ACL2VHDL Translator: A Simple Approach to Fill the Semantic Gap

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Abstract. We wrote an ACL2 to VHDL translator for our verification purpose. One major problem of translation between programming languages and the ACL2 language is the semantic gap caused by the translation; it is not easy to translate one language to another while precisely preserving its semantics. Our approach is to write a translator for a small subset of the ACL2 language for which there is no loss of semantic correctness. This seemingly restricted translator turned out to be a promising approach for combining ACL2 and VHDL testing/verification tools. This paper discusses the details of the translator and the bit-vector libraries used in the translatable ACL2 functions.

1 Introduction

The ACL2 theorem prover has been used for formally verifying many pieces of hardware and software. Typically, verification is performed on an abstract model which is manually written in the ACL2 language. This allows interesting analysis on various aspects of computation, including the hardware verification of pipelined machine controls[8], floating point algorithms[6], and cache coherence protocols. However, there is always a doubt whether a manually-written abstract model is a correct representation of an actual implementation. It is desirable to analyze actual programming and hardware description languages, rather than its manual translation.

However, formally verifying actual programs or RTL description of hardware using a theorem prover is not very popular. This is partially because real-world software and hardware are far more complicated than what can be easily handled by a theorem prover. But there is also a language barrier that prevents or at least discourages researchers and engineers from analyzing, for instance, C and VHDL using a theorem prover.

Typically, when ACL2 is used to verify software rewritten in some programming language, it must be translated to or interpreted in the ACL2 language. There are two types of approaches: deep embedding and shallow embedding. In a deep embedding approach, one writes an interpreter of the target programming language in ACL2, and proves theorems about the results of interpretation. In a shallow embedding approach, one writes a translator from the programming language to the ACL2 language, and proves properties about translated programs in the form of ACL2 functions. There are pros and cons in both approaches. In a deep embedding approach, it is often easy to define an interpreter with a precise semantic correctness. Data in the target programing language are typically represented by special data-types in ACL2. Liu and Moore[5] use this approach to precisely model the semantics of Java bytecode. On the downside of the deep embedding, the interpreter is slow when executed. Proof on the interpreter is sometimes cumbersome, as one has to reason in two steps about the interpreter that handles the target language.

The shallow embedding is opposite. Since we do not have to deal with an interpreter acting like a middleman, it is often straightforward to deal with a translated program. However, translation is often imprecise. One of the reasons is that data-types in programming languages are not isomorphic to ACL2 data-types. For example, nil in ACL2 means both Boolean false and empty list. Another example is that ACL2 integers are big numbers without limit, while most programming languages use 32-bit or 64-bit integers. There are also some concepts that are difficult to translate to ACL2, such as pointers in programming languages.

Both approaches have been used in hardware verification as well. In ACL2based verification projects, Hunt's DUAL-EVAL[4] is an example of deep embedding, while Russinoff's floating-point verification uses shallow embedding[7]. When translating hardware description language into the ACL2 language, there are additional problems deriving from the differences of computation models. For example, the concurrent signal assignment statement in VHDL is a collection of assignment statements which are not ordered. When evaluating, concurrent assignments must be repetitively applied until no more values are updated. In a sense, it is performing a fix-point calculation. Thus, translation of concurrent assignments in VHDL to ACL2 is not straightforward. Even if the translation can done, the resulting program may become too complex to analyze by a theorem prover.

Our goal in this paper is to define a translation that allows the analysis of VHDL code in ACL2. Our approach is rather simple but effective. First, we define a translator from ACL2 to VHDL, not the other way around. We rely on the VHDL-level verification tools in comparing translated ACL2 functions and VHDL implementations of hardware. We can also use ACL2 to prove theorems about the ACL2 functions before the translation. In this way, we never have to work on complicated VHDL source code at the ACL2 level. When the VHDL implementation of hardware is changed, as it happens so often, the VHDL-level verification tools have to simply be rerun. We use ACL2 in the analysis of the high-level concepts such as algorithms that do not change very often.

