Formal Verification of Financial Algorithms, Progress and Prospects

Grant Olney Passmore
ACL2-2017, Austin

Joint work with Denis Ignatovich and our incredible team at AI

Video
(see it on http://imandra.ai)

Problem

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Lack of Transparency: issues of misrepresentation (e.g. misleading marketing materials or regulatory filings) of trading algorithm behaviour (e.g., BATS/Direct Edge $14M settlement with the SEC)

Glitches: trading system errors in design or implementation, often causing significant losses (e.g., Knight Capital's loss of $400M)

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- **Regulations**: Transparency, safety and fairness (Reg ATS-N), best execution (Reg NMS)
- **Practice**: Be able to write a spec and analyse basic regulatory properties of a trading venue's matching logic

Goals for this talk

- **Intuitions**:
  - “Venue matching logics” = “ISA of the market”
  - Pressing need for:
    - venues to be bullet-proof w.r.t. safety and fairness regulations
    - matching logics to be formally described to regulators and market participants
    - matching logics to be formally analysed w.r.t. precise encodings of regulatory directives
    - financial mathematics (stochastic calculus) that takes precise discrete behaviour of matching logics into account

The Stack of Financial Algorithms

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Venues

The Stack of Financial Algorithms

Smart Order Routers

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Trading Algos

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Inventory Management

Algo Containers

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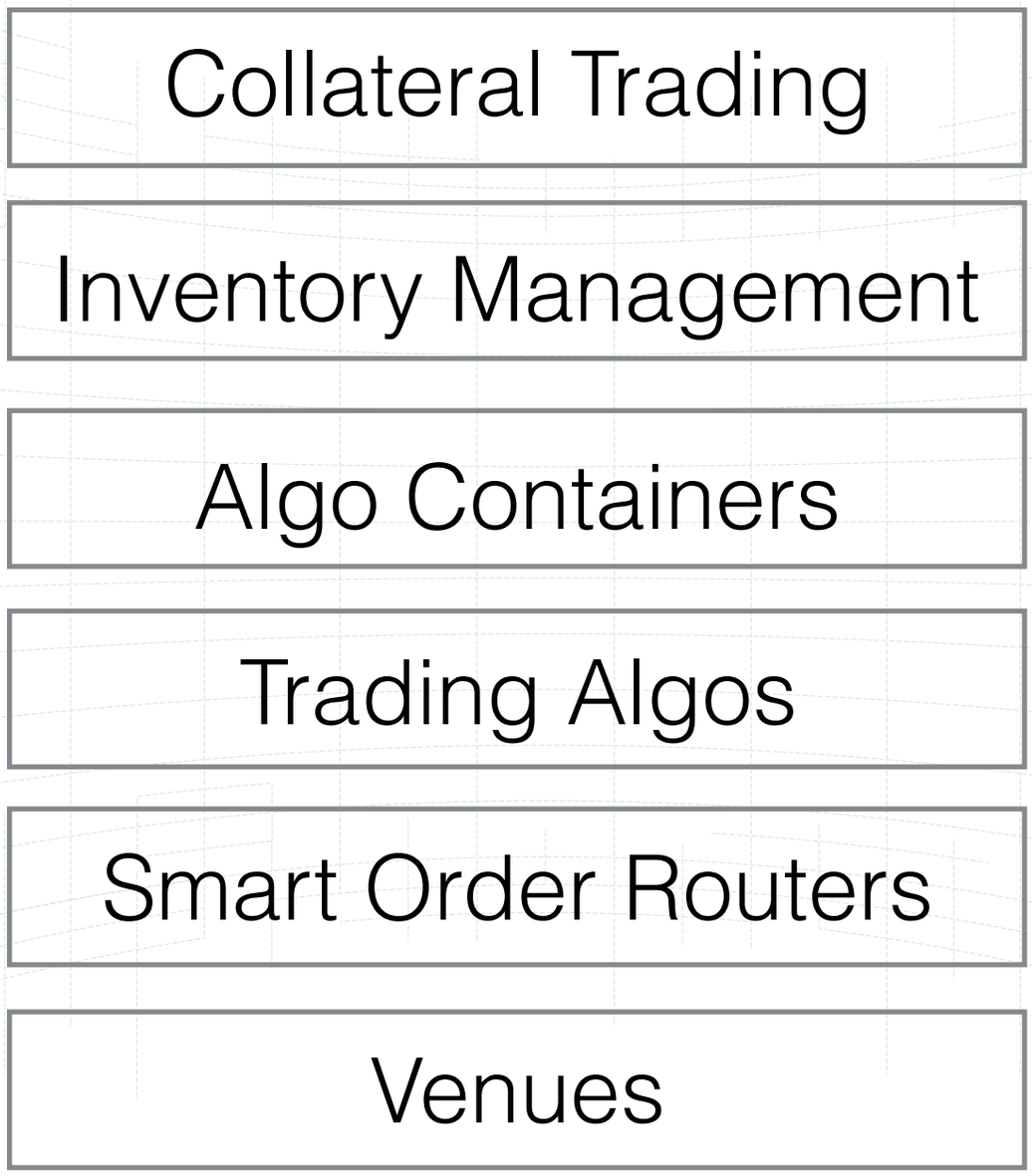

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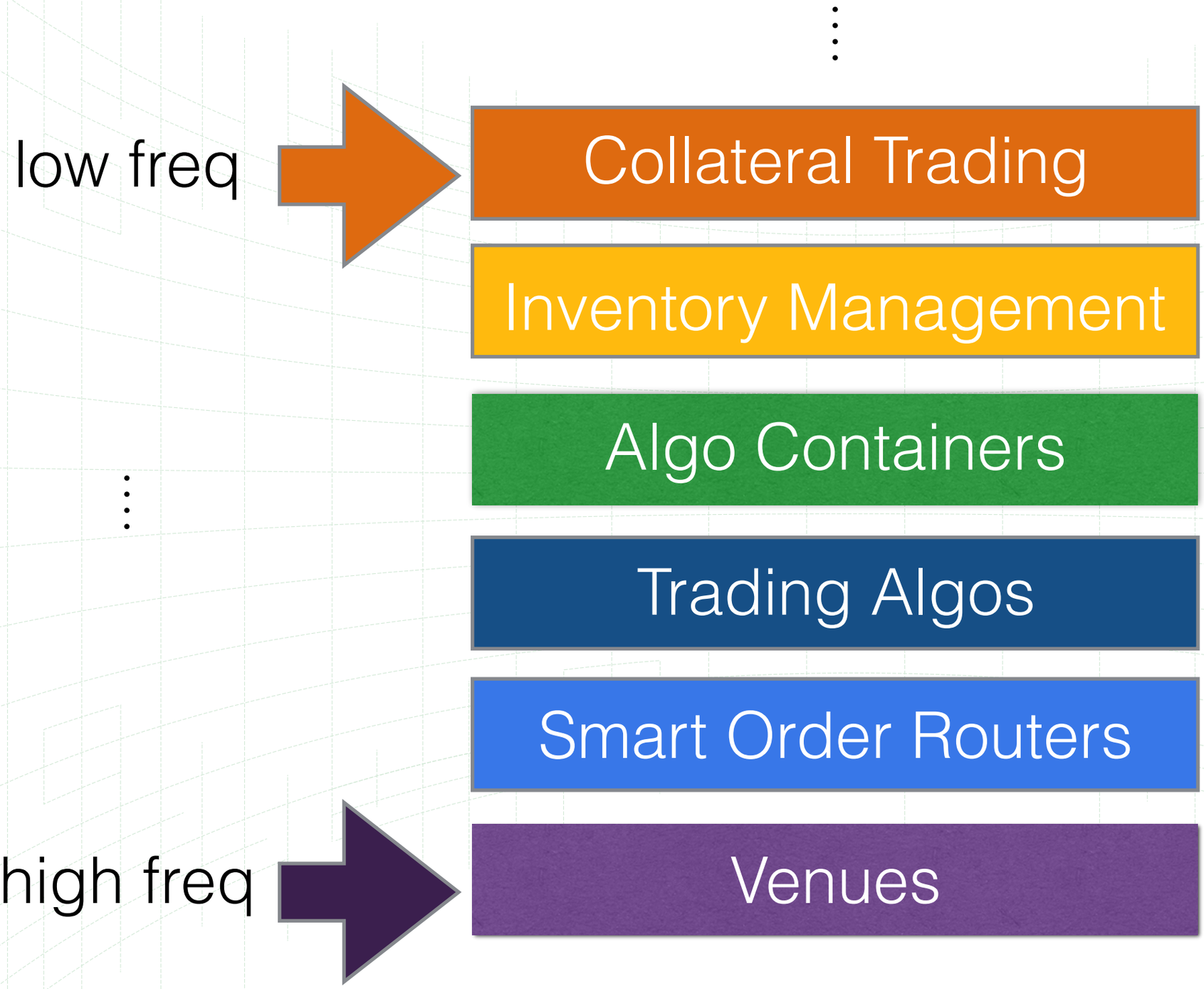

