

Problem Consider a game of Nim in which there are 3 piles with 11, 8 and 6 chips. What should be the first move of the first player? Does the first player have a winning strategy?

Solution The exclusive-or of the pile contents is computed as follows.

$$\begin{array}{rcl} & 1\ 0\ 1\ 1 & (=11) \\ \oplus & & \\ & 1\ 0\ 0\ 0 & (=8) \\ \oplus & & \\ & 0\ 1\ 1\ 0 & (=6) \\ = & 0\ 1\ 0\ 1 & (=u) \end{array}$$

Since u is non-zero, the first player has a winning strategy. The only pile that has a 1 in the leading bit position of u is the one with 6 chips. It will be replaced by $0110 \oplus u = 0110 \oplus 0101 = 0011 = 3$ chips. That is, the the only winning move by the first player is to remove 3 chips from the pile with 6 chips.