Exercises on Program Verification, Jayadev Misra

1. (3/3/98) Given a family of sets $in_j, 0 \leq j < N$ and $out_j, 0 \leq j < N$. Set a variable s to *true* iff

   $(\forall i, j : 0 \leq i < N \wedge 0 \leq j < N \wedge i \neq j : in_i \subseteq out_j)$.

   On first glance, a quadratic solution is obvious. We describe a linear solution below.

   Invariant::
   $cin = (\cup j : 0 \leq j < k : in_j)$,
   $cout = (\cap j : 0 \leq j < k : out_j)$,
   $s = (\forall i, j : 0 \leq i < k \wedge 0 \leq j < k \wedge i \neq j : in_i \subseteq out_j)$

   Program::
   $cin := \{\}$; $cout :=$ALL; $k := 0$; $s := true$;
   while $k \neq N$ do
   $s := s \wedge (in_k \subseteq cout) \wedge (cin \subseteq out_k)$;
   $cin := cin \cup in_k$; $cout := cout \cap out_k$;
   $k := k + 1$
   od

   Note: It may be simpler to use integer arrays X, Y and compute the predicate $s = (\forall i, j : 0 \leq i < N \wedge 0 \leq j < N \wedge i \neq j : X_i \leq Y_j)$. Then $\cup, \cap$ and can be replaced by min, max.

2. (3/3/98) Take Dijkstra's integer square root problem and solve it using binary search. From $s^2 \leq k \wedge (s+1)^2 > k$, use the invariant, $s^2 \leq k \wedge t^2 > k$ with the termination condition $t = s + 1$.