Open book and notes.
Max points = 50 Time = 50 min Do all questions.

1. (Programming, 32 points)

(a) Define function `min2` which accepts a list of two or more distinct integers as input and outputs the second smallest number in the list. Thus,

\[
\begin{align*}
\text{min2 } [7,3] &= 7 \\
\text{min2 } [7,3,5,1] &= 3
\end{align*}
\]

(b) Define function `apply` which has two inputs, a list of functions and a list of arguments, where each function can be applied to each argument. The output of `apply` is a list of lists, obtained by applying each function, in turn, to each argument. Shown below is the result of running `apply`, where `same` is the identity function, `sq`, `cube` and `quad` are the functions which compute the second, third and fourth power of the argument.

```
Main> apply [same, sq, cube, quad] [2,3,4]
[[2,3,4],[4,9,16],[8,27,64],[16,81,256]]
```

(c) Define a function which returns `True` if its argument string includes `''010''` as a substring (i.e., a contiguous segment), `False` otherwise.

(d) Function `flatten`, defined in your notes (section 10.1.3, page 34), takes a list of lists, like

```
[ [1,2,3], [10,20], [], [30] ]
```

and flattens it out by putting all the elements into a single list, like

```
[1,2,3,10,20,30]
```

Write a more efficient version, without using `+`.

Hint: Consider how you would do it manually.

2. (Types; 8 points) What are the types of the following expressions?

(a) `(3, "abc", ['a', 'b', 'c'])`

(b) Function `apply` of question (1b).

(c) A function which accepts two strings, `xs` and `ys` (of arbitrary elements), and returns a list of positions in `ys` such that `xs` occurs as a substring starting at each of these positions.

(d) `filter even [3,6,7]`. 

3. (Proofs; 10 points) Consider the prefix sum algorithm given in your notes (just before section 9.5, page 32). A function defined there is $pt$, which is reproduced below.

$$
pt \; [] \; c = [] \\
pt \; (x:xs) \; c = (c+x) \; : \; (pt \; xs \; (c+x))
$$

Consider the following function, $pd$.

$$
pd \; [] \; c = [] \\
pd \; (x:xs) \; c = (x-c) \; : \; (pd \; xs \; x)
$$

Show that

$$
pd \; (pt \; xs \; c) \; c = xs
$$

for any list of integers $xs$ and any integer $c$. 