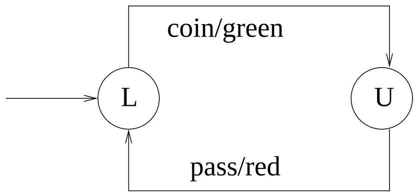
CS 337 **Solution to Test 2** **4/4/07**

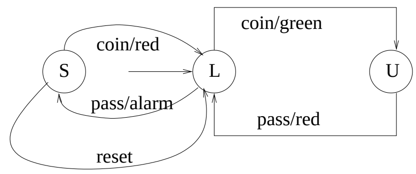
Open book and notes.

Max points = 75 Time = 75 min Do all questions.

1. (Finite State Machine)

   For part(1) the input alphabet is {coin,pass} and the output alphabet is {green,red}; the machine is shown in the left figure. For part (2), the input alphabet is {coin,pass,reset} and the output alphabet is {green,red,alarm}; the machine is shown in the right figure.





2. (Finite State Machine) Define the following predicates over any binary string x:

   p = every 0 is immediately followed by a 1 in x
   q = the last item of x is 0
   and every 0 in x, except the very last, is immediately followed by a 1
   r = there are two consecutive 0s in x

   I have attached the predicate names to the states in Fig 1.

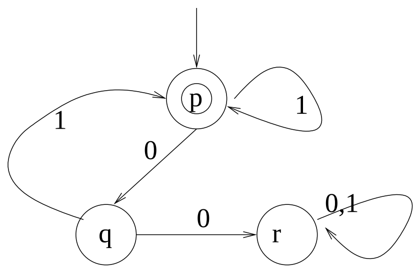


Figure 1: Accept binary strings in which a 0 is followed by a 1

   The theorems that have to be proved, one for each transition, are shown in Table 1. Additionally, p has to be proved for ϵ.

3. (Regular Expressions)

   (a) $0^*\ 1^*\ 2^*$ admits strings, such as 011, which are not strictly increasing.

	p	holds for ϵ		
if	p	holds for x then	q	holds for $x0$
if	p	holds for x then	p	holds for $x1$
if	q	holds for x then	r	holds for $x0$
if	q	holds for x then	p	holds for $x1$
if	r	holds for x then	r	holds for $x0$
if	r	holds for x then	r	holds for $x1$

Table 1: Verifications of state transitions

(b)
$$\begin{aligned} zero &= \epsilon|0 \\ one &= \epsilon|1 \\ two &= \epsilon|2 \end{aligned}$$

Observe that in an increasing string each symbol appears either once or none at all. Therefore, the desired expression is *zero one two*

4. (Types)

   (a) `charVal :: Int -> Char`

   (b) `parallel :: ((Int, Int),(Int, Int)) -> ((Int, Int),(Int, Int)) -> Bool`

   (c) `test :: (a -> Bool) -> a -> Bool`

   (d) `tower :: Int -> b -> b -> b -> [(Int,b,b)]`

   (e) `flatten :: [[a]] -> [a]`

5. (Haskell Programming)

   (a) The function takes a list of pairs as input and produces a pair of lists. Function *transpose*, given below, takes the first element of each input pair to form the first output list and the second elements to form the second output list.

```
transpose [] = ([],[])
transpose ((a,b): xs) = ((a: ys), (b: zs))
                  where (ys, zs) = try xs
```

   (b) Define `fib3` in terms of another function `fibtriple`.

```
fibtriple 0 = (0,1,2)
fibtriple n = (y, z, x+y+z)
                where (x,y,z) = fibtriple (n-1)
fib3 n = x  where (x,y,z) = fibtriple n
```

   (c)

```
close (x:(y:[])) = x-y
close (x:(y:ys))
 | x-y < close(y:ys) = x-y
 | otherwise = close(y:ys)
```