Second, the translator can handle a small subset of ACL2 language. We define an ACL2 book about bits, bit-vectors, and 32-bit integers. Our translator accepts ACL2 functions that use only the functions from this ACL2 book. This way, we can translate ACL2 functions to VHDL without loss of semantic correctness.

In this paper, we first introduce the ACL2 bit-vector library used in the translated functions, then go on to describe the details of the translation program. This paper assumes the minimum knowledge of VHDL. For a tutorial of VHDL, we recommend Bhasker's book[1].

2 Bit-Vector Library

In the standard ACL2 distribution, there is a library called the IHS (Integer Hardware Specification) library. It has been developed for specifying hardware in the verification project on a DSP processor[2]. In the IHS library, bits are represented by ACL2 integer 0 and 1. Bit-vectors are represented by integers. This simple representation allows quick execution of the model specified in the IHS library. Also the IHS library defines many operations over bits and bit-vectors, and provides related lemmas.

However, the IHS library is not adequate as a basis of ACL2-to-VHDL translation. First, the integer representation of bits and bit-vectors prevents us from defining a one-to-one mapping between the ACL2 and VHDL data-types. When you say 1, it could mean integer 1, bit 1, or a bit-vector whose value happens to be 1. And even if it is known to be a bit-vector of value 1, it could be a bit-vector of length 1, a bit-vector of length 10, or a bit-vector of any length.

Also, this ambiguity in the length of bit-vectors disallows the definition of certain functions. For example, let us consider a function msb that returns the most significant bit of a bit-vector. Should we define (msb 1) to be 0 or 1? It depends on the length of the bit-vector represented by 1.

We defined a new bit-vector library called BV library. In this library a bit is represented as list (BIT 0) or (BIT 1). A bit-vector is represented by a list (BV v n), where positive integer v is the value of the bit-vector, and n is its length. In other words, the corresponding bit-vector is the least significant n bits of the binary representations of v.

By building it on top of the IHS library, we simplified the definition of the BV library. For example, let us look at the definition of function bv&, which returns a concatenation of two bit-vectors:

Function bv is a constructor for bit-vectors; (bv val size) returns a list (BV val size). Function bv-val and bv-size returns the value and the length of a bit-vector, respectively. Function (logapp n bv1 bv2) is an IHS function that concatenates bit-vectors bv2 and bv1 assuming that the length of bv1 is n. In this definition, bv& calculates the value of the concatenated vectors by calling the IHS function. The length of the concatenated vector is the sum of the lengths of two vectors. In this way, we reused definitions and lemmas in the IHS library as much as possible in the BV library.

Bit constant *b0* and *b1* are defined to be (BIT 0) and (BIT 1), respectively. Type predicate (bitp x) is true if x is a bit. We define operators over bits such as b-not, b-ior, b-and, b-xor, and b-eqv.¹

Following are basic functions on bit-vectors defined in the BV library. A complete list is provided in Table 1 and 2 in Section 3. Function (lsb v) returns the least significant bit of bit-vector v, and (msb v) returns the most significant bit. Function (msbits v) returns v after removing the least significant bit, while (lsbits v) returns v except the most significant bit. Function (b&bv b v) and (bv&b v b) add bit b to the bit-vector v as the most significant bit and the least significant bit, respectively. Hence:

(msb (b&bv b v)) = b
(lsbits (b&bv b v)) = v
(lsb (bv&b v b)) = b
(msbits (bv&b v b)) = v

In a sense, msb and lsb act like car for bit-vectors, lsbits and msbits like cdr, and b&bv and bv&b like cons.

Function (bitn n v) returns the n'th bit of bit-vector v. The most significant bit is always indexed by 0, and the index increases toward the less significant bits. In VHDL, an IEEE standard ulogic vector defined to be std_ulogic_vector(0 to len-1) has the same indexing. Different vendors prefer different styles of indexing, i.e. indexing from the most significant bit or indexing from the least significant bit. Additionally, VHDL allows indexing starting from any integer. To keep the BV library simple, we decided to allow only the indexing from the most significant bit to the least significant bit starting from 0.

