



Using Axe to Reason About Binary Code

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Goal

- Lift binary code into logic
 - JVM bytecode
 - x86 binary code
- Then
 - verify against a spec
 - using Axe
 - or by constructing an APT derivation
 - analyze / prove properties
 - equivalence check two implementations
 - compare to malware
 - run on concrete data

Step 0: Parse the binary

- Parsers for Mach-O and PE (Windows) binaries.
- Build an ACL2 constant representing the binary.

Parsed Mach-O binary for TEA (Tiny Encryption Algorithm)

302 lines total

(DEFCONST *wiki* ((:MAGIC . :MH MAGIC 64) (:HEADER (:CPUTYPE . :CPU_TYPE_X86_64) (:CPUSUBTYPE . 2147483651) (:FILETYPE . :MH EXECUTE) (:NCMDS . 15) (:SIZEOFCMDS . 1360) (:FLAGS :MH_NOUNDEFS :MH_DYLDLINK :MH TWOLEVEL :MH PIE) (:RESERVED . 0)) (:CMDS ((:CMD . :LC_SEGMENT_64) (:SEGNAME . " PAGEZERO") (:VMADDR . 0) (:VMSIZE . 4294967296) (:MAXPROT . 0) (:INITPROT . 0) (:FLAGS) (:SECTIONS)) ((:CMD . :LC SEGMENT 64) (:SEGNAME . " TEXT") (:VMADDR . 4294967296) (:VMSIZE . 4096) (:MAXPROT . 7) (:INITPROT . 5) (:FLAGS) (:SECTIONS ((:SECTNAME . " text") (:TYPE . :S_REGULAR) (:SEGNAME . " TEXT") (:ADDR . 4294970560) (:ALIGN . 4) (:RELOFF . 0) (:NRELOC . 0) (:ATTRIBUTES :S_ATTR_PURE_INSTRUCTIONS :S_ATTR_SOME_INSTRUCTIONS) (:RESERVED1 . 0) (:RESERVED2 . 0) (:RESERVED3 . 0) (:CONTENTS 85 72 137 229 72 137 125 248 72 137 117 240 72 139 117 248 139 6 137 69 236 72 139 117 248 139 70 4 137 69 232 199 69 228 0 0 0 0 199 69 220 185 121 55 158 72 139 117 240 139 6 137 69 216 72 139 117 240 139 70 4 137 69 212 72 139 117 240 139 70 8 137 69 208 72 139 117 240 139 70 12 137 69 204 199 69 224 0 0 0 0 131 125 224 32 15 131 91 0 0 0 139 69 220 3 69 228 137 69 228 139 69 232 193 224 4 3 69 216 139 77 232 3 77 228 49 200 139 77 232 193

Parsed PE (Windows) binary for TEA

32,589 lines total !

Axe Tools

- Axe Rewriter
- Axe Prover
- Axe Equivalence Checker
- Lifter: JVM to logic
- Lifter: x86 to logic
- All built on ACL2
- All based on structure-shared terms (DAGs)

Axe Rewriter

- Represents terms as DAGs
 - Represent each sub-term only once
 - Allows massive sharing of structure
 - Can give exponential space/time savings
 - Manipulated using arrays under the hood.
 - Can be embedded in ACL2 terms
- Fast: 600K rewrite rule attempts per sec.
- Fancy features
 - conditional rules
 - assumptions and free variable matching
 - axe-syntaxp, axe-bind-free
 - axe-rewrite-objective
 - "work hard" like force
 - monitoring rules
 - memoization
 - limited use of content from overarching ifs
 - outside-in rewriting
- No forward chaining, linear, or type-prescription
- Does not produce proofs