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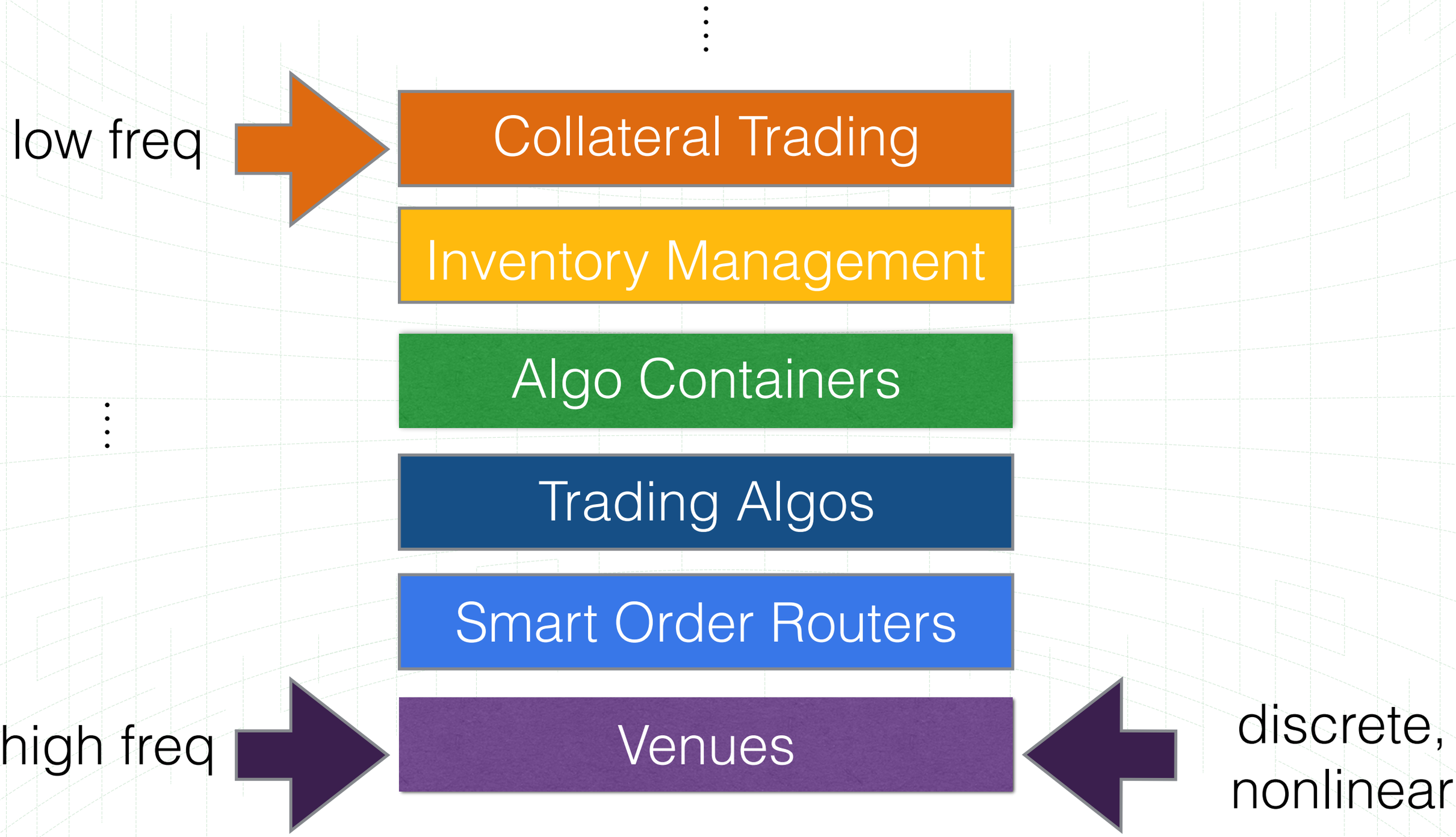

The Stack of Financial Algorithms

low freq

Collateral Trading

Inventory Management

Algo Containers

Trading Algos

Smart Order Routers

Venues

high freq

$$X_t = X_0 + \int_0^t \mu_s \, ds + \int_0^t \sigma_s \, dB_s$$

continuous, nonlinear

discrete, nonlinear

What is a venue?

London
Stock Exchange

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London Stock Exchange

NYSE

Nasdaq

Turquoise

iex

AQUIS EXCHANGE

eurex

BATS Making Markets Better

SIX Swiss Exchange

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eurex

BATS Making Markets Better

SIX Swiss Exchange

Deutsche Börse

Morgan Stanley

UBS

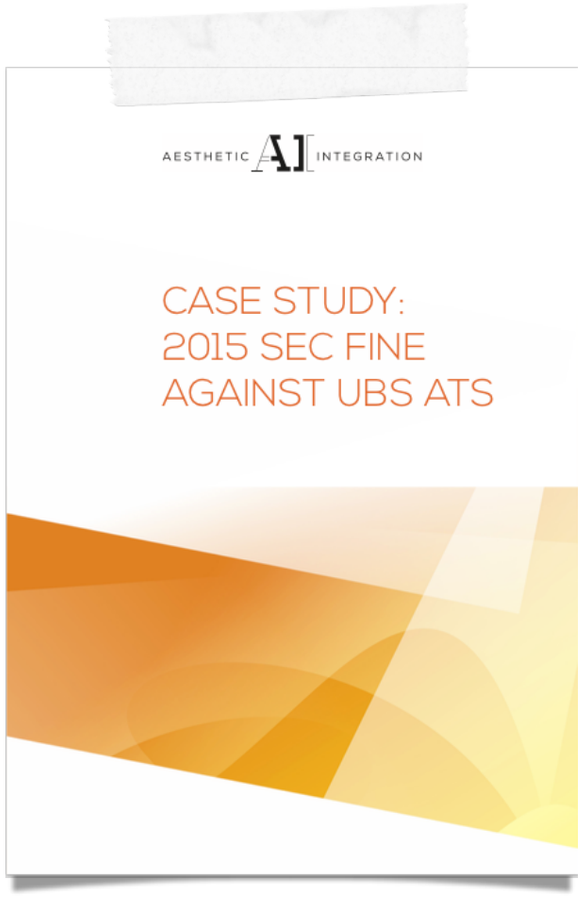
Goldman Sachs

What is a venue?

LIT LIQUIDITY

DARK LIQUIDITY

Running Example: UBS ATS



620 companies
52 countries

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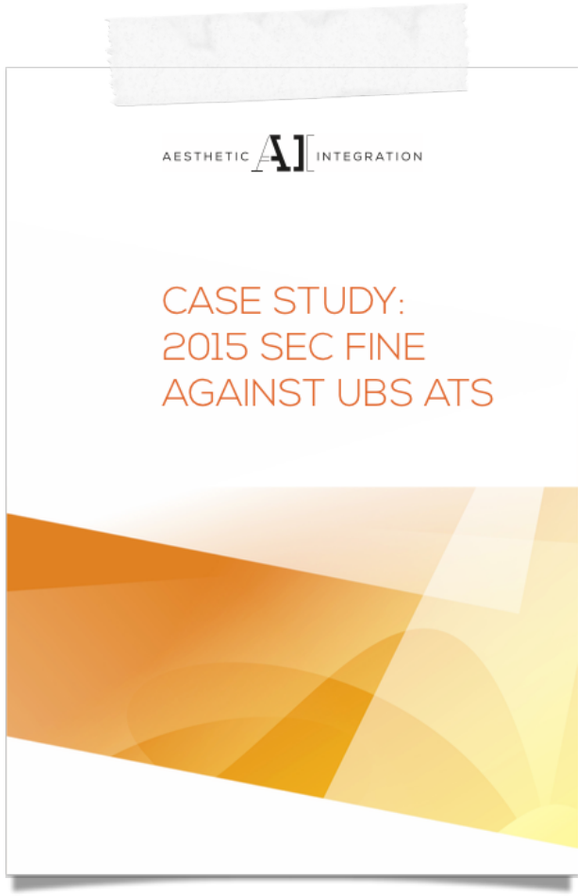

Jan, 2015: UBS fined $14M by the SEC for issues of unfairness in their dark pool design

We analysed it, found more issues

620 companies
52 countries

Running Example: UBS ATS

Form ATS Page 1 Execution Page	UNITED STATES SECURITIES AND EXCHANGE COMMISSION WASHINGTON, D.C. 20549 INITIAL OPERATION REPORT, AMENDMENT TO INITIAL OPERATION REPORT AND CESSATION OF OPERATIONS REPORT FOR ALTERNATIVE TRADING SYSTEMS	Date filed (MM/DD/YY): 6/1/15	OFFICIAL USE ONLY

WARNING: Failure to keep this form current and to file accurate supplementary information on a timely basis, or the failure to keep accurate books and records or otherwise to comply with the provisions of law applying to the conduct of alternative trading systems would violate the federal securities laws and may result in disciplinary, administrative or criminal action.
INTENTIONAL MISSTATEMENTS OR OMISSIONS OF FACTS MAY CONSTITUTE CRIMINAL VIOLATIONS

☐ INITIAL OPERATION REPORT ■ AMENDMENT TO INITIAL OPERATION REPORT ☐ CESSATION OF OPERATIONS REPORT

1. Exact name, principal business address, mailing address, if different, and telephone number of alternative trading system:

A. Full name of alternative trading system (if sole proprietor, last, first and middle name):
UBS ATS

B. Name(s) under which business is conducted, if different from Item 1A:
UBS Securities LLC

C. CRD Number: 7654 D. SEC File No.: 8- 22651

E. If this filing makes a name change on behalf of the alternative trading system, enter the previous name and specify whether the name change is of the _____ alternative trading system name (1A), or _____ business name (1B):
Previous name:

F. Alternative trading system's main street address (Do not use a P.O. Box):
1285 Avenue of the Americas, New York, NY 10019

G. Mailing address (if different):

H. Business telephone and facsimile number:
[redacted] [redacted]
(Telephone) (Facsimile)

I. Contact employee:
[redacted] [redacted]
(Name and Title) (Telephone Number) (Facsimile)

EXECUTION: The alternative trading system consents that service of any civil action brought by, or notice of any proceeding before, the SEC or a self-regulatory organization in connection with the alternative trading system's activities may be given by registeredor certified mail or confirmed telegram, to the alternative trading system's contact employee at the main address, or mailing address if differengiven in Items 1F and 1G. The undersigned, being first duly sworn, deposes and says that he/she has executed this form on behalf oand with the authority of, said alternative trading system. The undersigned and alternative trading system represent that the information and statements contained herein, including exhibits, schedules, or other documents attached hereto, and other information filed herewith, all of which are made a pa hereof, are current, true, and complete.