Other operations defined over bit-vectors include sub range of bit-vector (bits v i j), bit-wise negation (bv-not v), bit-wise AND (bv-and v1 v2), bit-wise OR (bv-ior v1 v2), logical shift to left (bv-<< v a), logical shift to right (bv->> v a), arithmetic-shift to right (bv-*> v a), binary addition (bv+ v1 v2), binary increment (bv++ v1), equality operation (bv-eq? v1 v2), and binary comparisons such as greater-than relation (bv-gt? v1 v2). We also defined a 0-length bit-vector *bv-nil* and a conversion function from bit to bit-vector of length 1, (b2bv b).

There are also a few functions for control. Function (bv-if b v1 v2) returns bit-vector v1 if b is 1, and returns v2 otherwise. We also define a macro bv-cond which works like the cond control structure in ACL2.

Using the functions described above, any functions over bit and bit-vectors can be defined. We found that the BV library can easily express most of the functions defined in hardware, such as when we described multipliers and floating point instructions.

¹ The IHS library has bit operators with the same name. However, we define our bit-vector library in a separate ACL2 package, thus they are different functions.

3 Conversion of ACL2 to VHDL

We wrote a translation program from the ACL2 language to VHDL. We call it ACL2VHDL translator. This translation program only handles bit and bitvectors and 32-bit integers. Bits are mapped to VHDL standard ulogic, bitvectors are mapped to standard ulogic vectors, and 32-bit integers are mapped to VHDL integers. Translatable ACL2 functions are those defined only in terms of let, let* and the functions in the BV library. Any other functions are not allowed in the translated function, including basic ACL2 functions if and cons. This limitation simplifies the implementation of the translation program and also helps to preserve the semantics during the function translation.

Some of the bit-vector functions in the BV library such as (bv& v0 v1) and (bv-and v0 v1) have corresponding VHDL operators, namely (v0 & v1) and (v0 AND v1). We defined additional VHDL functions so that every bit and bitvector functions from the BV library has a corresponding VHDL function. Such additional functions are defined in a VHDL file named acl2_support.vhdl. Some of them are msb(v), lsb(v), lsbits(v), msbits(v), bitn(n,v), bits(v,i,j), and bv_gt(v0,v1). The names of the BV functions and the corresponding VHDL functions are shown in Table 1 and 2. Due to the restriction of the characters that can be used in VHDL function names, the function names in the BV library and acl2_support.vhdl may not be identical, but their semantics should be.

Integers are also problems in translation. In ACL2, integers are big numbers without limit, but VHDL's integer is often implemented as 32-bit integers. In order to fill the gap, we define the 32-bit integer type (int32p i), and 32-bit integer operations such as (int32+ i j), (int32- i j) and (int32* i j). We allow only 32-bit integer and the associated operations in the translatable ACL2 functions, but not ordinary ACL2 arithmetic operations such as +, - and *.

The ACL2VHDL translator converts functions using 32-bit integers. For example

```
(defun plus (a b)
  (declare (xargs :guard (and (int32p a) (int32p b))))
  (int32+ a b))
```

is translated into

```
function plus(a : integer;b : integer)
return integer is
variable result : integer;
begin
result := (a + b);
return result;
end plus;
```

Since every BV function has a corresponding VHDL function, we can directly translate an ACL2 function whose definition only includes BV functions. However, in addition to the BV functions, we allow let and let* statements, which

Table 1. BV library functions that can be used in the translated functions and the corresponding VHDL functions. VHDL functions are defined in the standard IEEE 1164 library or defined in a new VHDL package for the conversion purpose. This table shows bit logical operations, bit-vector logical operations, and bit manipulation functions. Function bv-cond has no corresponding function, and it is translated to an expression using ite