(28 ACL2::BVPLUS '32 27 14) (27 ACL2::SLICE '31 '24 26) (26 ACL2::BVMULT '32 '16843009 25) (25 ACL2::BVAND '32 24 '252645135) (24 ACL2::BVPLUS '32 23 22) (23 ACL2::SLICE '31 '4 22) (22 ACL2::BVPLUS '32 21 19) (21 ACL2::BVAND '30 20 '858993459) (20 ACL2::SLICE '31 '2 18) (19 ACL2::BVAND '32 18 '858993459) (18 ACL2::BVPLUS '32 1 17) (17 ACL2::BVUMINUS '32 16) (16 ACL2::BVAND '31 15 '1431655765) (15 ACL2::SLICE '63 '33 0) (14 ACL2::SLICE '31 '24 13) (13 ACL2::BVMULT '32 '16843009 12) ACL2::BVAND '32 11 '252645135) (11 ACL2::BVPLUS '32 10 9) (10 ACL2::SLICE '31 '4 9) (9 ACL2::BVPLUS '32 8 6) (8 ACL2::BVAND '30 7 '858993459) (7 ACL2::SLICE '31 '2 5) ACL2::BVAND '32 5 '858993459) (5 ACL2::BVPLUS '32 0 4) (4 ACL2::BVUMINUS '32 3) (3 ACL2::BVAND '31 2 '1431655765) (2 ACL2::SLICE '31 '1 0) (1 ACL2::SLICE '63 '32 0) (0 . X))

Axe Equivalence Checker

- Tactic-based:
 - Rewriting
 - SMT solving
 - "sweeping and merging"
 - pruning dead branches (with STP and/or rewriting)
 - case-splitting
 - fancy handling of loops/recursions
- Can compare:
 - code to spec
 - code to code

Lifting Into Logic

- JVM Lifter
 - Based on our JVM model
 - Has been used on dozens of examples
 - Can lift loops to recursive functions

• X86 Lifter

- Based on Shilpi's x86 model
- Newer
- Support for loops still in progress
- Both lifters use the Axe rewriter for symbolic execution.

Prototype x86 Lifter

- Can lift small x86 binaries into logic
 - subroutine calls
 - conditional branches
 - data from data segment
 - unrollable loops
- Automatically adds lots of standard assumptions
 - especially if there is a symbol table
- Symbolic execution with Axe is orders of magnitude faster than with ACL2's rewriter
- No clock functions!
 - Partial function to "run until return" (run-until-rsp-greater-than)
 - Repeatedly open one step and simplify
- Currently can only lift unrollable loops
 - Loop lifter in progress, based on JVM lifter
- Does not produce proofs
 - Must trust Axe, etc.

Trivial Example: Lifting "add" (Mach-O) into Logic



Trivial Example: Lifting "add" (PE)

ADD1 (X86) (DECLARE (XARGS :NON-EXECUTABLE T :MODE :LOGIC)) (DECLARE (XARGS :STOBJS X86 :VERIFY-GUARDS NIL)) (PROG2\$ (ACL2::THROW-NONEXEC-ERROR 'ADD1 (LIST X86)) (XW ':RGF '0 (ACL2::BVPLUS '32 (XR ':RGF '2 X86) (XR ':RGF '1 X86)) (XW ':RGF '2 (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '1 X86)) (XW ':RGF '4 (BINARY-+ '8 (XR ':RGF '4 X86)) (XW ':RIP '0 (COMBINE-BYTES (MV-NTH '1 (RB (CREATE-CANONICAL-ADDRESS-LIST '8 (XR ':RGF '4 X86)) ':R X86))) (MV-NTH '1 (WB (CREATE-ADDR-BYTES-ALIST (CREATE-CANONICAL-ADDRESS-LIST '4 (BINARY-+ '16 (XR ':RGF '4 X86))) (BYTE-IFY '4 (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '2 X86)))) (MV-NTH '1 (WB (CREATE-ADDR-BYTES-ALIST (CREATE-CANONICAL-ADDRESS-LIST '4 (BINARY-+ '8 (XR ':RGF '4 X86))) (BYTE-IFY '4 (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '1 X86)))) (MV-NTH '1