Date: 06/01/2015 UBS ATS
(Name of applicant)
By: [redacted]
(Printed Name and Title)
Subscribed and sworn before me this 1 day of June 2015 by [redacted]
(Month) (Year)
My Commission expires 21 Dec 2017 County of [redacted] State of [redacted]

This page must always be completed in full with original, manual signature and notarization.
Affix notary stamp or seal where applicable.

DO NOT WRITE BELOW THIS LINE - FOR OFFICIAL USE ONLY

DEBBIE ANN PALOCIN
NOTARY PUBLIC-STATE OF NEW YORK
No. 01PA6215204
Qualified in New York County
My Commission Expires December 21, 2017

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Running Example: UBS ATS

Let's examine an actual regulatory disclosure (esp. Sec 4.1)

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C. CRD Num

E. If this filing
specify wh
business
Previous n

F. Alternative
1285 Aven

G. Mailing ad

H. Business telephone and facsimile number:
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(Telephone) (Facsimile)

I. Contact employee:
[redacted] [redacted]
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Affix notary stamp or seat where applicable.

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My Commission Expires December 21, 2017

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4

What is Imandra?

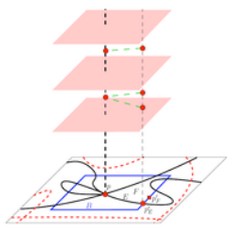
- Programming language
- Mathematical logic
- Reasoning engine

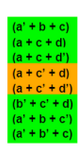
IMANDRA

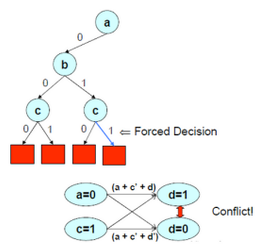
What is Imandra?

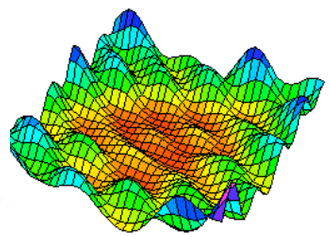
OCaml + Automated Reasoning

- Programming language
- Mathematical logic
- Reasoning engine
  - First-class counterexamples
  - Nonlinear + SE decomposition
  - Proof automation for various financial regulations
  - Test suite generation
  - Documentation generation









IMANDRA

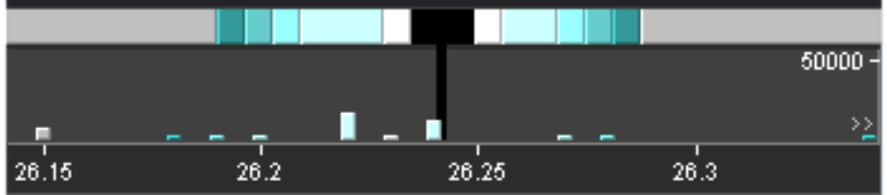
What does a venue do?

- maintain an **order book**
- process incoming **orders**
- **match** orders (‘trade’!)
- send `**fills**’
- route orders **away** (`best-ex’)
- **report** on market activity

…all according to a *(precisely?)* defined ‘spec’
…while obeying many complex regulations

What is an order book?

Symbol	MSFT	Volume	56,071,087
Last	26.23	High	26.49
Change	-0.25	Low	25.70



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Shares	MPID	Bid	Shares	MPID	Ask
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300	MSCO	26.07	100	FBCO	26.49
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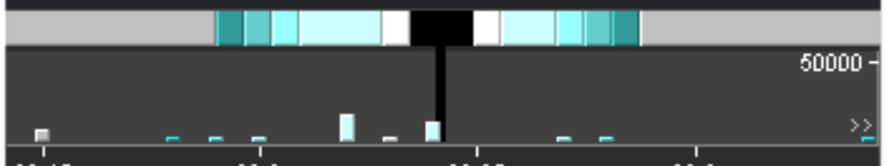

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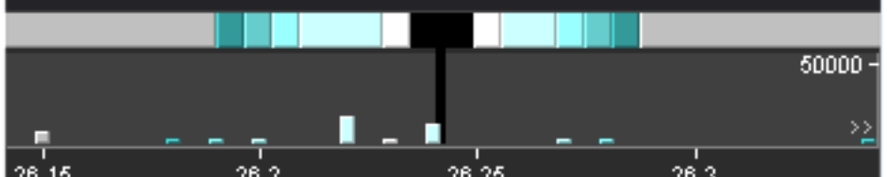

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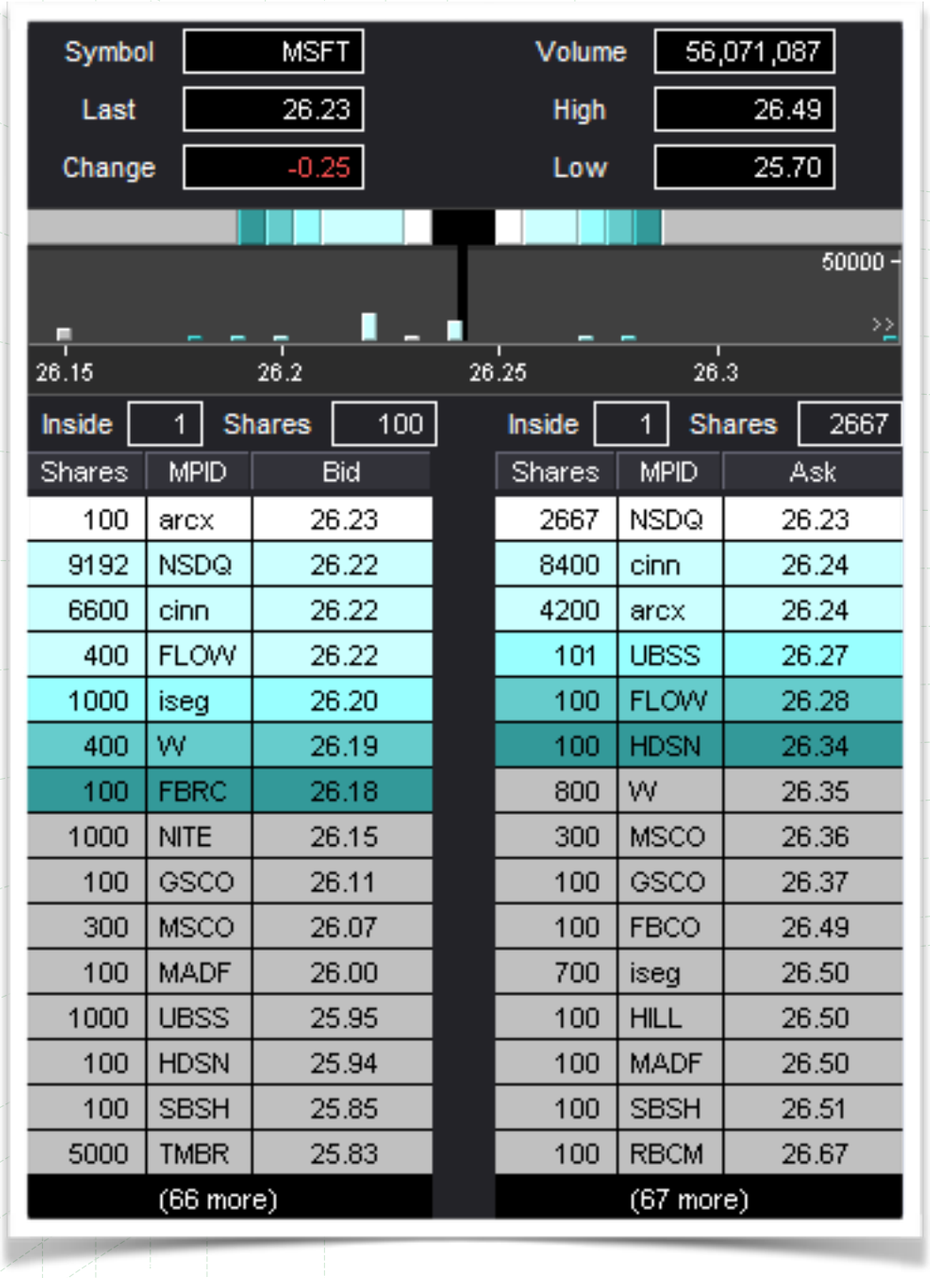
at each discrete time-step, the book is sorted.

What is an order book?



