq(x) \;\Rightarrow\; p1(x1) \wedge lt(x1)$

(c) $0(0|1)^*1$