CS 337 **Quiz 1** **2/14/03**

Open book and notes.

Max points = 50 Time = 50 min Do all questions.

1. (Game of Nim; 10 points)
   In a winning state in the game of nim let u be the exclusive-or of all the piles. There is a pile with y chips; the binary expansion of y has a 0 in the bit position where u has its leading 1 bit; see figure below.

$$\begin{array}{ccccc} u & = & \text{0s} & 1 & \alpha \\ y & = & \beta & 0 & \gamma \end{array}$$

   Suppose replacing pile y by pile x results in a losing state. Show that $x > y$.

   Hint: $p < q \;\equiv\; q$ has 1 in the leading bit position of $p \oplus q$

2. (Error Correction; 6 points)
   A consequence of the theorem on error detection/correction is that parity check code can detect at most one error, but correct none. In the RAID architecture, we are essentially using parity encoding (the extra disk keeps the check bit for each row). And, we are able to correct the failure of one disk. Is the theorem wrong? Answer in less than 4 sentences.

3. (Hadamard Matrix; 8 points)
   Show that each row of a Hadamard Matrix H_n, $n \geq 2$, has even parity.

4. (Finite State Machine; 26 points)

   (a) (8 points) Design a finite state machine whose input alphabet is the roman alphabet, A; it accepts any string which contains the substring "seuss" (a substring is a consecutive sequence of characters).

   (b) (8 points) Design a finite state machine whose input alphabet is $\{0, 1, 2, 3\}$. Accept a non-empty string of digits iff they are strictly increasing. So, 123, 023, 2 will be accepted, whereas 32 will be rejected.

   (c) (10 points) Design a finite state machine to accept a binary string which does not contain three consecutive 0s. So, 0010011110101 will be accepted whereas 010010001 will be rejected.

   Annotate the states with appropriate predicates for a correctness proof; but do not prove correctness.