Development of a Verified, Efficient Checker for SAT Proofs

Matt Kaufmann
(In collaboration with Marijn Heule and Warren Hunt, Jr.)

ACL2 Seminar
The University of Texas at Austin

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ABSTRACT

I’ll present a case study, consisting of a sequence of verified checkers that validate SAT proofs. These culminate in an efficient checker that can be used in SAT competitions and in industry. No background in SAT is assumed.
OUTLINE

INTRODUCTION
The Problem
Towards a Solution
Variables, Literals, Clauses, Formulas
Semantics: Assignments and Truth
Proofs
Formalizing Soundness
Efficient Proof-checking

A SEQUENCE OF CHECKERS
[drat]
The LRAT Proof Format
[lrat-1]
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CONCLUSION

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Underlining denotes links to the [ACL2+books online manual](https://acl2.sourceforge.io/Acl2/).
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- Checkers are typically simpler than solvers...
- ... but not *that* simple, and *inspection is error-prone*. 
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Background:
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**Example**: Is $F$ true under assignment $a$?

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A formula is satisfiable if it is true under some assignment; otherwise, it is unsatisfiable.
A proof (or clausal proof, or refutation) for a formula $F$ is a sequence $\Pi = \langle p_1, p_2, \ldots, p_k \rangle$ such that:
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PROOFS

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- All addition steps preserve satisfiability (see next slide).
PROOFS (2)

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Then $\Pi$ preserves satisifiability when for each addition step $p_i$, if $F_{i-1}$ is satisfiable then $F_i$ is satisfiable.
NOTE: The definition above of clausal proof is very general. A checker may impose more specific syntactic requirements that guarantee the property.
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The next slide shows Nathan’s formalization, for ITP 2013, based on the RAT (Resolution Asymmetric Tautology) check. Details on RAT are not the subject of today’s talk.
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All checkers discussed today use a formalization like the one on the next slide, based on RAT.
**FORMALIZING SOUNDNESS**

Below, `proofp` is a recognizer for proofs, and `solutionp` checks that a formula is true under a given assignment,

```
(defun refutationp (proof formula)
  (declare (xargs :guard (formulap formula)))
  (and (proofp proof formula)
       (member *empty-clause* proof)))
```

```
(defun-sk exists-solution (formula)
  (exists assignment
   (solutionp assignment formula)))
```

```
(defun-thm main-theorem
  (implies (and (formulap formula)
                (refutationp clause-list formula))
           (not (exists-solution formula))))
```
FORMALIZING SOUNDNESS (2)

The following is easily proved by induction.

**Lemma.** Suppose that $\Pi = \langle p_1, p_2, \ldots, p_k \rangle$ is a proof and $F_0$ is satisfiable. Then each $F_i$ is satisfiable.
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Soundness argument:
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**Lemma.** Suppose that $\Pi = \langle p_1, p_2, \ldots, p_k \rangle$ is a proof and $F_0$ is satisfiable. Then each $F_i$ is satisfiable.

Soundness argument:

1. Deletion steps clearly preserve satisfiability.
2. **Addition steps preserve satisfiability.** [Must be proved!]
3. By the lemma, if $F_0$ is satisfiable then $F_k$ is satisfiable.
4. Since $p_k$ adds the empty clause, $F_k$ is unsatisfiable.
5. It follows immediately that $F_0$ is unsatisfiable.
EFFICIENT PROOF-CHECKING

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- Its efficiency benefits in part from some techniques not yet invented at the time of Nathan’s work.
Efficient Proof-checking (2)

The flow for **efficient**, verified SAT proof-checking:
EFFECTIVE PROOF-CHECKING (2)

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2. \textit{DRAT-trim} [2] takes inputs $\Pi_0$ and $F$; outputs alleged proof $\Pi_1$ for checker, \textit{in a format amenable to efficient checking}.

3. Verified ACL2 checker validates that $\Pi_1$ is a proof for $F$. 

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3. [lrat-1] Avoid search and delete clauses efficiently, using fast-alists (applicative hash tables) and a linear proof format, and with soundness proved from scratch
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1. [rat] Nathan’s ITP 2013 RAT checker: no deletion
2. [drat] Added deletion (thus implementing DRAT)
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▶ Of course, many ACL2 features were crucial, including proof procedures (many inherited from earlier Boyer-Moore provers) and fast-alists (initially Boyer/Hunt, later Davis/Swords).
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  (profile-all) ; or just profile specific functions
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But the [drat] checker is still slow. **Why?**
[drat]: Why It’s Slow
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THE LRAT PROOF FORMAT

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Example LRAT proof step $p_i$:  

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For the RAT check on clause 87, restrict UP to the clauses 308, 117, ..., and 310, in order.

