CS 337	**Solution to Test 3**	**5/5/06**

Open book and notes.

Max points = 50 Time = 50 min Do all questions.

1. Relational Algebra

   (a) The natural join of CT and CR, $CT \bowtie CR$, is shown in Table 1.

Course	Day	Hour	Room
Phy313K	T	9AM	Phi 1.021
Phy313K	Th	9AM	Phi 1.021
CS380D	F	9AM	Wel 2.304
CS337	M	2PM	Tay 2.106
CS337	W	2PM	Tay 2.106
CS337	F	2PM	Tay 2.106

Table 1: $CT \bowtie CR$

   (b) The Day-Hour pairs during which Tay 2.106 is occupied is computed by

$$\pi_{Day,Hour}(\sigma_{Room=\text{``}Tay2.106''}(CT \bowtie CR))$$

   (c) The Courses which meet on Friday(F) in Tay 2.106 is computed by

$$\pi_{Course}(\sigma_{Day=\text{``}F'' \wedge Room=\text{``}Tay2.106''}(CT \bowtie CR))$$

2. (String Matching)

   (a) One possible s is $(01)^{2n}$.

   (b)

$$
\begin{array}{ll}
 & pal(v) \;\wedge\; u \preceq v \\
\equiv & \{\text{expand definitions of } pal \text{ and } \preceq\} \\
 & (v = rev(v)) \wedge (u \sqsubseteq v \;\wedge\; rev(u) \sqsubseteq rev(v)) \\
\Rightarrow & \{\text{replace } rev(v) \text{ by } v\} \\
 & u \sqsubseteq v \;\wedge\; rev(u) \sqsubseteq v \\
\equiv & \{u \text{ and } rev(u) \text{ have the same length. They are both prefixes of } v\} \\
 & u = rev(u) \\
\equiv & \{\text{definition of } pal\} \\
 & pal(u)
\end{array}
$$

   (c) First, we show $c(u) \prec u$. In $x \preceq c(y) \;\equiv\; x \prec y$, substitute $c(u)$ for x and u for y to get $c(u) \preceq c(u) \;\equiv\; c(u) \prec u$. That is, $c(u) \prec u$. Next,

$$
\begin{array}{ll}
 & u \preceq v \\
\Rightarrow & \{\text{As shown, } c(u) \prec u\} \\
 & c(u) \prec v \\
\equiv & \{\text{in } x \preceq c(y) \;\equiv\; x \prec y, \text{ substitute } c(u) \text{ for } x \text{ and } v \text{ for } y \text{ to get} \\
 & \quad c(u) \preceq c(v) \;\equiv\; c(u) \prec v\} \\
 & c(u) \preceq c(v)
\end{array}
$$

(a)
$$f\langle x\ y\rangle\langle 0\rangle \;=\; \langle x\ y\rangle$$
$$f\langle x\ y\rangle\langle 1\rangle \;=\; \langle y\ x\rangle$$
$$f(u \mid v)(r \mid s) \;=\; (f\ u\ r) \mid (f\ v\ s)$$

(b) We show $rev(p \bowtie q) = (rev\ \ q) \bowtie (rev\ \ p)$. Proof is by structural induction on p and q.

- Base case:

$$
\begin{array}{ll}
 & rev(\langle x\rangle \bowtie \langle y\rangle) \\
= & \{\text{From Law L0}: \langle x\rangle \bowtie \langle y\rangle = \langle x\rangle \mid \langle y\rangle\} \\
 & rev(\langle x\rangle \mid \langle y\rangle) \\
= & \{\text{definition of } rev\} \\
 & rev\langle y\rangle \mid rev\langle x\rangle \\
= & \{rev\langle x\rangle = \langle x\rangle, rev\langle y\rangle = \langle y\rangle.\ \text{Thus, they are singletons. Apply Law L0}\} \\
 & rev\langle y\rangle \bowtie rev\langle x\rangle
\end{array}
$$

- Inductive case: Let $p = r \mid s$ and $q = u \mid v$

$$
\begin{array}{ll}
 & rev(p \bowtie q) \\
= & \{p = r \mid s \text{ and } q = u \mid v\} \\
 & rev((r \mid s) \bowtie (u \mid v)) \\
= & \{\text{commutativity of } \mid,\ \bowtie\} \\
 & rev((r \bowtie u) \mid (s \bowtie v)) \\
= & \{\text{definition of } rev\} \\
 & rev(s \bowtie v) \mid rev(r \bowtie u) \\
= & \{\text{induction}\} \\
 & (rev\ v \bowtie rev\ s) \mid (rev\ u \bowtie rev\ r) \\
= & \{\mid,\ \bowtie\ \text{commute}\} \\
 & (rev\ v \mid rev\ u) \bowtie (rev\ s \mid rev\ r) \\
= & \{\text{apply definition of } rev \text{ to both sides of } \bowtie\} \\
 & rev(u \mid v) \bowtie rev(r \mid s) \\
= & \{p = r \mid s \text{ and } q = u \mid v\} \\
 & (rev\ q) \bowtie (rev\ p)
\end{array}
$$