A Useful Recurrence for Division

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We show a recurrence that is useful for division. To compute $1/y$ where $y = 1 - x$, and $0 \leq x < 1$, we start with the identity

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$$

For finite precision, we compute $1 + x + x^2 + x^3 + .. + x^N$, for some N. Observe that, for all $n \geq 0$

$$\begin{aligned} & 1 + x + x^2 + x^3 + .. + x^{2^{n+1}-1} \\ = & (1+x)(1+x^2)(1+x^4)...(1+x^{2^j})...(1+x^{2^n}) \end{aligned}$$

This result that can be proven by induction on n.

We use the right side of the above identity for computation. At the start of the j^{th} iteration, where $j \geq 0$, variables *prod* and *term* are given by

$$\begin{aligned} & term = x^{2^j} \\ & prod = \prod_{0 \leq i < j} (1 + x^{2^i}) \end{aligned}$$

For $j = 0$, we get $term = x$ and $prod = 1$. The iterative step is:

$$term := term * term \mid\mid prod := prod * (1 + term)$$

This iterative structure is easily implemented in hardware. Each iteration computes *term* and *prod* independently.

Note: Another possibility is to first compute x^{2^j}, for all j, $0 < j < n$, in n steps, each step involving one multiplication. Next, compute $1 + x^{2^j}$, for all j in one step. Then compute the product of these terms in $\log n$ steps.