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The LRAT Proof Format (The Big Take-away)

Hints direct exactly where unit propagation is done – no search!
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Clause indices help solve the second problem.
The next checker implements these efficiencies.
[lrat-1]
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- This list is a fast-alist: ACL2 uses a hash-table to find \(c\) from \(i\) in essentially constant time.

- A formula is a pair \((\max \ . \ \text{fal})\), where \(\text{fal}\) is its fast-alist and \(\max\) is an upper bound on its indices.
How do fast-alists help with efficiency?
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How do fast-alists help with efficiency?

- Unit propagation benefits from fast lookup to obtain a clause from its index; and

- Deletion of clause $i$ simply extends the fast-alist with a pair $(i, *\text{deleted-clause}*).$
  
  - The value of $*\text{deleted-clause}*\text{ is a non-nil atom,}$ hence not a clause.
Soundness Proof Problem:
How to manage the substantial change from [drat] to [lrat-1].
[lrat-1] (3)

**Soundness Proof Problem:**
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Soundness Proof Problem:
How to manage the substantial change from [drat] to [lrat-1].

- Painful to rework another’s proof
- Decision: Sketch hand proof and manage a fresh proof
- Used top-down approach (see my 1999 ACL2 Workshop paper)
satisfiable-add-proof-clause.lisp

<hand proof in comment>
(in-package "ACL2")
(include-book "lrat-checker")

(local (encapsulate ()
  (local (include-book "satisfiable-add-proof-clause-rup"))
  (local (include-book "satisfiable-add-proof-clause-drat"))
  (set-enforce-redundancy t)
  (defthm satisfiable-add-proof-clause-rup
    ...)
  (defthm satisfiable-add-proof-clause-drat
    ...)))

(defthm satisfiable-add-proof-clause
  ...
  :hints
  ("Goal" :use (satisfiable-add-proof-clause-rup
                satisfiable-add-proof-clause-drat)
            :in-theory (union-theories '(verify-clause
                                         (theory 'minimal-theory))))
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The RAT check visits *every* clause in the formula $F_i$. 
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The RAT check visits *every* clause in the formula $F_i$.

The [lrat-2] checker improves on [lrat-1] in two ways:

- Shrink the formula’s fast-alist when heuristics say to do so.
- RAT check recurs through the fast-alist instead of recurring down from the max index.
[lrat-2]: SHRINKING

Two counts maintained on the formula:
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- ▶ \textit{ndel}: number of pairs (i.e. deleted-clause*)
Two counts maintained on the formula:

- \( n_{del} \): number of pairs \((i \cdot \text{deleted-clause})\)
- \( n_{cls} \): the number of pairs \((i \cdot c)\) representing clauses that have not been deleted
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Heuristically shrink the fast-alist at an addition proof step, based on experimentation:

- whenever \( ndel > 10 \times ncls \);
- when RAT check is necessary, shrink first if \( ndel > 1/3 \times ncls \).
To shrink a fast-alist (will discuss only if time):

(defun remove-deleted-clauses (fal acc)
  (declare (xargs :guard (alistp fal)))
  (cond ((endp fal) (make-fast-alist acc))
        (t (remove-deleted-clauses
            (cdr fal)
            (if (deleted-clause-p (cdar fal))
                acc
                (cons (car fal) acc))))))

(defun shrink-formula-fal (fal)
  (declare (xargs :guard (formula-fal-p fal)))
  (let ((fal2 (fast-alist-clean fal)))
    (fast-alist-free-on-exit
     fal2
     (remove-deleted-clauses fal2 nil))))
[lrat-2]: PROOF

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Proved soundness by tweaking the [lrat-1] proof:

- Disabled the top-level “maybe shrink” function
- Re-ran the [lrat-1] proof on [lrat-2]
- Looked at key checkpoints on failure to determine lemmas to prove (about shrinking).
[lrat-3]

Changed formula from \((\text{max} . \text{fal})\) to simply \text{fal}. 
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- This simplification seemed useful before starting the next checker, and it saves consing.
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- Soundness proof for [lrat-2] was easy to modify for [lrat-3].
A bottleneck in [lrat-3]: evaluation of a literal $n$ requires a linear-time search for either $n$ or $-n$ in the assignment.
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- Lookup is a constant-time array reference.
A bottleneck in [lrat-3]: evaluation of a literal $n$ requires a linear-time search for either $n$ or $-n$ in the assignment.


