ACL2 Code Proofs

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Lecture 3
Today

partial correctness

partial correctness and clocks

Dave Greve’s wormhole abstraction

inductive invariant style proofs
State Based Partial Correctness

\[(\text{pre } s) \land (\text{haltedp (m1 } s \ k))) \rightarrow (\text{post } s (\text{m1 } s \ k))\]
Demo

Clock-functions can be used to prove such theorems.

In this demo, we’ll prove a non-termination result, which is a special case of partial correctness:

\[
\begin{align*}
((\text{pre } s) \\
\land \\
(\text{haltedp} (m1 s k))) \\
\rightarrow \\
(\text{post } s (m1 s k))
\end{align*}
\]
Demo

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\[(\text{pre } s) \land \neg (\text{post } s (\text{m1 } s k)) \rightarrow \neg (\text{haltedp (m1 } s k))\]
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Demo

Clock-functions can be used to prove such theorems.

In this demo, we’ll prove a non-termination result, which is a special case of partial correctness:

\[(\text{pre } s)\]

\[\rightarrow\]

\[\neg (\text{haltedp (m1 } s \ k))\]
Conventional Mechanized Code Proofs

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<td>$\gamma$</td>
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VC1. $P(s) \rightarrow R(f(s)),$
Conventional Mechanized Code Proofs

VC1. $P(s) \rightarrow R(f(s))$

VC2. $R(s) \land t \rightarrow R(g(s))$, and
Conventional Mechanized Code Proofs

VC1. $P(s) \rightarrow R(f(s))$

VC2. $R(s) \land t \rightarrow R(g(s))$, and

VC3. $R(s) \land \neg t \rightarrow Q(h(s))$. 
Conventional Mechanized Code Proofs

Code is annotated with assertions.

A special-purpose tool is used to generate verification conditions (VCs)

This tool, called a verification condition generator (VCG), contains the language semantics, e.g., encoded as Hoare-triples.

Most practical VCGs do a lot of simplification as they build VCs.

A theorem prover is used to prove the VCs.
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We assume the program in $s$, $\pi$, does not change during execution.

We assume there is exactly one \texttt{HALT}, at $\gamma$.

Let $s_0$ be the initial state of program $\pi$.

$$pc(s_0) = \alpha$$

Let $s_k$ denote $m1(s_0, k)$.

Partial Correctness:
$$P(s_0) \land haltedp(s_k) \rightarrow Q(s_k).$$
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Let $s_k$ denote $m1 (s_0, k)$.

Partial Correctness:

$P (s_0) \land pc (s_k) = \gamma \rightarrow Q(s_k)$. 
Theorem: \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

Proof: Define

\[
Inv(s) \equiv \begin{cases} 
P(s) & \text{if } pc(s) = \alpha \\
R(s) & \text{if } pc(s) = \beta \\
Q(s) & \text{if } pc(s) = \gamma \\
Inv(step(s)) & \text{otherwise}
\end{cases}
\]

(Actually, we assert \( \text{prog}(s) = \pi \) at \( \alpha, \beta \) and \( \gamma \), but we omit that here by our convention that the program is always \( \pi \).)
**Theorem:** \( P (s_0) \land pc (s_k) = \gamma \rightarrow Q (s_k) \)

**Proof:** Define

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Inv (\text{step} (s)) & \text{otherwise}
\end{cases}
\]

Objection: Is this definition consistent? Yes: Every tail-recursive definition is witnessed by a total function [Manolios and Moore, 2000]. See ACL2 Community Books misc/defpun and misc/defp.
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\end{cases}$$

Assume we’ve proved

$$Inv(s) \rightarrow Inv(step(s)).$$

(We’ll see the proof in a moment.)
**Theorem:** \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

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\[
Inv(s_0) \rightarrow Inv(s_k) \quad (By \ induction)
\]
**Theorem:** \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

**Proof:** Define

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Inv(s_0) \rightarrow Inv(s_k)
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pc(s_0) = \alpha \quad (\text{By def of } s_0)
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Theorem: \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

Proof: Define

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  Inv(\text{step}(s_0)) & \text{otherwise}
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\[Inv(s_0) \rightarrow Inv(s_k)\]

\[pc(s_0) = \alpha \quad (By \ def \ of \ s_0)\]
**Theorem:** \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

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\( P(s_0) \rightarrow Inv(s_k) \)
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\[P(s_0) \rightarrow Inv(s_k)\]

\[P(s_0) \quad (Given)\]
**Theorem:** \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

**Proof:** Define

\[
Inv(s) \equiv \begin{cases} 
  P(s) & \text{if } pc(s) = \alpha \\
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  Inv(step(s)) & \text{otherwise}
\end{cases}
\]

\( Inv(s_k) \)
Theorem: $P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k)$

Proof: Define

$$
Inv(s) \equiv \begin{cases} 
    P(s) & \text{if } pc(s) = \alpha \\
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    Inv(step(s)) & \text{otherwise}
\end{cases}
$$

$$
Inv(s_k)
$$

$$
pc(s_k) = \gamma \quad (Given)
$$
Theorem: \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

Proof: Define

\[
\text{Inv}(s_k) \equiv \begin{cases} 
P(s_k) & \text{if } pc(s_k) = \alpha \\
R(s_k) & \text{if } pc(s_k) = \beta \\
Q(s_k) & \text{if } pc(s_k) = \gamma \\
\text{Inv(step}(s_k)) & \text{otherwise}
\end{cases}
\]

\[\text{Inv}(s_k)\]

\[pc(s_k) = \gamma \quad (\text{Given})\]
Theorem: \( P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \)

Proof: Define

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  Inv(step(s)) & \text{otherwise}
\end{cases}
\]

