1. Relational Algebra

(a) The natural join of $CT$ and $CR$, $CT \bowtie CR$, is shown in Table 1.

<table>
<thead>
<tr>
<th>Course</th>
<th>Day</th>
<th>Hour</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phy313K</td>
<td>T</td>
<td>9AM</td>
<td>Phi 1.021</td>
</tr>
<tr>
<td>Phy313K</td>
<td>Th</td>
<td>9AM</td>
<td>Phi 1.021</td>
</tr>
<tr>
<td>CS380D</td>
<td>F</td>
<td>9AM</td>
<td>Wel 2.304</td>
</tr>
<tr>
<td>CS337</td>
<td>M</td>
<td>2PM</td>
<td>Tay 2.106</td>
</tr>
<tr>
<td>CS337</td>
<td>W</td>
<td>2PM</td>
<td>Tay 2.106</td>
</tr>
<tr>
<td>CS337</td>
<td>F</td>
<td>2PM</td>
<td>Tay 2.106</td>
</tr>
</tbody>
</table>

Table 1: $CT \bowtie CR$

(b) The Day-Hour pairs during which Tay 2.106 is occupied is computed by

$$\pi_{Day,Hour}(\sigma_{Room=“Tay2.106”}(CT \bowtie CR))$$

(c) The Courses which meet on Friday(F) in Tay 2.106 is computed by

$$\pi_{Course}(\sigma_{Day=“F”\land Room=“Tay2.106”}(CT \bowtie CR))$$

2. (String Matching)

(a) One possible $s$ is $(01)^2n$.

(b) $pal(v) \land u \preceq v$

$$\equiv \{\text{expand definitions of } pal \text{ and } \preceq\}$$

$$\text{(v = rev(v)) } \land (u \sqsubseteq v \land rev(u) \sqsubseteq rev(v))$$

$$\Rightarrow \{\text{replace } rev(v) \text{ by } v\}$$

$$u \sqsubseteq v \land rev(u) \sqsubseteq v$$

$$\equiv \{u \text{ and } rev(u) \text{ have the same length. They are both prefixes of } v\}$$

$$u = rev(u)$$

$$\equiv \{\text{definition of } pal\}$$

$$pal(u)$$

(c) First, we show $c(u) \prec u$. In $x \preceq c(y) \equiv x \prec y$, substitute $c(u)$ for $x$ and $u$ for $y$ to get $c(u) \preceq c(u) \equiv c(u) \prec u$. That is, $c(u) \prec u$. Next,

$$u \preceq v$$

$$\Rightarrow \{\text{As shown, } c(u) \prec u\}$$

$$c(u) \prec v$$

$$\equiv \{\text{in } x \preceq c(y) \equiv x \prec y, \text{ substitute } c(u) \text{ for } x \text{ and } v \text{ for } y \text{ to get}\}$$

$$c(u) \preceq c(v) \equiv c(u) \prec v$$

$$c(u) \preceq c(v)$$
(a) \[ f(\langle x \ y \rangle | 0) = \langle x \ y \rangle \]
\[ f(\langle x \ y \rangle | 1) = \langle y \ x \rangle \]
\[ f(u \ | \ v)(r \ | \ s) = (f(u \ r) \ | \ (f(v \ s)) \]

(b) We show \( rev(p \otimes q) = (rev \ q) \otimes (rev \ p) \). Proof is by structural induction on \( p \) and \( q \).

- **Base case:**
  \[
  rev(\langle x \otimes y \rangle) = \{ From \ Law \ L0 : \langle x \otimes y \rangle = \langle x \ | \ \langle y \rangle \} \]
  \[
  = \{ definition \ of \ rev \} \]
  \[
  rev(\langle x \rangle) | rev(\langle y \rangle) \]
  \[
  = \{ rev(x) = \langle x \rangle, rev(y) = \langle y \rangle \}. \ Thus, \ they \ are \ singletons. \ Apply \ Law \ L0 \}
  \[
  rev(\langle y \rangle) \otimes rev(\langle x \rangle) \]

- **Inductive case:** Let \( p = r \ | \ s \) and \( q = u \ | \ v \)
  \[
  rev(\langle x \rangle \otimes \langle y \rangle) \]
  \[
  = \{ p = r \ | \ s \ \text{and} \ q = u \ | \ v \} \]
  \[
  rev((r \ | \ s) \otimes (u \ | \ v)) \]
  \[
  = \{ \text{commutativity of } | , \otimes \} \]
  \[
  rev((r \otimes u) \ | \ (s \otimes v)) \]
  \[
  = \{ definition \ of \ rev \} \]
  \[
  rev(s \otimes v) | rev(r \otimes u) \]
  \[
  = \{ \text{induction} \}
  \[
  (rev v \otimes rev s) | (rev u \otimes rev r) \]
  \[
  = \{ | , \otimes \ text{ commute} \}
  \[
  (rev v | rev u) \otimes (rev s | rev r) \]
  \[
  = \{ \text{apply definition of rev to both sides of } \otimes \} \]
  \[
  rev(u | v) \otimes rev(r | s) \]
  \[
  = \{ p = r \ | \ s \ \text{and} \ q = u \ | \ v \} \]
  \[
  (rev q) \otimes (rev p) \]