

1 Problem #4 on page 433 of Rosen

Let R be any antisymmetric relation on a set A . This means that for any $x, y \in A$ with $x \neq y$, R does not contain both (x, y) and (y, x) .

1.1 Direct proof

A subset R' of R cannot contain elements that are not R , consequently R' will not contain both pairs neither. Consequently R' is antisymmetric also.

1.2 Proof by contraddiction

Suppose that a subset R' of R is not antisymmetric. Then there are $x, y \in A$ with $x \neq y$ such that both (x, y) and (y, x) are in R' . But as R' is a subset of R , then R would contain both pairs too. Consequently R is not antisymmetric neither. This is a contraddiction.