ACL2 Function	VHDL Function	Descriptions
b0	,0,	Bit one (constant).
b1	'1'	Bit zero(constant).
b-not	not	Bit negation.
b-ior	or	Bit OR.
b-nor	nor	Bit NOR.
b-and	and	Bit AND.
b-nand	nand	Bit NAND.
b-xor	xor	Bit exclusive OR.
b-andc1	andc1	Bit AND with the first argument complemented.
b-andc2	andc2	Bit AND with the second argument complemented.
b-orc1	orc1	Bit OR with the first argument complemented.
b-orc2	orc2	Bit OR with the first argument complemented.
b-maj	maj	Majority function.
b-if	ite	IF-statement returning a bit.
bv-nil		Bit-vector of length zero(constant).
bv-not	not	Bit-wise negation.
bv-and	and	Bit-wise AND.
bv-ior	or	Bit-wise OR.
bv-xor	xor	Bit-wise XOR.
bv-neg	bv_neg	2's complement.
bv-if	ite	IF-statement returning a bit-vector.
bv-cond		Cond-statement returning a bit-vector.
b2bv	b2bv	Conversion from bit to bit-vector of length 1.
bitn	bitn	N'th bit from the most significant bit.
msb	msb	The most significant bit.
lsb	lsb	The least significant bit.
msbits	msbits	Removing the least significant bit.
lsbits	lsbits	Removing the most significant bit.
bv&b	&	Concatenating a bit after a bit-vector.
b&bv	&	Concatenating a bit before a bit-vector.
bv&	&	Concatenating two bit-vectors.
bits	bits	Subrange of a bit-vector.
bv-right	bv_right	The rightmost n bits of a bit-vector.
bv-left	bv_left	The leftmost n bits of a bit-vector.
zeros	pad0	A bit-vector with every bit set to 0.
ones	pad1	A bit-vector with every bit set to 1.
pad0	pad0	Identical to zeros.
pad1	pad1	Identical to ones.
fill-0	fill_0	Bit zeros filling a subrange.
fill-1	fill_1	Bit ones filling a subrange.

Table 2. A continued list of BV library functions and the corresponding VHDL functions. Here we define shifting operations, arithmetic operations, arithmetic relations, and logical operations over bits in a vector.

ACL2 Function	VHDL Function	Descriptions
bv-uextend	bv_uexnted	Unsigned extension of a bit-vector.
bv-sextend	$bv_sexnted$	Signed extension of a bit-vector.
bv-lsh	sll	Shift left logical with an integer shift amount.
bv-ash	sla	Shift left arithmetic with an integer shift amount.
BV-<<	bv_sll	Shift left with a bit-vector shift amount.
BV->>	bv_srl	Shift right logical with a bit-vector shift amount.
BV-*>>	bv_sra	Shift right arithmetic with a bit-vector shift amount.
bv+	+	Binary addition.
bv+carry	bv_carry	Carry out from a binary addition.
bv-	-	Binary subtraction.
bv*	*	Binary multiplication.
bv++	increment	Binary increment.
bv	decrement	Binary decrement.
bv-inc	add_bit	Binary increment if a bit input is set.
bv-eq?	bv_eq	bit-vector equality.
bv-neq?	bv_neq	bit-vector inequality
bv-gt?	bv_gt	Binary greater than.
bv-ge?	bv_ge	Binary greater than or equal to.
bv-lt?	bv_lt	Binary less than.
bv-le?	bv_le	Binary less than or equal to.
bv-all-ones?	and_reduce	AND'ing all bits in a bit-vector.
bv-all-zeros?	nor_reduce	NOR'ing all bits in a bit-vector.
bv-some-zeros?	$\mathtt{nand}_{-}\mathtt{reduce}$	NAND'ing all bits in a bit-vector.
bv-some-ones?	or_reduce	OR'ing all bits in a bit-vector.
bv-and-all	and_reduce	Identical to by-all-ones?.
bv-ior-all	or_reduce	Identical to by-all-zeros?.
bv-right-ior	bv_right_or	OR'ing the rightmost n bits of a bit-vector.
bv-left-ior	bv_left_or	OR'ing the leftmost n bits of a bit-vector.
bv-lz	leadz	Number of zero bits in a bit-vector.

requires a special treatment. Basically, variable bindings in a let expression are translated into VHDL sequential assignments.