(DEFUN

(WB (CREATE-ADDR-BYTES-ALIST (CREATE-CANONICAL-ADDRESS-LIST '8 (BINARY-+ '-8 (XR ':RGF '4 X86))) (BYTE-IFY '8 (ACL2::LOGHEAD\$INLINE '64 (XR ':RGF '5 X86)))) (!FLGI '0 (CF-SPEC32\$INLINE (BINARY-+ (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '2 X86)) (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '1 X86)))) (!FLGI '2 (PF-SPEC32\$INLINE (ACL2::BVPLUS '32 (XR ':RGF '2 X86) (XR ':RGF '1 X86))) (!FLGI '4 (ADD-AF-SPEC32\$INLINE (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '2 X86)) (ACL2::LOGHEAD\$INLINE '32 (XR ':RGF '1 X86))) (!FLGI '6 (IF (EQUAL '0 (ACL2::BVPLUS '32 (XR ':RGF '2 X86) (XR ':RGF '1 X86))) '1 'Ø) (!FLGI '7 (ACL2::GETBIT '31 (ACL2::BVPLUS '32 (XR ':RGF '2 X86) (XR ':RGF '1 X86))) (!FLGI '11 (OF-SPEC32\$INLINE (BINARY-+ (LOGEXT '32 (XR ':RGF '2 X86)) (LOGEXT '32 (XR ':RGF '1 X86))))

Using / Extending the x86 Model

- Adding many rewrite rules
 - Some adjustments for Axe rewriter
 - Rules about disjointness
 - Connecting to our bit vector library
 - Every operator has an explicit size
 - Hundreds of rewrite rules
 - Used in our specs for crypto code
 - Used in translation to STP SMT solver
 - Used in the Axe equivalence checker
- Adding for 32-bit instructions to x86 model.

Examples

- Popcount
- TEA

Example: popcount

- Count the number of 1's in a bit vector
- Optimized C program
 - Correctness non-obvious!
- Lift to a structure-shared "DAG"
- Lifting takes ~1 second.

```
int popcount_32 (unsigned int v)
{
    v = v - ((v >> 1) & 0x55555555);
    v = (v & 0x33333333) + ((v >> 2) & 0x33333333);
    v = ((v + (v >> 4) & 0xF0F0F0F) * 0x1010101) >> 24;
    return(v);
    }
    int popcount_64 (long unsigned int v)
{
        long unsigned int v1, v2;
        // v1: lower 32 bits of v
        v1 = (v & 0xFFFFFFFF);
        // printf ("\n v1: %lu", v1);
        // v2: upper 32 bits of v
        v2 = (v >> 32);
        // printf ("\n v2: %lu", v2);
        return (popcount_32(v1) + popcount_32(v2));
}
```

Example: popcount

```
(def-lifted-x86-axe popcount "_popcount_64" acl2:: |*popcount-64.o*|
 :output (:register 0)
 :assumptions '((equal (xr ':rqf '7 x86) x))
  :enable (lifter-rules)
 :print nil)
 int popcount_32 (unsigned int v)
   v = v - ((v >> 1) \& 0x55555555);
   v = (v \& 0x33333333) + ((v >> 2) \& 0x33333333);
   v = ((v + (v >> 4) \& 0 \times F0F0F0F) * 0 \times 1010101) >> 24;
   return(v);
  }
                                                    Lift
 int popcount 64 (long unsigned int v)
   long unsigned int v1, v2;
   // v1: lower 32 bits of v
   v1 = (v \& 0 \times FFFFFFF);
   // printf ("\n v1: %lu", v1);
   // v2: upper 32 bits of v
   v^2 = (v >> 32);
   // printf ("\n v2: %lu", v2);
   return (popcount_32(v1) + popcount_32(v2));
```

