(Problem 11) A tree is a <u>*maple*</u> *if and only the degree of every vertex is either one or five. Given a maple on n vertices, find a formula to express the maximum and minimum number of pendants it contains. Prove your result is correct.*

Let's start from an easy case: if $n = 2$, then the number of pendants vertices is 2.

Now consider $n > 2$. As the maple is a tree, the number of edges is $n - 1$. Let i be the number of pendant vertices of the maple. This is also the number of edges that connect a pendant to a non-pendant vertex. So the number of non-pendant vertices is $n - i$, and the number of edges that connect exclusively non-pendant vertices are $n - 1 - i$.

If we count the number of edges that come out from all the non-pendant vertices, as they have degree five, the sum is $5(n - i)$. In this sum each edge that connect two non-pendant vertices is counted twice, while those that connect a pendant to a non-pendant one only once.

Consequently $5(n - i) = n - 1 + (n - 1 - i)$. This contraint, that can be simplified to $i = \frac{3n+2}{4}$, indicates that there only one possible value for i given n. In addition to this, as i has to be an integer, n should be divisible by 2 but not by four.