Distributed Execution

William Cook and Jayadev Misra

Department of Computer Science
University of Texas at Austin

Email: {cook,misra}@cs.utexas.edu
web: http://www.cs.utexas.edu/users/psp

Centralized Execution Model

- semantics (and execution) described for one machine.
- Then,
  - immediate sites respond instantaneously.
  - Rtimer is exact.
- We develop a theory to permit distributed execution.

Distributed Execution Model

- Assign subexpressions to different machines.
- Execution starts on a goal machine.
- A machine requests another machine to start evaluation.
- The calling machine supplies the context and values of variables as they become defined.
- The calling machine orders all tokens to be killed, and asks for ack.
- All messages among these machines are delivered after arbitrary but finite delay.

When can we distribute?

- $Rtimer(1) \gg let(0) \mid Rtimer(2) \gg let(1)$

  Neither branch can be executed on another machine.

- $$\begin{array}{lll} P(c,e) & \underline{\Delta} & c.get >x> Compute(x) >y> e.put(y) \gg P(c,e) \\ N & \underline{\Delta} & P(c,e) \mid P(d,e) \end{array}$$

  $c.get$ and $e.put$ are on different machines.
  $P(c,e)$ and $P(d,e)$ can be evaluated on different machines.

- $Rtimer(1) \gg M \mid Rtimer(2) \gg N$

  The two branches can be executed on different machines.

The theory identifies subexpressions which can be distributed.

Distributed Execution has to be faithful

- A distributed execution is valid if it mimics a centralized execution.
- The sites can not distinguish between the two execution styles.

An example

- Site $keyboard$ responds at arbitrary times, or never.
  Its response is received immediately.

- Site $screen$ responds immediately and displays a message.

$$\cdots keyboard \gg screen \cdots$$

can not be distributed.

Video-game:
A human responds sometime on the keyboard.

Expects to see an echo immediately.

Punctual Site

- A site is punctual if communication delay with it is zero.
- A site is unpunctual if communication delay with it is arbitrary.
- Simplification: Assume all sites are either punctual or unpunctual.
- A punctual site has to be implemented on the caller's machine.
- An unpunctual site may be implemented on a remote machine.

Punctual: $Rtimer$, let, if, $\cdots$, all immediate sites, $\cdots$, $keyboard$, $screen$

Unpunctual: 0, CNN, $PostWeb$, $\cdots$ Generic symbol M

Punctual Expression

- Any delay in starting it or processing its response can be detected.
- Some timing information for site calls or publications is known.
- A punctual expression has to be implemented on the caller's machine.
- An unpunctual expression may be implemented on another machine.
- A punctual expression may have unpunctual subexpressions.

  The subexpression may be implemented on another machine.

Definition of Punctual Expression

A punctual expression is of the form

- $S(x)$: S is punctual,
- $f \mid g$: either f or g is punctual,
- $f > x > g$: both f and g are punctual,
- $f \text{ where } x {:\in}\ g$: both f and g are punctual, or $f \backslash x$ is punctual.

Obtain $f \backslash x$ from f:
Replace all site calls which have parameter x by 0.

Examples: Punctual Expressions

$let(x)$	let is punctual (immediate)
$let(x) > x > Rtimer(x)$	both sites are punctual
$Rtimer(1)$ where $x :\in g$	$f \backslash x = Rtimer(1)$ is punctual
$let(x)$ where $x :\in Rtimer(1)$	$let(x)$ and $Rtimer(1)$ are both punctual
$Rtimer(1) \mid h$ where $x :\in g$	$f \backslash x = Rtimer(1) \mid h \backslash x$ is punctual
$let(x) > x > Rtimer(1)$ where $x :\in Rtimer(1)$	$let(x) > x > Rtimer(1)$ and $Rtimer(1)$ are both punctual