(28 ACL2::BVPLUS '32 27 14) (27 ACL2::SLICE '31 '24 26) (26 ACL2::BVMULT '32 '16843009 25) (25 ACL2::BVAND '32 24 '252645135) (24 ACL2::BVPLUS '32 23 22) (23 ACL2::SLICE '31 '4 22) (22 ACL2::BVPLUS '32 21 19) (21 ACL2::BVAND '30 20 '858993459) (20 ACL2::SLICE '31 '2 18) (19 ACL2::BVAND '32 18 '858993459) (18 ACL2::BVPLUS '32 1 17) (17 ACL2::BVUMINUS '32 16) (16 ACL2::BVAND '31 15 '1431655765) (15 ACL2::SLICE '63 '33 0) (14 ACL2::SLICE '31 '24 13) (13 ACL2::BVMULT '32 '16843009 12) (12 ACL2::BVAND '32 11 '252645135) (11 ACL2::BVPLUS '32 10 9) (10 ACL2::SLICE '31 '4 9) (9 ACL2::BVPLUS '32 8 6) (8 ACL2::BVAND '30 7 '858993459) (7 ACL2::SLICE '31 '2 5) (6 ACL2::BVAND '32 5 '858993459) (5 ACL2::BVPLUS '32 0 4) (4 ACL2::BVUMINUS '32 3) (3 ACL2::BVAND '31 2 '1431655765) (2 ACL2::SLICE '31 '1 0) (1 ACL2::SLICE '63 '32 0) (0 . X))

Example: popcount

• Spec: (acl2::bvcount 64 x)

Unrolls to naive algorithm (check each bit and count the 1's)

• Equivalence proof by unrolling spec, rewriting, calling SMT (most work done by SMT).

– Proof takes a few minutes

 Shows spec and code equivalent, for all 2⁶⁴ inputs.

Example: TEA Block Cipher (Tiny Encryption Algorithm)

Formal spec:

```
(defconst *delta* #x9e3779b9)
                                                                                                         Delta:
(defun tea-encrypt-loop (n y z sum k)
  (declare (xargs :guard (and (unsigned-byte-p 32 n) ;n<=32
                               (unsigned-byte-p 32 y)
                               (unsigned-byte-p 32 z)
                               (unsigned-byte-p 32 sum)
                                                                                                       K[1]
                               (bv-arrayp 32 4 k))))
  (if (zp n)
                                                                                                       K[2]
      (mvyz)
   (let* ((n (+ -1 n)))
           (sum (bvplus 32 sum *delta*))
                                                                                                         -Delta
           (y (bvplus 32 y (bvxor 32 (bvplus 32 (shl 32 z 4) (bv-array-read 32 4 0 k))
                                   (bvxor 32 (bvplus 32 z sum)
                                          (bvplus 32 (shr 32 z 5) ;unsigned right-shift
                                                  (bv-array-read 32 4 1 k))))))
           (z (bvplus 32 z (bvxor 32 (bvplus 32 (shl 32 y 4) (bv-array-read 32 4 2 k))
                                                                                                       K[3]
                                   (bvxor 32 (bvplus 32 y sum)
                                          (bvplus 32 (shr 32 y 5) ;unsigned right-shift
                                                  (bv-array-read 32 4 3 k)))))))
      (tea-encrypt-loop n y z sum k))))
;; encrypt value V with key K
(defun tea-encrypt (v k)
  (declare (xargs :quard (and (bv-arrayp 32 2 v)
                              (bv-arrayp 32 4 k))))
  (let* ((y (bv-array-read 32 2 0 v))
         (z (bv-array-read 32 2 1 v))
         (sum 0)
         (n 32))
    (mv-let (y z)
      (tea-encrypt-loop n y z sum k)
      (bv-array-write 32 2 0 y (bv-array-write 32 2 1 z '(0 0))))))
```

Example: TEA

- Lifting the binary requires assuming nonoverlap in memory of:
 - Params (v, k) and next stack slots
 - Params (v, k) and code
 - v param and stored return address

Example: TEA

• Stats on lifted TEA (after extracting the result):

Showing info for DAG: 557 unique nodes 71154924322843234631020052204138214 total nodes 24 Variables: INØ IN1 IN2 IN3 IN4 IN5 IN6 IN7 KEYØ KEY1 KEY2 KEY3 KEY4 KEY5 KEY6 KEY7 KEY8 KEY9 KEY10 KEY11 KEY12 KEY13 KEY14 KEY15 5 Functions: ACL2::BVCAT ACL2::SLICE ACL2::BVXOR ACL2::BVPLUS CONS Function counts: CONS: 2 ACL2::SLICE: 64 ACL2::BVCAT: 83 ACL2::BVXOR: 128 ACL2::BVPLUS: 256

