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Thanks, Ali!
OUTLINE

Overview

ACL2 Introduction

Foundations

Conclusion
Overview

ACL2 Introduction

Foundations

Conclusion
OVERVIEW

Quoting the ACL2 home page:
ACL2 is a logic and programming language in which you can model computer systems, together with a tool to help you prove properties of those models. "ACL2" denotes "A Computational Logic for Applicative Common Lisp".

Goal for this talk:
▶ The focus will be on mechanizing logic for a practical proof assistant.
▶ Boring or not, logical challenges must be addressed! (Note: ACL2 does not generate formal proofs.)
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Let’s start with some context.
OVERVIEW: FORMAL VERIFICATION

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There are also users in the U.S. Government and universities.
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There are also users in the **U.S. Government** and **universities**.

▶ UT Austin: x86 interpreter defined in ACL2, validation by co-simulation, proofs about x86 machine code
OVERVIEW: ITP SYSTEMS

Just a few words about interactive theorem proving...
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Just a few words about interactive theorem proving . . .

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- Many ITP systems (e.g., ACL2) can send sub-problems to automatic proof tools, e.g., SAT solvers for Boolean problems.

REMARK (thanks to J Moore for this): All industrial-scale deduction tools are, in a deep sense, interactive, even the ones that claim to be automatic. The issue is HOW MUCH interaction is required to do interesting things.
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That talk mentions this link to several demos and their logs:

OUTLINE

Overview

ACL2 Introduction

Foundations

Conclusion
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ACL2 Introduction

Foundations

Conclusion
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  - Boyer-Moore Theorem Provers go back to the start of their collaboration in 1971.
ACL2 Demos

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- Interfaces include Emacs, ACL2 Sedan (Eclipse-based), none.
Some ACL2 features *not* discussed further today:

- **Prover algorithms**
  - Waterfall, linear arithmetic, Boolean reasoning, …
  - Rewriting: Conditional, congruence-based, rewrite cache, syntaxp, bind-free, …

- **Using the system effectively**
  - The-method and introduction-to-the-theorem-prover
  - Theories, hints, rule-classes, …
  - Accumulated-persistence, brr, proof-checker, dmr, …

- **Programming support, including (just a few):**
  - Guards
  - Hash-cons and function memoization
  - Packages
  - Mutable State, stobjs, arrays, applicative hash tables, …

- **System-level:** Emacs support, books and certification, abbreviated printing, parallelism (ACL2(p)), …
Overview

ACL2 Introduction

Foundations

Conclusion
Outline

Overview

ACL2 Introduction

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Conclusion
FOUNDATIONS (1)

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But all ACL2 theories extend a given ground-zero theory, which is essentially Peano Arithmetic with $\varepsilon_0$-induction, extended with data types for:

- characters,
- strings,
- symbols,
- complex numbers with rational coefficients, and
- closure under a pairing operation ($\text{cons}$).
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FOUNDATIONS (2)

Evolving theories: conservative extensions
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  - . . . even with recursive definitions, since “termination” must be provable.

- Importance: One may want to introduce new concepts to carry out some proofs, but this must be done conservatively in order to believe the results.
FOUNDATIONS (3)

Fun example in **ACL2(r)**, an extension of ACL2 that supports the real numbers due to Ruben Gamboa:
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The Overspill Principle of non-standard analysis.

- `overspill.lisp`: Nice result
- `overspill-proof.lisp`: Ugly proof, but local to the main proof, by conservativity
Foundations (4)

Many “simple” logical issues require care in the implementation. While \texttt{LOCAL} is a great example, there are others.
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Many “simple” logical issues require care in the implementation. While LOCAL is a great example, there are others.

We’ll look at just a few on the next slides.
Defattach allows non-conservative extensions. Example:
**Defattach (1)**

*Defattach* allows non-conservative extensions. **Example:**

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- **Attach $\text{impl}$ to $\text{spec}$**:
  
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**Result not provable from axioms for** $f$ **and** $\text{spec}$:

ACL2 !(f 3 4)
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ACL2 !>(f 3 4)

70

ACL2 !>
DEFATTACH (2)

Issues to consider:

▶ Is (local (defattach ...)) supported?
YES, local is supported.

▶ Then how do we deal with conservativity?
Two theories: The usual current theory and a stronger evaluation theory, extended using defattach.

▶ Ah, but what about this?
(thm (equal (f 3 4) 70))
The proof fails! (Whew!)

▶ Why is the evaluation theory consistent?
A key requirement is that the attachment relation is suitably acyclic.

For details, including issues pertaining to evaluation, see the Essay on Defattach comment in the ACL2 sources.
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Quantification, Choice, & Induction (1)

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ACL2 generates the following.

Conservatively introduce \( w(y, z) \) and \( r(y, z) \) using local witness
\[ w(y, z) = (\varepsilon x)(p(x, y, z) \land q(xyz)) \]
to prove these axioms:

\[ r(y, z) = (p(w(y, z), y, z) \land q(w(y, z), y, z)) \]
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\[ (p(x, y, z) \land q(x, y, z)) \implies r(y, z) \]
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. . . **IF** we ignore induction!
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. . . IF we ignore induction!

Conservativity *with* induction follows from a *model-theoretic forcing argument*. 
Meta-theoretic Reasoning (1)

In ACL2, you can:
META-THEORETIC REASONING (1)

In ACL2, you can:

- code a simplifier,
Meta-theoretic Reasoning (1)

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▶ code a simplifier,
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Meta-theoretic Reasoning (1)

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More complex forms are supported, including:

- extended-metafunctions that take `STATE` and contextual inputs;
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For details, including issues pertaining to evaluation, see the Essay on Correctness of Meta Reasoning comment in the ACL2 sources.

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- One can specify a *measure* in order to admit a recursive definition. But what if the measure is defined in terms of a function whose definition is LOCAL?
OUTLINE

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Foundations

Conclusion
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THANK YOU!