Dijkstra's Shortest Path as a Simulation Problem

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Dijkstra's Shortest path problem can be seen as a simulation of a physical system, as follows. Let a ray of light start from the source and travel along its outgoing edges; length of an edge is the amount of time taken by the ray to traverse that edge. Whenever a ray is received for the *first time* at a node rays are sent along all of its outgoing edges. (This description is due to Papadimitriou.)

Claim: The shortest path to a node is the time at which the first ray arrives at that node. This is proven by induction on the number of edges on the shortest path.

Simulation: The algorithm is described as concurrent physical processes evolving in time. Let an event be the arrival of the first ray at a node. The processes can be simulated by an event-driven simulator. Let (t, v) denote that at node v the first ray arrives at time t.

initially: the event list contains the pair $(0, source)$, (∞, v), for all other nodes v. All nodes are *unscanned*.

iterate: as long as there is an unscanned node. Remove the earliest event (t, v) from the event-list. Node v is then *scanned*. For each outgoing edge of length c to u where u is unscanned: if the event list contains (t', u) then add $(t + c, u)$ to the event list provided $t + c < t'$.