

## Problem #40 page 305 of Rosen

As suggested by the book, we can consider that

$$E = (E \cap F) \cup (E \cap \bar{F}).$$

Consequently

$$p(E) = p((E \cap F) \cup (E \cap \bar{F})).$$

$F$  and  $\bar{F}$  are distinct events, then  $E \cap F$  and  $E \cap \bar{F}$  are distinct also. Consequently the probability of their union is the sum of the single probabilities:

$$p(E) = p(E \cap F) + p(E \cap \bar{F}). \quad (1)$$

We also know, from the definition of conditional probability, that for any pair of events  $A$  and  $B$ ,

$$p(A \cap B) = p(A|B) p(B).$$

Applying this formula to (1), we finally obtain

$$p(E) = p(E|F) p(F) + p(E|\bar{F}) p(\bar{F}).$$