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50000

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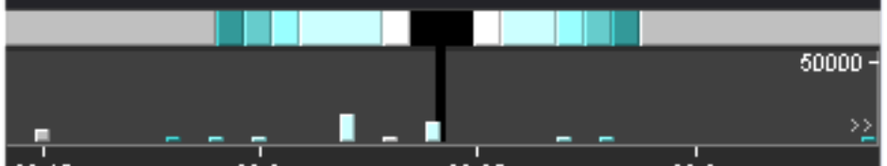

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VERY COMPLEX ANSWER!

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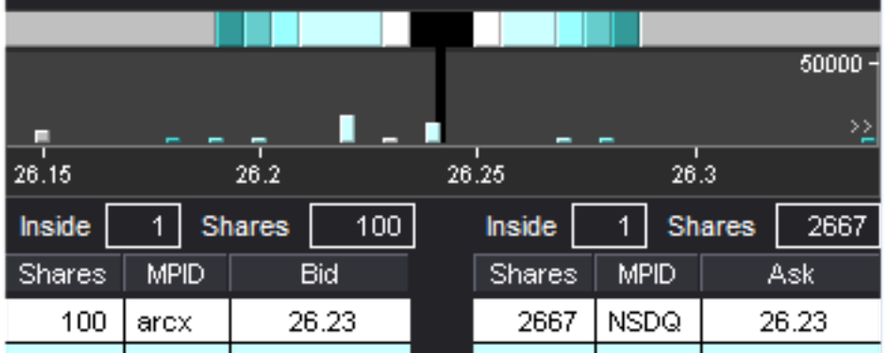

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INTUITION:
Price/Time Priority

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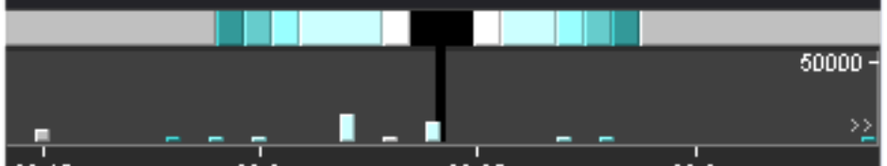

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100	FBRC	26.18		800	W	26.35	
1000	NITE	26.15		300	MSCO	26.36	
100	GSCO	26.11		100	GSCO	26.37	
300	MSCO	26.07		100	FBCO	26.49	
100	MADF	26.00		700	iseg	26.50	
1000	UBSS	25.95		100	HILL	26.50	
100	HDSN	25.94		100	MADF	26.50	
100	SBSH	25.85		100	SBSH	26.51	
5000	TMBR	25.83		100	RBCM	26.67	
(66 more)				(67 more)			

```
type order_book = { buys  : order list;
                    sells : order list }
```

how is it sorted?

INTUITION:
Price/Time Priority

REALITY:
Let's see!

What is an order?

an instruction to

- **buy or sell** a given security
- in a **specified manner**,
- subject to **market constraints**, and
- order **parameters**.

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"buy 100 shares of MSFT, with price at most $50"

What is an order?

an instruction to

- **buy or sell** a given security
- in a **specified manner**,
- subject to **market constraints**, and
- order **parameters**.

"buy 100 shares of MSFT, with price at most $50"

"buy 100 shares of MSFT"

What is an order?

```
o1 = { id = 437;
       peg = NEAR;
       client_id = 5;
       order_type = FIRM_UP_LIMIT;
       qty = 2438;
       min_qty = 60;
       leaves_qty = 0;
       price = 2533;
       time = 8857;
       src = 7;
       order_attr = RESIDENT;
       capacity = Principal;
       category = C_ONE;
       cross_restrict = { cr_self_cross = false;
                          cr_ubs_principal = false;
                          cr_round_lot_only = true;
                          cr_no_locked_nbbo = false;
                          cr_pegged_mid_point_mode = 3;
                          cr_enable_conditionals = true;
                          cr_min_qty = false;
                          cr_cat_elig = { c_one_elig = false;
                                          c_two_elig = false;
                                          c_three_elig = false;
                                          c_four_elig = false; }; };
        locate_found = false;
        expiry_time = 8; };
```

What is an order type?

MARKET ORDER

What is an order type?

MARKET ORDER

LIMIT ORDER

What is an order type?

MARKET ORDER

LIMIT ORDER

ICEBERG ORDER

What is an order type?

MARKET ORDER

LIMIT ORDER

ICEBERG ORDER

STOP LOSS ORDER

What is an order type?

Simplicity Is the Goal of Nasdaq's New Order Type, CEO Says

by Annie Massa
antoniabmassa

August 15, 2016 — 11:26 PM CEST

- CEO Greifeld expects to release new order by end of year
- Makes an appeal to investors, as IEX prepares for exchange

Nasdaq Inc. is responding to a competitor preparing to enter the exchange arena.

Nasdaq plans to offer a new order type aimed at long-term investors, the company announced Monday. The exchange operator expects to have the new order available for use by the end of year, said Nasdaq Chief

What is an order type?

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'Hide Not Slide' Orders Were Slippery and Hidden

12 JAN 12, 2015 7:35 PM EST

By Matt Levine

Today, the Securities and Exchange Commission fined the Direct Edge stock exchanges $14 million for violations involving their "Hide Not Slide" order types.[1] Here's a 2012 Wall Street Journal article that comes with basically a graphic novel devoted to how a "Hide Not Slide" order works, and I refer you to there if you want to know how it works. The thing is that you probably don't want to know how it works. But here's the basic idea, without the cartoon of a jumping man in a suit:[2]

What is an order type?

MarketWatch

NEWS

4:00 A.M. ET This flying egg is actuall

3:47 A.M. ET Oil prices slide after jun

NEWS VIEWER | MARKETS | INVESTING | TRADING DECK | PERSONAL FINANCE | RETIREMENT | ECONOMY

Home › Economy & Politics

SEC fines exchange over 'queue-jumping' orders

Published: Jan 13, 2015 9:11 a.m. ET

2

WASHINGTON (MarketWatch) – BATS Global Markets Inc. agreed to a $14 mil
settlement with federal regulators over charges that two exchanges it acquir
year did not accurately describe order types to customers, officials said Mon

Nasdaq plans to offer a new order type aimed at long-term investors, the company announced Monday. The exchange operator expects to have the new order available for use by the end of year, said Nasdaq Chief

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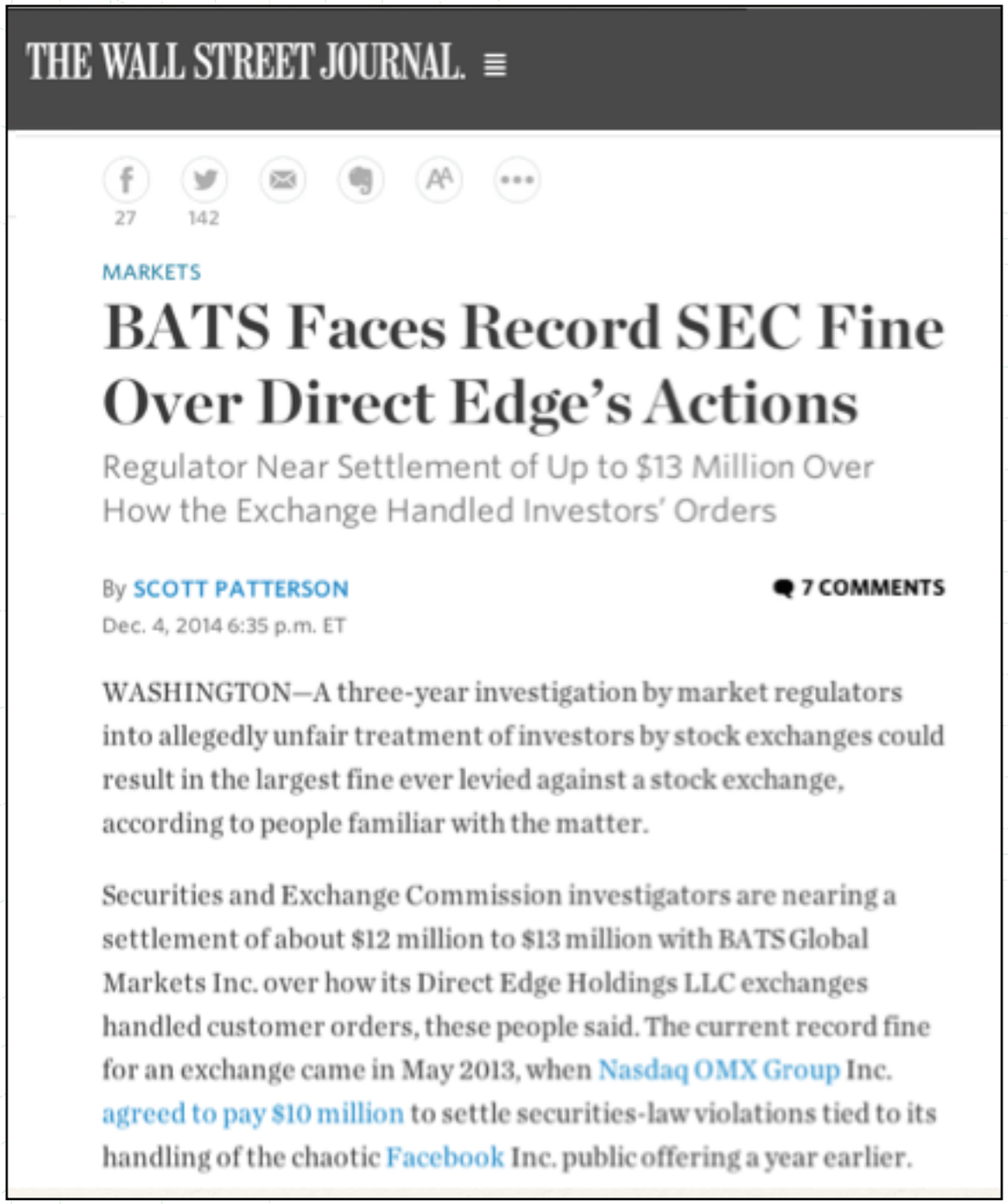
Is your venue fair?