- Lookup is a constant-time array reference.
- Avoids memory allocation (consing) when pushing new literals onto assignment.
(defstobj a$
  (a$ptr :type (integer 0 *) ; stack pointer
   :initially 0)
  (a$stk :type (array t (1)) ; stack of a$arr indices
   :resizable t)
  (a$arr :type (array t (1)) ; array of 0, t, nil
   :initially 0
   :resizable t)
  :renaming ((a$arrp a$arrp-weak)
    (a$p a$p-weak)))
[lrat-4]: ASSIGNMENTS (2)

Operations on assignments:
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- \((\text{push-literal~lit~a$})\) extends assignment \(a$\) with literal \(\text{lit}\) (writes to \(a$\text{stk}\), increments \(a$\text{ptr}\)).
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Operations on assignments:

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KEY OBSERVATION: These operations generate calls to nth and update-nth, but for [lrat-3], they are implemented with cons and cdr.
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KEY OBSERVATION: These operations generate calls to nth and update-nth, but for [lrat-3], they are implemented with cons and cdr.

Tweaking the [lrat-3] proof seemed difficult! Instead....
[lrat-4]: PROOF

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- Then I derived the soundness of [lrat-4] directly from those correspondence theorems and the soundness of [lrat-3].
(defthm main-theorem-list-based
  (implies (and (formula-p formula)
                (refutation-p proof formula))
           (not (satisfiable formula)))
  :hints ...) 

(defthm refutation-p-equiv
  (implies (and (formula-p formula)
                (refutation-p$ proof formula)
                (refutation-p proof formula)))

(defthm main-theorem-stobj-based
  (implies (and (formula-p formula)
                (refutation-p$ proof formula))
           (not (satisfiable formula)))
  :hints ("Goal"
           :in-theory '(refutation-p-equiv
                        :use main-theorem-list-based)))
[lrat-4]: PROOF (3)

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3. Proved correspondence theorems
I’ll very briefly discuss the invariant:

```lisp
(defun a$p (a$)
  (declare (xargs :xstobjs a$))
  (and (a$p-weak a$)
       (<= (a$ptr a$) (a$stk-length a$))
       (equal (a$arr-length a$)
              (1+ (a$stk-length a$)))
       (good-stk-p (a$ptr a$) a$)
       (a$arrp a$)
       (arr-matches-stk (a$arr-length a$) a$)))
```
[lrat-4]: PROOF (5)

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- One [lrat-3] function, negate-clause-or-assignment, did not match up with its corresponding [lrat-4] function.

The [lrat-2] function (originally used in [lrat-3]):

```lisp
(defun negate-clause-or-assignment (clause)
  (declare (xargs :guard (clause-or-assignment-p clause)))
  (if (atom clause)
      nil
    (cons (negate (car clause))
          (negate-clause-or-assignment (cdr clause)))))
```
[lrat-4]: PROOF (6)

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Then completed correspondence theorems, which yielded soundness for [lrat-4].
OUTLINE

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The Problem
Towards a Solution
Variables, Literals, Clauses, Formulas
Semantics: Assignments and Truth
Proofs
Formalizing Soundness
Efficient Proof-checking

A SEQUENCE OF CHECKERS
[drat]
The LRAT Proof Format
[lrat-1]
[lrat-2]
[lrat-3]
[lrat-4]

CONCLUSION

REFERENCES
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Other checkers are available via links from the seminar page.
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A SEQUENCE OF CHECKERS
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- The LRAT Proof Format
- [lrat-1]
- [lrat-2]
- [lrat-3]
- [lrat-4]

CONCLUSION

REFERENCES
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This work can be found in the community books, with the latest version on github:

books/projects/sat/lrat/
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books/projects/sat/proof-checker-itp13/
bibliography/proof-checker-array/
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The next slide has references for citations in this talk.


