1. (Programming)

(a) Function \texttt{minn} returns the smallest and the second smallest values as a pair, and \texttt{min2} extracts the second value.

\[
\text{min2 } xs = \text{snd (minn } xs) \\
\text{minn } [x,y] = ((\text{min } x \ y), (\text{max } x \ y)) \\
\text{minn } (x:xs) \\
| \ x < m = (x, m) \\
| \ x < n = (m, x) \\
| \ True = (m, n) \\
\text{where } (m, n) = \text{minn } xs
\]

(b) \texttt{apply } [] xs = [] \\
\texttt{apply } (f:fs) xs = (\text{map } f \ xs) : (\text{apply } fs \ xs)

(c) Use three functions: \texttt{f} is the main one, \texttt{f0} is called after detecting the string "0", and \texttt{f1} after detecting "01". Useful to draw a finite state machine diagram.

\[
f [] = \text{False} \\
f('0':xs) = (f0 \ xs) \\
f('1':xs) = (f \ xs) \\
\text{where} \\
f0 [] = \text{False} \\
f0('0':xs) = (f0 \ xs) \\
f0('1':xs) = (f1 \ xs)
\]

\[
f1 [] = \text{False} \\
f1('0':xs) = \text{True} \\
f1('1':xs) = (f \ xs)
\]

(d) \texttt{flatten } [] = [] \\
\texttt{flatten } ([]: \text{xss}) = \text{flatten } \text{xss} \\
\texttt{flatten } ([x:xs]:\text{xss}) = x : \text{flatten } (xs: \text{xss})

2. (Types)

(a) \(3,\"abc\",[\'a\',\'b\',\'c\']\) :: Num a => (a,[Char],[Char])

(b) \texttt{apply } :: [a -> b] -> [a] -> [[b]]

(c) \texttt{search } :: [a] -> [a] -> [Int]

(d) \texttt{filter even } [3,6,7] :: Integral a => [a]
3. (Proofs) We show that

\[ \text{pd (pt xs c) c} = \text{xs} \]

for any list of integers \( \text{xs} \) and integer \( \text{c} \), by induction on \( \text{xs} \).

- Base case: Show that \( \text{pd (pt [] c) c} = [] \).

\[
\begin{align*}
\text{pd (pt [] c) c} &= \{ \text{definition of pt} \} \\
\text{pd [] c} &= \{ \text{definition of pd} \} \\
[]
\end{align*}
\]

- Inductive case: Show that \( \text{pd (pt (x:xs) c) c} = (x:xs) \), given that \( \text{pd (pt xs c) c} = \text{xs} \), for any \( \text{c} \). To simplify notation, we use \( \text{d} \) as an abbreviation for \( \text{c+x} \).

\[
\begin{align*}
\text{pd (pt (x:xs) c) c} &= \{ \text{definition of pt; also d = c+x} \} \\
\text{pd (d:(pt xs d)) c} &= \{ \text{definition of pd} \} \\
(d-c) : (\text{pd (pt xs d) d}) &= \{ \text{induction hypothesis applied to (pd (pt xs d) d)} \} \\
(d-c) : \text{xs} &= \{ \text{simplify first term: d-c = c+x-c = x} \} \\
\text{x:xs}
\end{align*}
\]