THE WALL STREET JOURNAL.

27 142

MARKETS

BATS Faces Record SEC Fine Over Direct Edge's Actions

Regulator Near Settlement of Up to $13 Million Over How the Exchange Handled Investors' Orders

By SCOTT PATTERSON

7 COMMENTS

Dec. 4, 2014 6:35 p.m. ET

WASHINGTON—A three-year investigation by market regulators into allegedly unfair treatment of investors by stock exchanges could result in the largest fine ever levied against a stock exchange, according to people familiar with the matter.

Securities and Exchange Commission investigators are nearing a settlement of about $12 million to $13 million with BATS Global Markets Inc. over how its Direct Edge Holdings LLC exchanges handled customer orders, these people said. The current record fine for an exchange came in May 2013, when Nasdaq OMX Group Inc. agreed to pay $10 million to settle securities-law violations tied to its handling of the chaotic Facebook Inc. public offering a year earlier.

Difficult questions:

- Is your venue *fair*?
- Can you *prove* it?
- If it's not fair, how can you *fix* it?
- Can your collection of order-types ever *violate* regulatory directives?
- Does your high-performance *implementation* conform to your high-level design specification?
- Does your *documentation* of your order-types truly match your implementation?
- How can you *automate* both *testing* and *compliance*?
- What is the *strongest possible evidence* you can give to regulators?

IMANDRA VENUES

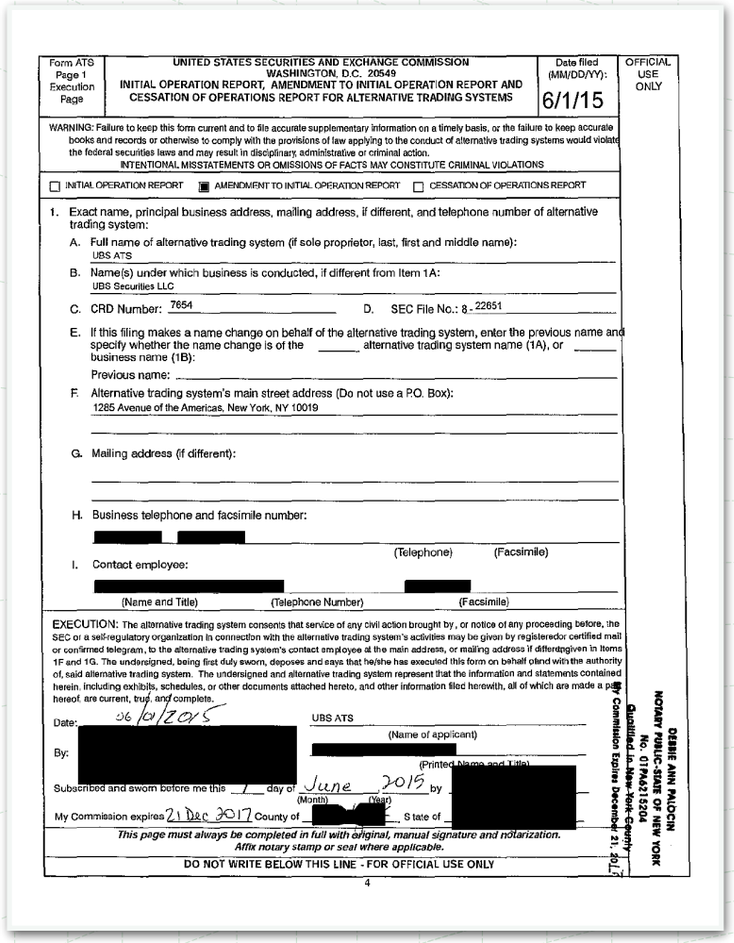
Formal analysis of trading venues

- Analysis of safety and fairness properties of trading venues (dark pools, exchanges, etc.)
- In use at top global investment banks
- Principled way to manage growing order-type proliferation
- Exciting developments in regulatory space (more soon…!)

```
| (Quote,  Limit)   ->
   if bb.order_time > bs.order_time then
     (* incoming quote *)
     begin
       let nextSellLimit = next_sell s in
       if bb.order_qty < bs.order_qty then Known bs.order_price
       else if bb.order_qty = bs.order_qty then
         match nextSellLimit with
         | None      -> Known bb.order_price
         | Some ord  -> Known ord.order_price
       else Unknown
     end
   else
     (* existing quote's price is used *)
     Known bb.order_price

| (Quote, Market) ->
   if bb.order_time > bs.order_time then
     (* incoming quote *)
     begin
       let nextSellLimit = next_sell s in
       if bb.order_qty < bs.order_qty then Known bs.order_price
       else if bb.order_qty = bs.order_qty then
         match nextSellLimit with
         | None      -> Known bb.order_price
         | Some ord  -> Known ord.order_price
       else Unknown
     end
   else
     (* existing quote's price is used *)
     Known bb.order_price
```

Running Example: UBS ATS



Form ATS Page 1 Execution Page

UNITED STATES SECURITIES AND EXCHANGE COMMISSION
WASHINGTON, D.C. 20549
INITIAL OPERATION REPORT, AMENDMENT TO INITIAL OPERATION REPORT AND CESSATION OF OPERATIONS REPORT FOR ALTERNATIVE TRADING SYSTEMS

Date filed (MM/DD/YY): 6/1/15

OFFICIAL USE ONLY

WARNING: Failure to keep this form current and to file accurate supplementary information on a timely basis, or the failure to keep accurate books and records or otherwise to comply with the provisions of law applying to the conduct of alternative trading systems would violate the federal securities laws and may result in disciplinary, administrative or criminal action.
INTENTIONAL MISSTATEMENTS OR OMISSIONS OF FACTS MAY CONSTITUTE CRIMINAL VIOLATIONS

☐ INITIAL OPERATION REPORT ■ AMENDMENT TO INITIAL OPERATION REPORT ☐ CESSATION OF OPERATIONS REPORT

1. Exact name, principal business address, mailing address, if different, and telephone number of alternative trading system:

A. Full name of alternative trading system (if sole proprietor, last, first and middle name):
UBS ATS

B. Name(s) under which business is conducted, if different from Item 1A:
UBS Securities LLC

C. CRD Number: 7654 D. SEC File No.: 8- 22651

E. If this filing makes a name change on behalf of the alternative trading system, enter the previous name and specify whether the name change is of the _____ alternative trading system name (1A), or _____ business name (1B):
Previous name:

F. Alternative trading system's main street address (Do not use a P.O. Box):
1285 Avenue of the Americas, New York, NY 10019

G. Mailing address (if different):

H. Business telephone and facsimile number:
[redacted] [redacted] (Telephone) (Facsimile)

I. Contact employee:
[redacted] [redacted] [redacted]
(Name and Title) (Telephone Number) (Facsimile)

EXECUTION: The alternative trading system consents that service of any civil action brought by, or notice of any proceeding before, the SEC or a self-regulatory organization in connection with the alternative trading system's activities may be given by registered or certified mail or confirmed telegram, to the alternative trading system's contact employee at the main address, or mailing address if different given in Items 1F and 1G. The undersigned, being first duly sworn, deposes and says that he/she has executed this form on behalf of and with the authority of, said alternative trading system. The undersigned and alternative trading system represent that the information and statements contained herein, including exhibits, schedules, or other documents attached hereto, and other information filed herewith, all of which are made a part hereof, are current, true, and complete.

Date: 06/01/2015 UBS ATS
(Name of applicant)
By: [redacted]
(Printed Name and Title)
Subscribed and sworn before me this 1 day of June 2015 by [redacted]
(Month) (Year)
My Commission expires 21 Dec 2017 County of [redacted] State of [redacted]

This page must always be completed in full with original, manual signature and notarization.
Affix notary stamp or seal where applicable.