When the ACL2VHDL translator first parses an ACL2 function definition, it extracts a list of bindings introduced by let. In order to translate the ACL2 let semantics correctly, this extraction algorithm renames a variable every time it is rebound by a let expression. For example, let us look at the translation of function foo:

```
(defun foo (a)
  (declare (xargs :guard (and (bvp a) (equal (bv-size a) 1))))
  (let ((x (b&bv *b1* a)))
     (let ((x (b&bv *b0* x)))
        (let ((x (b&bv *b1* x)))
              (msb x)))))
```

The ACL2VHDL translator converts it into a VHDL function:

```
function foo(a : std_ulogic_vector)
return std_ulogic is
variable x : std_ulogic_vector(0 to 1);
variable x_1 : std_ulogic_vector(0 to 2);
variable x_2 : std_ulogic_vector(0 to 3);
variable result : std_ulogic;
begin
x := (b1 & a);
x_1 := (b0 & x);
x_2 := (b1 & x_1);
result := msb(x_2);
return result;
end foo;
```

In this translation,² bit *b0* and *b1* are translated to VHDL constant b0 and b1. ACL2 function b&bv and msb are translated into corresponding VHDL functions & and msb, respectively. The scope analysis introduced new variables x_1 and x_2 to the VHDL code to store the value of x bound in the second and third let expressions. (Note we convert each ACL2 function to an individual VHDL function, and we do not have to worry about name collisions with existing VHDL variables or signals.) The scope analysis was performed to follow the LISP scope rules, thus we allow let expressions inside the binding of another let expression. For example,

```
(defun bar (a)
  (declare (xargs :guard (and (bvp a) (equal (bv-size a) 1))))
  (let ((x (let ((x (b&bv *b0* a))) (b&bv *b1* x))))
      (bv& x x)))
```

² For readability of the paper, the output from the ACL2VHDL translator is indented properly, although the raw output is not pretty-printed.

is translated into:

```
function bar(a : std_ulogic_vector)
  return std_ulogic_vector is
  variable x_4 : std_ulogic_vector(0 to 1);
  variable x : std_ulogic_vector(0 to 2);
  variable result : std_ulogic_vector(0 to 5);
begin
  x_4 := (b0 & a);
  x := (b1 & x_4);
  result := (x & x);
  return result;
end bar;
```

In this translation, x_4 is used to store the value of x in the inside let expression, while x represents the x of the outside let.

The ACL2VHDL translator also performs type inference on the ACL2 functions. Note in the examples above, function **foo** and **bar** return a bit and a bit-vector of length 6, respectively. In VHDL, such type information of results and intermediate values are needed to define variables. The ACL2VHDL translator infers such required type information from the type of the input arguments. The user must provide a guard expression to each function in order to annotate the type of all the input arguments.

The translated ACL2 function can take only bits, bit-vectors, and 32-bit integers as arguments, and the types of all input variables must be determined from the guard expression. The guard expression must be a conjunction of form (and $cond_1 \ cond_2 \ ... \ cond_n$). For a bit type argument b, (bitp b) should appear as a conjunct. 32-bit integer type argument i should have a conjunct (int32p i) in the guard expression. bit-vector argument v of length n has to have conjunct expressions (bvp v) and (equal (bv-size v) n) in the guard expression. The length of n can be either a verbatim positive number or an expression of type int32p, which might use 32-bit integer type arguments. For example,

```
result := (b1 & x);
```

```
return result;
end test1;
```

The ACL2VHDL translator ignores any conjuncts in the guard expression that are none of the forms discussed above.

One exception to the restriction of input argument types are big integers as a first argument of bv. This is because we sometimes need to specify a constant bit-vector which is longer than 32 bits. The ACL2VHDL translator directly converts such a bv expression into a bit-vector constant in VHDL.

Type inference algorithm knows the type of basic functions defined in the BV library. It also stores the type of a newly defined function. From the type information of functions and arguments, the type inference algorithm deduces result types. Since most functions in the BV library can take bit-vectors of variable lengths, our type inference system is implemented to handle parametrized types. For example, the type of function b&bv is represented by a list ((bit (bv a)) (bv (+ a 1))), which means b&bv takes a bit and a bit-vector of length a as arguments and returns a bit-vector of length a+1.