Examples: Unpunctual Expressions

$M \gg Rtimer(1)$

$Rtimer(1) \gg M$

$N(x) \; {>}x{>} \; Rtimer(x)$

$Rtimer(x)$ where $x{:}{\in}\; M$

$N(x)$ where $x{:}{\in}\; Rtimer(1)$

$let(x)$ where $x{:}{\in}\; M$

punctual within unpunctual, and vice versa

$keyboard \gg screen \mid JoyStick \gg (M \mid N)$, where

$keyboard$, $screen$, $JoyStick$ are punctual
M and N are unpunctual.

$$\underbrace{\underbrace{keyboard \gg screen}_{punctual} \mid \underbrace{\underbrace{JoyStick}_{punctual} \gg \underbrace{(M \mid N)}_{unpunctual}}_{unpunctual}}_{punctual}$$

Example: Distributed Execution

$$\underbrace{keyboard \gg screen}_{punctual} \mid \underbrace{JoyStick \gg (M \mid N)}_{unpunctual}$$

$$\begin{aligned} \equiv \quad & keyboard \gg screen \mid f \\ & f \;\underline{\Delta}\; JoyStick \gg \underbrace{(M \mid N)}_{unpunctual} \end{aligned}$$

$$\begin{aligned} \equiv \quad & keyboard \gg screen \mid f \\ & f \;\underline{\Delta}\; JoyStick \gg g \\ & g \;\underline{\Delta}\; M \mid N \end{aligned}$$

Some properties

- If all sites in an expression are punctual, the expression is punctual.
- If all sites in an expression are unpunctual, the expression is unpunctual.
- Monotonicity: An unpunctual expression remains unpunctual if you replace any site by an unpunctual site.

M punctual, $f(M)$ unpunctual, N unpunctual
$\Rightarrow$ $f(N)$ unpunctual.

$M \sqsubseteq N \Rightarrow f(M) \sqsubseteq f(N)$,
where punctual $\sqsubset$ unpunctual

Proofs by structural induction.

Punctuality is conservative

- $$\begin{array}{lll} & let(\textsf{true}) & \\ >b> & (\ \ if(b) \gg g & \equiv \quad g \\ & \mid if(\neg b) \gg Rtimer(1) & \\ &) & \end{array}$$

  Though the execution can be distributed, the expression is punctual.

- $Random \; >x> \; Rtimer(x)$

  $Random$ publishes a random natural number within some bounds.
  The expression is punctual, and it can not be distributed.

- $Nat \; >x> \; Rtimer(x)$

  Nat publishes any natural number.
  Though the execution can be distributed, the expression is punctual.

Recursively defined Expressions

$Metronome \;\underline{\Delta}\; Signal \mid Rtimer(1) \gg Metronome$, i.e.
$Metronome \;\underline{\Delta}\; if(\textsf{true}) \mid Rtimer(1) \gg Metronome$

$Signal$ is punctual, so the rhs is punctual. $Metronome$ is punctual.

$$Met \;\underline{\Delta}\; Signal \mid \underbrace{M \gg Met}_{unpunctual}$$

Met is punctual. Its subexpression $M \gg Met$ may be distributed.

$$Signal \mid M \gg Met$$

$$\equiv \quad Signal \mid f$$
$$f \;\underline{\Delta}\; M \gg Met$$

Least fixed point

$$f \;\underline{\Delta}\; M \mid Rtimer(1) \gg f$$

Assume f punctual.

$$f \;\underline{\Delta}\; \underbrace{\underbrace{M}_{unpunctual} \mid \underbrace{\underbrace{Rtimer(1)}_{punctual} \gg \underbrace{f}_{punctual}}_{punctual}}_{punctual}$$

Assume f unpunctual.

$$f \;\underline{\Delta}\; \underbrace{\underbrace{M}_{unpunctual} \mid \underbrace{\underbrace{Rtimer(1)}_{punctual} \gg \underbrace{f}_{punctual}}_{unpunctual}}_{unpunctual}$$

The least fixed point is f is punctual. punctual $\sqsubset$ unpunctual.

Existence of least fixed point

Least fixed point exists for any defined expression f.

Assume f is punctual.

- definition of f is punctual: f is punctual.
- definition of f is unpunctual: f is unpunctual.

  Monotonicity: assuming f unpunctual, definition of f is unpunctual.