DO NOT WRITE BELOW THIS LINE - FOR OFFICIAL USE ONLY

4

DEBBIE ANN PALOCIN
NOTARY PUBLIC-STATE OF NEW YORK
No. 01PA6215204
Qualified in New York County
My Commission Expires December 21, 2017

Demo:

Transitivity of order ranking

Example: SIX Swiss Exchange

Trading Period Overview

			Pre-Opening	Opening	Continuous Trading	End of Trading	
						Without Closing Auction	With Closing Auction
	Duration		06:00 CET until Opening	not applicable	Opening until End of Trading	not applicable	10 Minutes
	Random Time		not applicable	2 Minutes	not applicable	None	2 Minutes at Run Auction and Close
Order Entry	STI Orders	Day	Yes	No	Yes	No	Yes
		Immediate or Cancel	No	No	Yes	No	No
		Fill or Kill	No	No	Yes	No	No
		At the Opening	Yes	No	No	No	No
		At the Close	Yes	No	Yes	No	Yes
		Good for Date	Yes	No	Yes	No	Yes
	OTI Orders	Day	Yes	No	Yes	No	Yes
		Immediate or Cancel	No	No	Yes	No	No
		Fill or Kill	No	No	Yes	No	No
		At the Opening	Yes	No	No	No	No
		At the Close	Yes	No	Yes	No	Yes
	QTI Quotes		Yes	No	Yes	No	Yes
Order Deletion & Expiry	STI Orders	Day	Yes	No	Yes	No	Yes Order expiry after End of Trading
		Immediate or Cancel	No	No	Order deletion of no execution	No	No
		Fill or Kill	No	No	Order deletion of no execution	No	No
		At the Opening	Yes	Order expiry if not executed in Opening	No	No	No
		At the Close	Yes	No	Yes	No	Yes Order expiry if not executed in Auct
		Good for Date	Yes	No	Yes	No	Yes
	OTI Orders	Day	Yes	No	Yes	No	Yes Order expiry after End of Trading
		Immediate or Cancel	No	No	Order deletion of no execution	No	No
		Fill or Kill	No	No	Order deletion of no execution	No	No
		At the Opening	Yes	Order expiry if not executed in Opening	No	No	No
		At the Close	Yes	No	Yes	No	Yes Order expiry if not executed in Auct
	QTI Quotes		Yes	No	Yes	No	Yes
	Executions		No	Executions according to largest best execution principle	Continuous execution of orders and quotes	No	*During Closing Auction* No *At Run Auction and Close* Executions according to largest best execution principle
	Price Calculation		Theoretical Opening Price	Opening Price	Reference Price	Closing Price	Closing Price
	Exceptional Order Book Situations		None	Delay Opening Non Opening	Stop Trading Non Opening Underlying Condition	None	Non Opening
	Off Order Book Reporting		Yes Limit to trade on order book not applicable	Yes Observe "Limit to trade on order book" where applicable	Yes Observe "Limit to trade on order book" where applicable	Yes Observe "Limit to trade on order book" where applicable	Yes Observe "Limit to trade on order bo where applicable

			Pre-Opening
	Duration		06:00 CET until Opening
	Random Time		not applicable
Order Entry	STI Orders	Day	Yes
		Immediate or Cancel	No
		Fill or Kill	No
		At the Opening	Yes
		At the Close	Yes
		Good for Date	Yes
	OTI Orders	Day	Yes
		Immediate or Cancel	No
		Fill or Kill	No
		At the Opening	Yes
		At the Close	Yes
	QTI Quotes		Yes
Order Deletion & Expiry	STI Orders	Day	Yes
		Immediate or Cancel	No
		Fill or Kill	No
		At the Opening	Yes
		At the Close	Yes
		Good for Date	Yes

Pricing Logic: Informal

5. Example: Execution of an incoming quote in order book at best quote or at best remaining limit when quote volume < bid/ask volume and book contains limit orders and quotes.

Example 5.1: Incoming quote, buy 550 units, limit CHF 75.00

Type	Bid Size	Bid	Ask	Ask Size	Type
➔Quote	550	75.00	71.00	200	Order
			72.00	200	Order
			73.00	100	Quote
			74.00	100	Order

Type	Bid Size	Price	Ask Size	Type
➔Quote	550	75.00		
		74.00	100	Order
		73.00	100	Quote
		72.00	200	Order
		71.00	200	Order

Pricing Logic: Formal

```
let match_price (s : exchange_state) =
  let bb = best_buy s in
  let bs = best_sell s in
  match bb, bs with
    Some bb, Some bs ->
    begin
      match bb.order_type, bs.order_type with
      | (Limit, Limit) ->
         Known (older_price (bb, bs))

      | (Quote, Quote) ->
         Known (older_price (bb, bs))

      | (Market, Market) ->
         if bb.order_qty <> bs.order_qty then Unknown
         else
           (* need to look at other orders in the order book *)
           let bestBuy = next_buy s in
           let bBid =
             match bestBuy with
               Some bestBuy ->
               if bestBuy.order_type = Market then None
               else Some bestBuy.order_price
             | _ -> None in

           let bestSell = next_sell s in
           let bAsk =
             match bestSell with
               Some bestSell ->
               if bestSell.order_type = Market then None
               else Some bestSell.order_price
             | _ -> None in
           begin
             match bBid, bAsk with
             | (None, None) -> Known s.ref_price
             | (None, Some ask) ->
               if ask < s.ref_price then Known ask
               else Known s.ref_price
             | (Some bid, None) ->
               if bid > s.ref_price then Known bid
               else Known s.ref_price
             | (Some bid, Some ask) ->
               if bid > s.ref_price then Known bid
               else
                 if ask < s.ref_price then Known ask
                 else Known s.ref_price
           end

      | (Market, Limit)   -> Known bs.order_price

      | (Limit, Market)   -> Known bb.order_price

      | (Quote,  Limit)   ->
        if bb.order_time > bs.order_time then
          (* incoming quote *)
          begin
            let nextSellLimit = next_sell s in
            if bb.order_qty < bs.order_qty then Known bs.order_price
            else if bb.order_qty = bs.order_qty then
              match nextSellLimit with
              | None      -> Known bb.order_price
              | Some ord  -> Known ord.order_price
            else Unknown
          end
        else
          (* existing quote's price is used *)
          Known bb.order_price

      | (Quote, Market) ->
        if bb.order_time > bs.order_time then
          (* incoming quote *)
          begin
            let nextSellLimit = next_sell s in
            if bb.order_qty < bs.order_qty then Known bs.order_price
-:---  Exchange.ml    45% (264,69)  Git-master  (Tuareg) 11:08AM 1.92
```

```
| (Quote,  Limit)   ->
   if bb.order_time > bs.order_time then
     (* incoming quote *)
     begin
       let nextSellLimit = next_sell s in
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       else Unknown
     end
   else
     (* existing quote's price is used *)
     Known bb.order_price
```

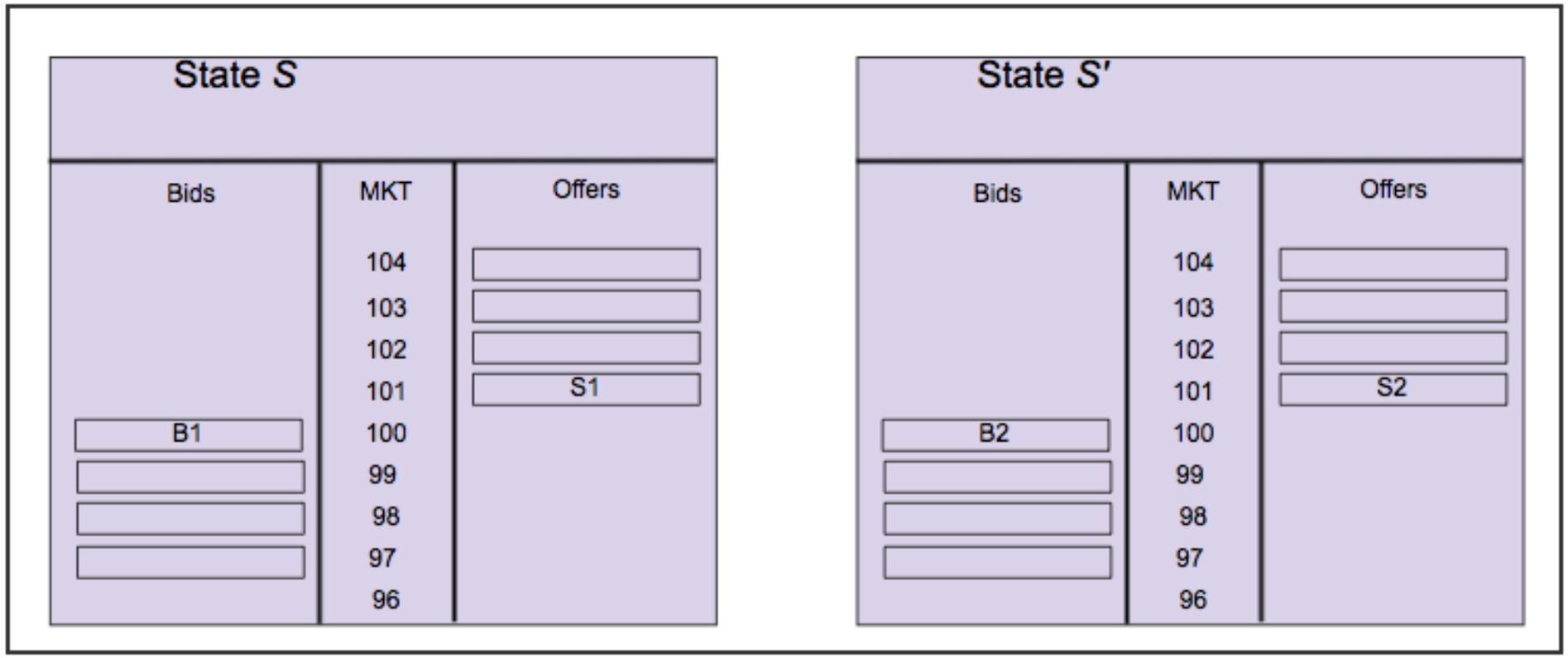
Pricing Fairness Example

- Consider specification of the SIX Swiss matching logic - we will prove that client ID, although used by the system, does not factor into pricing and matching decisions
- We will use IML to encode both the matching logic and the fairness principle, and then use Imandra to reason about the model
- Our example highlights iterative nature of the specification process: We discover, through a non-trivial counter-example, that our original hypothesis is incorrect. We then update our specification or model accordingly and iterate.

