CS313K: Logic, Sets, and Functions

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Lecture 23 – Chap 7 (7.6, 7.7)

Final Exam Date

The Final Exam will be held on Tuesday, May 18, 9:00 – 12:00 noon, WEL: 2.224

Midterm Grades

Recall that the curve on Midterm 1 spilled-over to Midterm 2.

We miscalculated some of that spill-over and it was fixed on the gradesheet this weekend.

We've also added a column to show the spill-over points carrying into the Final Exam.

Induction and Quantifiers

One way to prove

$$(\forall\ \texttt{x} : (\forall\ \texttt{a} : (\psi\ \texttt{x}\ \texttt{a})))$$

is to prove

$$(\psi\ \texttt{x}\ \texttt{a})$$

And you might do that by induction. And when you induct on `x` you can instantiate `a` any way you want.

But you don't have to remove all the quantifiers from the formula before induction. To prove

$(\forall\ \mathtt{x}\ :\ (\psi\ \mathtt{x}))$:

Base:
$(\mathtt{endp\ x}) \rightarrow (\psi\ \mathtt{x})$.

Induction Step:
$((\neg(\text{endp x})) \wedge (\psi\ (\text{rest x}))) \rightarrow (\psi\ \mathtt{x})$

This works even if ψ has other quantifiers in it.

```
(defun len (x)
  (if (endp x)
      0
      (+ 1 (len (rest x)))))

(defun len2 (x a)
  (if (endp x)
      a
      (len2 (rest x) (+ 1 a))))
```

You should be able to prove

`(len2 x a)` $=$ `(len x)+a`

Induct on `x` but use:

σ: { `x` $\leftarrow$ `(rest x)`, `a` $\leftarrow$ `(+ 1 a)`.

[Note: I've omitted the necessary hyp `(natp a)`.]

$$(\forall\ \texttt{x} : (\forall\ \texttt{a} : ((\texttt{len2 x a}) = (\texttt{len x})\texttt{+a})))$$

$$(\forall\ \texttt{x} : (\forall\ \texttt{a} : ((\texttt{len2 x a}) = (\texttt{len x})\texttt{+a})))$$

Base:

$$(\texttt{endp x}) \rightarrow (\forall\ \texttt{a} : ((\texttt{len2 x a}) = (\texttt{len x})\texttt{+a}))$$

$(\forall$ `x` $:$ $(\forall$ `a` $:$ `((len2 x a)` $=$ `(len x)+a)))`

Induction Step:

$(($`¬`$($`endp x))` $\wedge$

$(\forall$ `a` $:$ `((len2 (rest x) a)`

$=$

`(len (rest x))+a)))`

$\longrightarrow$

$(\forall$ `a` $:$ `((len2 x a)` $=$ `(len x)+a))`

$(\forall$ x : $(\forall$ a : ((len2 x a) $=$ (len x)+a)))

Induction Step:

$(($$\neg$(endp x)) $\wedge$

$(\forall$ a : ((len2 (rest x) a)

$=$

(len (rest x))+a)))

$\longrightarrow$

((len2 x a) $=$ (len x)+a)

$$(\forall\ \texttt{x} : (\forall\ \texttt{a} : ((\texttt{len2 x a}) = (\texttt{len x})\texttt{+a})))$$

Induction Step:

$$((\neg(\texttt{endp x})) \wedge$$
$$(\forall\ \texttt{a} : ((\texttt{len2 (rest x) a}) = (\texttt{len (rest x)})\texttt{+a})))$$
$$\rightarrow$$
$$((\texttt{len2 (rest x) (+ 1 a)}) = (\texttt{len x})\texttt{+a})$$

```
(∀ x : (∀ a : ((len2 x a) = (len x)+a)))

Induction Step:
((¬(endp x)) ∧
 (∀ a : ((len2 (rest x) a)
            =
            (len (rest x))+a)))
→
 ((len2 (rest x) (+ 1 a)) = (len x)+a)
```

$(\forall$ x : $(\forall$ a : ((len2 x a) $=$ (len x)+a)))

Induction Step:

$((\neg$(endp x)) $\wedge$

((len2 (rest x) (+ 1 a))

$=$

(len (rest x))+(+ 1 a)))

$\longrightarrow$

((len2 (rest x) (+ 1 a)) $=$ (len x)+a)