A Specialization of *detects*

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1 Introduction

For a given program, p *detects* q, for predicates p, q is:

$$p \;\Rightarrow\; q \quad \text{and} \quad q \;\mapsto\; p$$

It is shown in [1] how certain kinds of detection problems—detection of termination, deadlock, stable properties—can be accomplished.

It turns out that a certain special case of *detects*, which we call *trails*, is the concept that arises in almost all situations. To motivate this concept, consider a program in which q is a stable property and p *detects* q. It cannot then be asserted that p is stable: Once p becomes true it need not remain true; the only requirement is that it become true eventually if q holds. In p *trails* q, we require that p *detects* q and, furthermore, once p becomes true it remain true as long as q remains true. Formally, in a given program

$$p \;\textit{trails}\; q \;\equiv\; p \;\Rightarrow\; q \;,\; q \;\mapsto\; p \;,\; p \;\textit{unless}\; \neg q$$

Theorem: For a given program, *trails* is reflexive, antisymmetric and transitive.

Proof: Reflexivity and antisymmetry are straightforward (for antisymmetry, prove that p *trails* q and q *trails* p implies $p \equiv q$). For transitivity suppose

$$p \;\textit{trails}\; q \;,\; q \;\textit{trails}\; r \;.$$

We show p *trails* r

- $p \;\Rightarrow\; r$: From $p \;\Rightarrow\; q$ {from p *trails* q} and
  $q \;\Rightarrow\; r$ {from q *trails* r}.
- $r \;\mapsto\; p$:
  $r \;\mapsto\; q$, from q *trails* r

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$q \mapsto p$, from p *trails* q
$r \mapsto p$, transitivity on the above two

- p *unless* $\neg r$:
  p *unless* $\neg q$, from p *trails* q
  q *unless* $\neg r$, from q *trails* r
  $p \wedge q$ *unless* $\neg r$, conjunction and weakening the rhs
  p *unless* $\neg r$, $p \wedge q \equiv p$ since $p \Rightarrow q$ $\square$

2 References

1. *Parallel Program Design: A Foundation*, K. Mani Chandy and Jayadev Misra, Addison-Wesley, 1988.