\( Q(s_k) \)

Q.E.D.
So it’s trivial to prove the theorem

\[ P(s_0) \land pc(s_k) = \gamma \rightarrow Q(s_k) \]

if we can prove

\[ Inv(s) \rightarrow Inv(step(s)). \]
\[ Inv(s) \equiv \begin{cases} 
  P(s) & \text{if } pc(s) = \alpha \\
  R(s) & \text{if } pc(s) = \beta \\
  Q(s) & \text{if } pc(s) = \gamma \\
  Inv(step(s)) & \text{otherwise} 
\end{cases} \]
$\text{Inv}(s) \rightarrow \text{Inv}(\text{step}(s))$

**Proof.**

Expanding $\text{Inv}(s)$ generates four cases:

- **Case** $\text{pc}(s) = \alpha$:
- **Case** $\text{pc}(s) = \beta$:
- **Case** $\text{pc}(s) = \gamma$:
- **Case** otherwise:
$Inv(s) \equiv \begin{cases} 
  P(s) & \text{if } pc(s) = \alpha \\
  R(s) & \text{if } pc(s) = \beta \\
  Q(s) & \text{if } pc(s) = \gamma \\
  Inv(step(s)) & \text{otherwise}
\end{cases}$

$Inv(s) = Inv(step(s)) = Inv(step(step(s))) \ldots$

as long as the $pc \notin \{\alpha, \beta, \gamma\}$. 
\[ \text{Inv}(s) \rightarrow \text{Inv}(\text{step}(s)) \quad \text{[Case } pc(s) = \alpha]\]
\[ P(s) \rightarrow Inv(step(s)) \]  

[Case \( pc(s) = \alpha \)]

- **labels**
  - \( \alpha \)
  - \( \beta \)
  - \( \gamma \)

- **program \( \pi \)**
  - \( f(s) \)
  - \( g(s) \)
  - \( h(s) \)

- **paths**
  - \( t \)

- **assertions**
  - \( P(s) \) pre-condition
  - \( R(s) \) loop invariant
  - \( Q(s) \) post-condition

\[ \text{RETURN} \]
\[ P(s) \rightarrow R(f(s)) \quad \text{[Case } pc(s) = \alpha \text{]} \]
\[ \text{Inv}(s) \rightarrow \text{Inv}(\text{step}(s)) \quad \text{[Case } pc(s) = \beta] \]
\((R(s) \land t \rightarrow R(g(s))))

\((R(s) \land \neg t \rightarrow Q(h(s))))

[Case \(pc(s) = \beta\)]

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\[ Inv(s) \rightarrow Inv(step(s)) \quad [\text{Case } pc(s) = \gamma] \]
$$Inv(s) \rightarrow Inv(s)$$

[Case $pc(s) = \gamma$]

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\[ \text{Inv}(s) \rightarrow \text{Inv}(\text{step}(s)) \]  

[Case otherwise]

\[
\text{assertions} \\
\begin{array}{ll}
P(s) & \text{pre-condition} \\
R(s) & \text{loop invariant} \\
Q(s) & \text{post-condition}
\end{array}
\]

\[
\text{paths} \\
\alpha \\
\beta \\
\gamma
\]

\[
\text{program } \pi \\
\text{labels}
\]

\[
\begin{array}{c}
f(s) \\
g(s) \\
h(s)
\end{array}
\]

\[
\text{return}
\]

\[
\alpha \\
\beta \\
\gamma
\]

\[
\text{RETURN}
\]
$$Inv(\text{step}(s)) \rightarrow Inv(\text{step}(s)) \text{ [Case otherwise]}$$

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Recap: Given the definition of $\text{Inv}$, the “natural” proof of

$$\text{Inv}(s) \rightarrow \text{Inv}(\text{step}(s))$$

generates the standard verification conditions

VC1. $P(s) \rightarrow R(f(s))$,

VC2. $R(s) \land t \rightarrow R(g(s))$, and

VC3. $R(s) \land \neg t \rightarrow Q(h(s))$

as subgoals from the operational semantics!

It generates no other non-trivial proof obligations.

The VCs are simplified as they are generated.
Demo
Discussion

We did not write a VCG for M1.

The VCs were generated directly from the operational semantics by the theorem prover.

Since VCs are generated by proof, the paths explored and the VCs generated are sensitive to the pre-condition specified.

The VCs are simplified (and possibly proved) by the same process.
We did not count instructions or define a clock function.

We did not constrain the inputs so that the program terminated.

Indeed, we can deal with non-terminating programs.
Demo
Total Correctness via Inductive Assertions

We have also handled total correctness via the VCG approach.

An ordinal measure is provided at each cut point and the VCs establish that it decreases upon each arrival at the cut point.

Clock functions can be automatically generated and admitted from such proofs.

See Sandip Ray’s proofstyles/ books in the Community Books.
Inductive Assertion Tools

See the following Community Books:

symbolic/generic/defsimulate.lisp

workshops/2011/krug-et-al/support/Symbolic/defsimulate+.lisp

See also paper
which describes the defsimulate book.
Citations


Additional Material


and also ACL2 Community Books models/jvm/m5/ for examples of subroutine call/return, heap manipulation, and multi-threading.
Subroutine Call Tips

• define (poised-to-involve-name s)

• if using clocks, define (clk-name s) to count instructions from call through return

• specify correctness to include restoration of (relevant) registers and stack, preservation of the program segment, and advancement of pc to the next instruction
• use (whatever-it-is—... s ...) wormhole abstraction to specify “don’t care” values for those parts of the state that you will never care about

• specify correct answer
Multi-Threading Tip

Try to reduce it to sequential correctness in a slightly chaotic environment