Pricing Fairness Example

- The following is an overview of the SIX Swiss Matching Engine
- Three market models: CLOB, MMB, and MMB-FoK
- Six business periods, 5 order types with numerous attributes
- 22 product types across equities, bonds, funds, and structured products with specific trading parameters
- Randomised auction times, volatility circuit breakers, order validities, and regulatory reporting requirements
- Complexity stems from the need to meet diverse client needs while operating a heavily regulated business

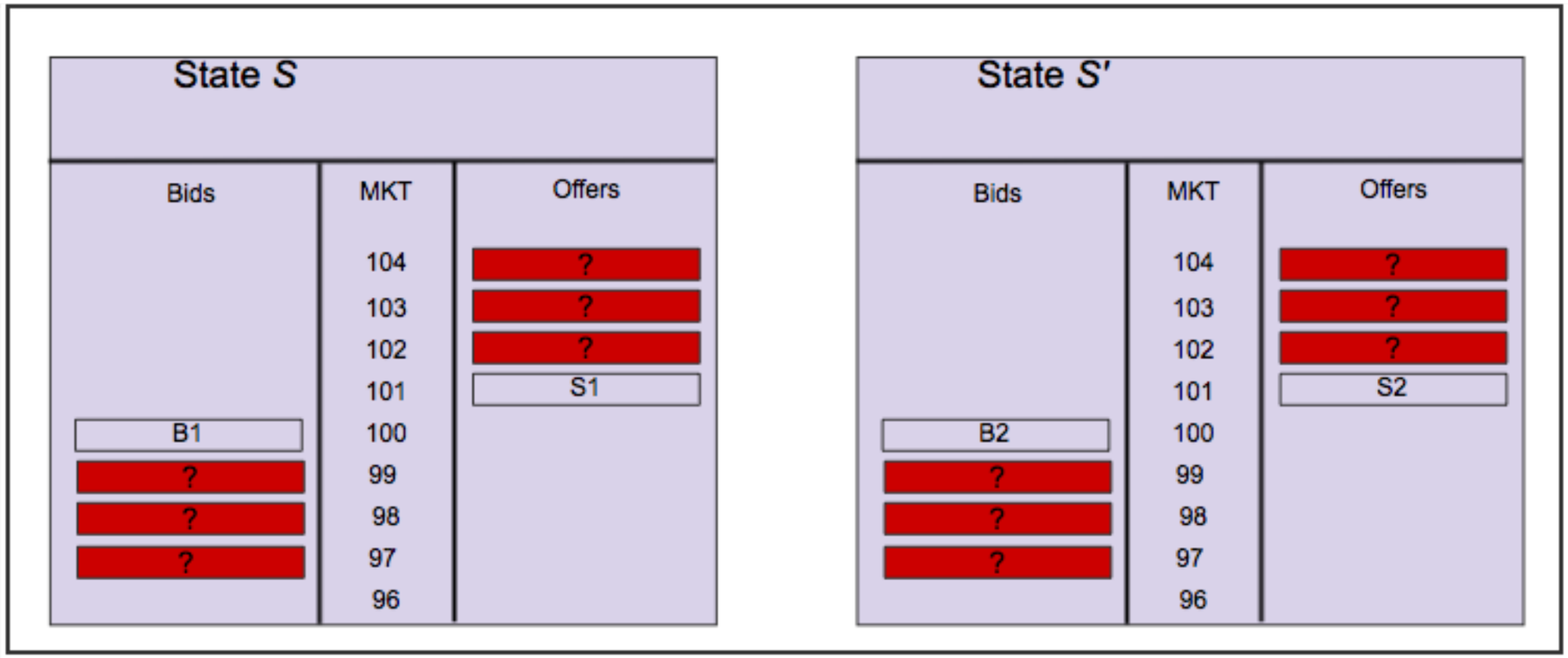
Pricing Fairness Example



- B_1 == B_2 except Client ID
- S_1 == S_2 except Client ID

? → MatchPrice(S) = MatchPrice(S′)

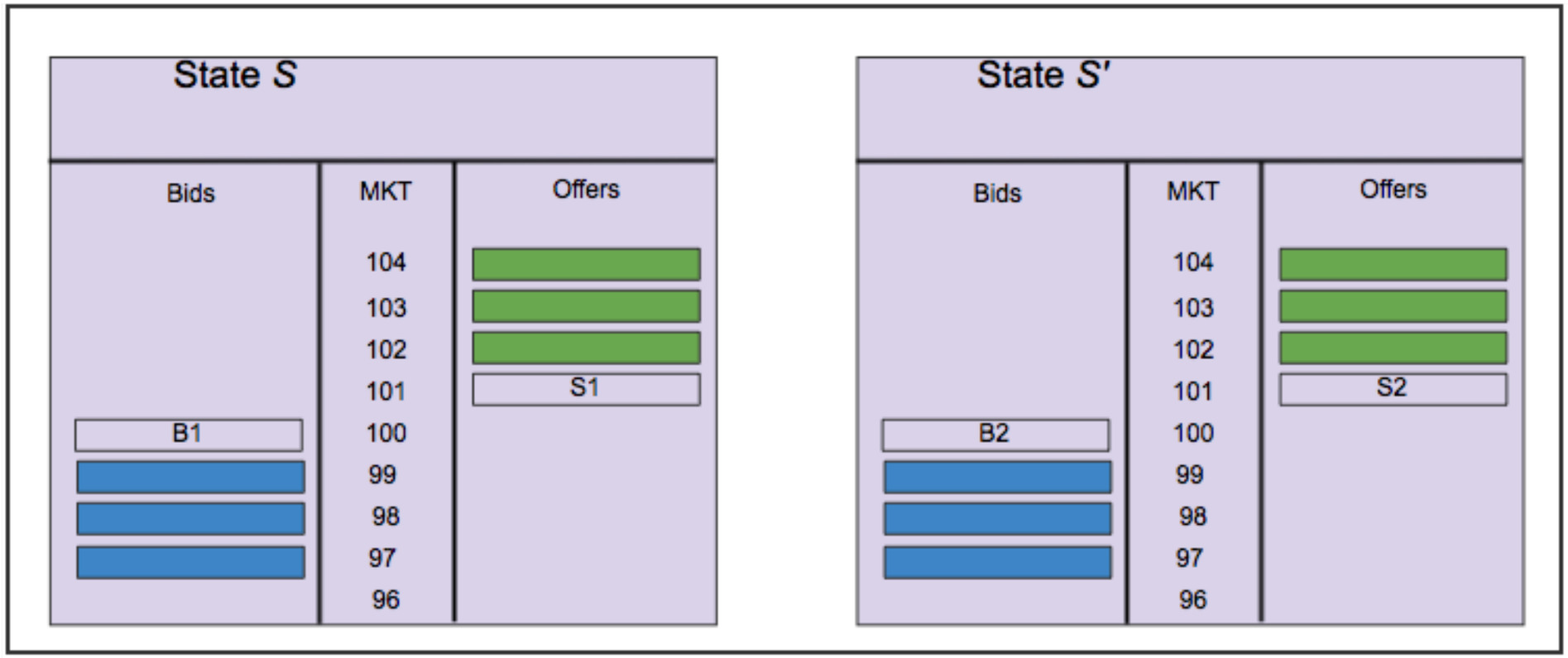
Pricing Fairness Example



- B_1 == B_2 except Client ID
- S_1 == S_2 except Client ID

NO → MatchPrice(S) = MatchPrice(S′)

