Problem  Prove that if every element of $L$ is zero then $\text{sum} L = 0$. Use the following definition of $\text{sum}$.  

$$\text{sum} [] = 0$$

$$\text{sum} (x:xs) = x + (\text{sum} \ xs)$$
Solution  Proof is by induction on the length of $L$. If $L$ is an empty list then,

\[
\text{sum}_{L} = \begin{cases} 
L = [] & \Rightarrow \text{sum}_{[]} = 0 \\
\text{definition of sum}_{L} 
\end{cases}
\]

If $L$ is non-empty, then it is of the form $(x : xs)$ where $x = 0$ and every element of $xs$ is zero. Then,

\[
\text{sum}_{L} = \begin{cases} 
L = x : xs & \Rightarrow \text{sum}_{(x : xs)} = x + (\text{sum}_{xs}) \\
\text{definition of sum}_{L} \\
x = 0 & \Rightarrow \text{sum}_{xs} \\
\text{induction: the length of xs is smaller than that of L} \\
0 
\end{cases}
\]