After variable renaming for let expressions and type inference, the process is rather simple to generate the VHDL functions. The ACL2VHDL translator converts the body expression of an ACL2 function into a sequence of assignments. For each let binding, it creates a separate variable assignment in VHDL. The translator does not generate VHDL control constructs such as IF-THEN-ELSE statements. Even ACL2 function bv-if is translated into the corresponding VHDL if-then-else function ite. For example

```
(defun test3 (a x y)
  (declare (xargs :guard (and (bitp a)
        (bvp x) (equal (bv-size x) 32)
        (bvp y) (equal (bv-size y) 32))))
  (bv-if a (bv& x y) (bv& y x)))
```

is translated into

Since the ACL2VHDL translator does not accept *if*, translated ACL2 functions cannot be recursive. Generally speaking, not all recursive functions can be translated into VHDL. Recursion is allowed for VHDL functions only if the compiler can statically unroll a recursive function to a flat definition. In other words, functions can recurse only on statically known values, such as the length of bitvectors or constant integers. It is possible to translate such a limited recursive function. However, the recursive VHDL function must use VHDL IF-THEN-ELSE statements, and thus requires a significant modification of our translator which currently outputs only sequential assignment statements in the function body.

4 Discussion

We implemented a small translator from the ACL2 language to VHDL using the BV library. Because the translated language is limited, we can translate ACL2 functions to VHDL functions while minimizing the semantic gap between two completely different languages.

Although we can use only the BV library functions in the definition of translated functions, we found our BV library expressively enough to describe many algorithms used in hardware. We specified multipliers and the behavior of floating-point instructions using this library. We found that the BV library supplies frequently used functions for such work. The ACL2VHDL translator also allows a kind of parametrized function by permitting variable length bit-vectors as arguments. This enables us to convert generalized functions. For example, we can specify a single function that rounds a floating-point mantissa to a bit vector of any length, such as single-precision and double-precision lengths.

It is true that the ACL2VHDL translator is limited with various features, and the most notable missing feature is recursion. Although recursive functions are not frequently used in hardware descriptions, it is useful in describing algorithms.

However, recursive functions are disallowed only in the function to be translated. We can freely define a recursive function in ACL2 using the BV library function. We can also "translate" such a recursive function to a VHDL function by taking a two-step approach. First, we define a non-recursive version of ACL2 function and prove the equivalence between the two ACL2 functions. Then, we translate the non-recursive function into VHDL.

Although the BV library only defines the bit and bit-vectors, it does not provide any mechanisms to define data-structures like records and lists. We did not include them in the translation because we wanted to keep our translator simple, and also because there are no corresponding data-structures in VHDL. One exception is an array. Arrays are used to define memory and register files, and it is desirable to implement it in the future version of the ACL2VHDL translator.

Also our translator does not handle features used to describe hardware in VHDL, such as the concept of entity and ports, signal delays, and state holding elements such as latches. These features are excluded from our translator to keep it simple. Our approach is remarkably different from some projects that tries to model many details of VHDL, such as the work by Georgelin et. al. [3]. Their translator converts VHDL concepts of entity and process into the ACL2, assuming synchronous behavior and transformation on the concurrent

assignments. We do not handle any of those, instead, we rely on VHDL-level verification tools to handle them.

One might ask, then, whether such a simple translator is actually useful in hardware verification. Our experience shows that it is, especially when the translator is combined with VHDL-level verification tools. In one approach, one can compare the translated ACL2 function and actual VHDL implementation by simulation or formal verification tools. For example, we wrote the specification of floating-point conversion instructions, translated it into VHDL, and compared it against the actual VHDL implementation of a floating-point unit using a SAT solver. For simple instructions like a floating point conversion instruction, we can completely verify that a VHDL implementation of floating point unit is equivalent to the ACL2 specification.

Yet another application is translating properties written as ACL2 functions into VHDL, and check them using a VHDL-level formal verification tool. Once those properties are verified at the VHDL level, we can use verified properties as lemmas and construct a verification proof of a large system. For example, we may use such an approach for compositional verification of a multiplier.

Either way, the ACL2VHDL translator is useful when it is combined with VHDL-level verification/testing tools. For future work, we would like to integrate the ACL2 system and VHDL-level verification tools using the ACL2VHDL translator.

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