Pricing Fairness Example

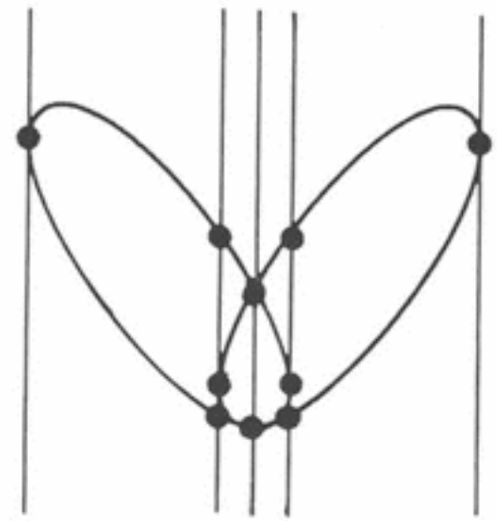


- B_1 == B_2 except Client ID
- S_1 == S_2 except Client ID

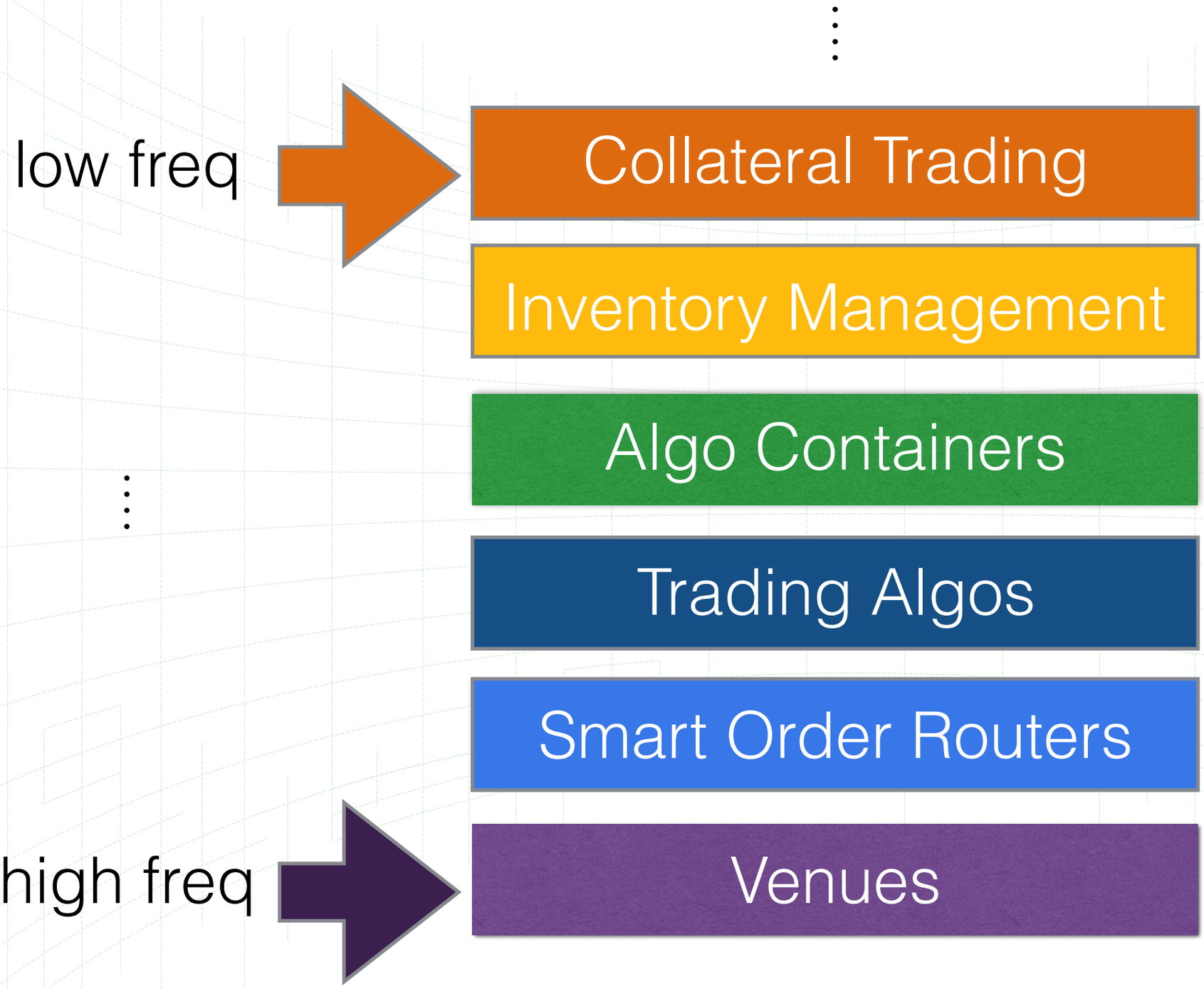
YES → MatchPrice(S) = MatchPrice(S′)

Principal Region Decomposition

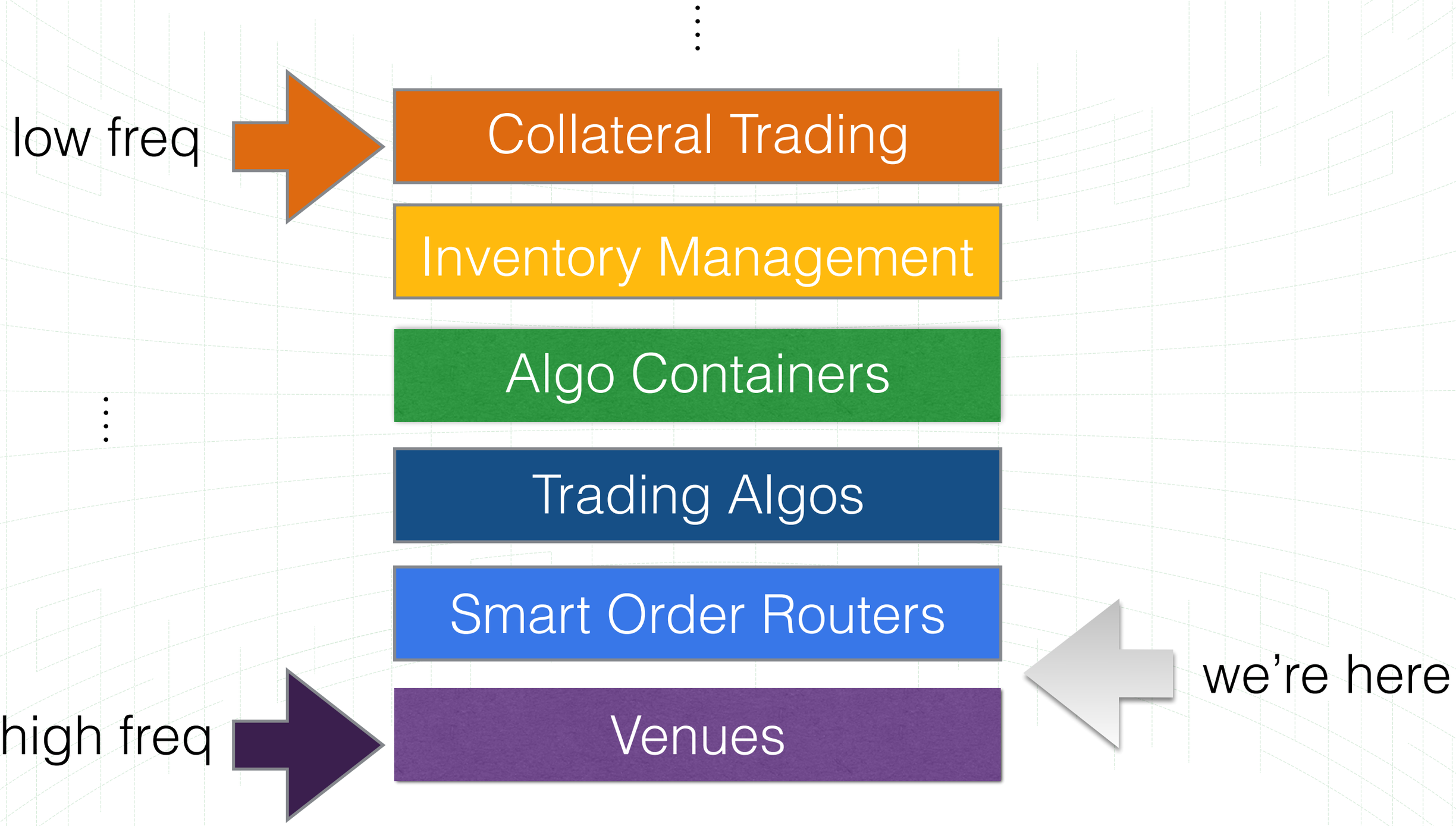
- symbolic execution modulo nonlinear sign-invariance
- intuitively, a generalisation of *cylindrical algebraic decomposition* to programs
- the basis of IMANDRA's test-suite and documentation generation



The Stack of Financial Algorithms



The Stack of Financial Algorithms



The Stack of Financial Algorithms

low freq

Collateral Trading

Inventory Management

Algo Containers

Trading Algos

Smart Order Routers

Venues

high freq

$$X_t = X_0 + \int_0^t \mu_s \, ds + \int_0^t \sigma_s \, dB_s$$

how do we get here?

we're here

Formalized Financial Mathematics

$$dX(t) = rX(t)\ dt + \sigma X(t)\ dB(t).$$

$$p(\tau;\ x, y) = \frac{1}{\sigma y\sqrt{2\pi\tau}} \exp\left\{-\frac{1}{2\tau\sigma^2}\left[\log\frac{y}{x} - (r - \tfrac{1}{2}\sigma^2)\tau\right]^2\right\}.$$

Assuming every order is a MARKET ORDER is ridiculous.

We need *new* financial mathematics that takes the precise discrete market microstructure into account.

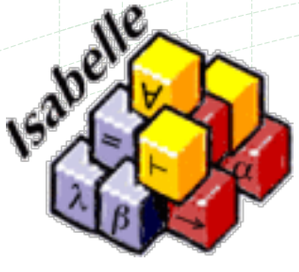
Formalized Financial Mathematics

$$dX(t) = rX(t)\ dt + \sigma X(t)\ dB(t).$$

$$p(\tau;\ x, y) = \frac{1}{\sigma y\sqrt{2\pi\tau}} \exp\left\{-\frac{1}{2\tau\sigma^2}\left[\log\frac{y}{x} - (r - \tfrac{1}{2}\sigma^2)\tau\right]^2\right\}.$$

Assuming every order is a MARKET ORDER is ridiculous.

We need *new* financial mathematics that takes the precise discrete market microstructure into account.



Stochastic Control

Stochastic Calculus

Brownian Motion

Wiener Processes

Martingales

Conclusion

- Pressing need for:
  - venues to be bullet-proof w.r.t. safety and fairness regulations
  - matching logics to be formally described to regulators and market participants
  - matching logics to be formally analysed w.r.t. precise encodings of regulatory directives
  - financial mathematics (stochastic calculus) that takes precise discrete behaviour of matching logics into account
  - this is a ***killer app*** for formal methods!