CS 337 **Solution to Quiz 2** **3/28/03**

1. (Programming)

   (a) Function `minn` returns the smallest and the second smallest values as a pair, and `min2` extracts the second value.

   ```
   min2 xs = snd (minn xs)
   minn [x,y] = ((min x y), (max x y))
   minn (x:xs)
    | x < m = (x,m)
    | x < n = (m,x)
    | True  = (m,n)
               where (m,n) = minn xs
   ```

   (b)
   ```
   apply [] xs = []
   apply (f:fs) xs = (map f xs) : (apply fs xs)
   ```

   (c) Use three functions: `f` is the main one, `f0` is called after detecting the string `"0"`, and `f1` after detecting `"01"`. Useful to draw a finite state machine diagram.

   ```
   f []      = False
   f('0':xs) = (f0 xs)
   f('1':xs) = (f xs)
    where
    f0 []      = False
    f0('0':xs) = (f0 xs)
    f0('1':xs) = (f1 xs)

    f1 []      = False
    f1('0':xs) = True
    f1('1':xs) = (f xs)
   ```

   (d)
   ```
   flatten [] = []
   flatten ([]: xss) = flatten xss
   flatten ((x:xs):xss) = x: flatten(xs:xss)
   ```

2. (Types)

   (a) `(3,"abc",['a','b','c']) ::  Num a => (a,[Char],[Char])`

   (b) `apply ::  [a -> b] -> [a] -> [[b]]`

   (c) `search ::  [a] -> [a] -> [Int]`

   (d) `filter even [3,6,7] ::  Integral a => [a]`

3. (Proofs) We show that

```
pd (pt xs c) c = xs
```

for any list of integers `xs` and integer `c`, by induction on `xs`.

- Base case: Show that `pd (pt [] c) c = []`.

```
     pd (pt [] c) c
=  {definition of pt}
     pd [] c
=  {definition of pd}
     []
```

- Inductive case: Show that `(pd (pt (x:xs) c) c) = (x:xs)`, given that `(pd (pt xs c) c) = xs`, for any `c`. To simplify notation, we use `d` as an abbreviation for `(c+x)`.

```
     pd (pt (x:xs) c) c
=  {definition of pt; also d = c+x}
     pd (d:(pt xs d)) c
=  {definition of pd}
     (d-c) :  (pd (pt xs d) d)
=  {induction hypothesis applied to (pd (pt xs d) d)}
     (d-c) :  xs
=  {simplify first term: d-c = c+x-c = x}
     x:xs
```