

(Problem 7) Use the provided definitions, and prove or disprove:  $n^3 \in \Theta((3 - n)^3)$ .

I will start simplifying the problem. I will consider only  $n > 3$ , such that the absolute value of  $(3 - n)^3$  is  $(n - 3)^3$ .

Now let's see if  $(n - 3)^3 \in O(n^3)$ . Well,  $n > n - 3$ , so we can choose  $c = 1$ , and then  $(n - 3)^3 < cn^3$  for any  $n > 3$ .

Finally, let's see if  $n^3 \in O((n - 3)^3)$ .

$$2(n - 3)^3 = 2(n^3 - 9n^2 + 27n - 27) = n^3 + n^3 - 18n^2 + 54n - 54 = n^3 + n^2(n - 18) + 54(n - 1).$$

Note that the two parentheses in the last expression are greater than zero when  $n > 18$ , so that for  $n > 18$ , we have that  $2(n - 3)^3 > n^3$ .

So we can choose  $c = 2$ , and  $k = 18$ , and we have that for any  $n > k$ ,  $n^3 < c(n - 3)^3$ . Then  $n^3 \in O((n - 3)^3)$ .

We can conclude that  $n^3 \in \Theta((3 - n)^3)$ .