

**Problem** Prove that if every element of  $L$  is zero then  $\text{suml } L = 0$ . Use the following definition of  $\text{suml}$ .

```
suml [] = 0
suml (x:xs) = x + (suml xs)
```

**Solution** Proof is by induction on the length of  $L$ . If  $L$  is an empty list then,

$$\begin{aligned}
 & \text{suml } L \\
 = & \{L = []\} \\
 & \quad \text{suml } [] \\
 = & \{\text{definition of suml}\} \\
 & \quad 0
 \end{aligned}$$

If  $L$  is non-empty, then it is of the form  $(x : xs)$  where  $x = 0$  and every element of  $xs$  is zero. Then,

$$\begin{aligned}
 & \text{suml } L \\
 = & \{L = x : xs\} \\
 & \quad \text{suml}(x : xs) \\
 = & \{\text{definition of suml}\} \\
 & \quad x + (\text{suml } xs) \\
 = & \{x = 0\} \\
 & \quad \text{suml } xs \\
 = & \{\text{induction: the length of } xs \text{ is smaller than that of } L\} \\
 & \quad 0
 \end{aligned}$$