- Unrolled spec is similar
- Equivalence proof via rewriting
 - 4,540 rule hits of 229,625 tries
 - 0.23 seconds

((556 CONS 546 555) (555 CONS 554 'NIL) (554 ACL2::BVPLUS '32 553 538) (553 ACL2::BVXOR '32 551 552) (552 ACL2::BVXOR '32 549 548) (551 ACL2::BVPLUS '32 550 41) (550 ACL2::SLICE '31 '5 546) (549 ACL2::BVPLUS '32 '3337565984 546) (548 ACL2::BVPLUS '32 547 38) (547 ACL2::BVCAT '28 546 '4 '0) (546 ACL2::BVPLUS '32 545 530) (545 ACL2::BVXOR '32 543 544) (544 ACL2::BVXOR '32 541 540) (543 ACL2::BVPLUS '32 542 35) (542 ACL2::SLICE '31 '5 538) (541 ACL2::BVPLUS '32 '3337565984 538) (29 ACL2::BVCAT '24 28 '8 7) (28 ACL2::BVCAT '16 27 '8 6) (27 ACL2::BVCAT '8 4 '8 5) (26 ACL2::BVCAT '24 25 '8 3) (25 ACL2::BVCAT '16 24 '8 2) (24 ACL2::BVCAT '8 0 '8 1) (23 , KEY15) (22 . KEY14) (21 . KEY13) (20 . KEY12) (19 . KEY11) (18 . KEY10) (17 . KEY9) (16 . KEY8) (15 . KEY7) (14 . KEY6) (13 . KEY5) (12 . KEY4) (11 . KEY3) (10 . KEY2) (9 . KEY1) 8 . KEY0 7 . IN7) 6 . IN6) . IN5 (4 . IN4) (3 . IN3) . IN2) (2 (1 . IN1)(0 . IN0))

Challenges / Next Steps

- Lifting loops in x86 binaries
 - Approach similar to our JVM lifter
 - May do some things differently:
 - Have lifted functions still traffic in x86 memories
 - Don't require all aliasing to be resolved
 - Allow lifted functions to represent exceptions / errors
 - Don't require proving absence of errors

Bonus Example: TEA in Java

TEA in Java (bouncycastle)

```
private int encryptBlock(
                       byte[] in,
                        int
                                                                     inOff,
                       byte[] out,
                                                                    outOff)
                        int
{
                       // Pack bytes into integers
                       int v0 = bytesToInt(in, inOff);
                       int v1 = bytesToInt(in, inOff + 4);
                       int sum = 0;
                       for (int i = 0; i != rounds; i++)
                       {
                                               sum += delta;
                                             v0 += ((v1 << 4) + a)^{(v1 + sum)^{(v1 + sum)^{(v1 >>> 5)} + b)};
                                             v1 += ((v0 << 4) + c)^{(v0 + sum)^{(v0 + sum^{(v0 + sum)^{(v0 + sum)^{(v0 + sum^{(v0 + sum^{(v0 + sum)^{(v0 + sum^{(v0 + sum^{(v)}}} sum^{(v0 + sum^{(v)}} sum^{(v0 + sum^{(v0 + sum^{(v) + sum^{(v0 + sum^{(v) + sum^{(v)}} sum^{(v0 + sum^{(v)} sum^{(v0 + sum^{(v0 + sum^{(v + sum^{(v) + sum^{(v)} sum^{(v)}} sum^{(v)} sum^{(v)} 
                        }
                       unpackInt(v0, out, outOff);
                       unpackInt(v1, out, outOff + 4);
                       return block size;
 }
```

TEA in Java

- Lifting into logic
- Reconstruct a derivation
 - Proof-emitting transformation steps
 - Link the code and the spec

