A Brief Introduction to Linear Programming

Jeremy Stober

How do we answer the following problem?

- Suppose you run a refinery with the capacity to produce two products, fuel oil and gasoline.
- Fuel oil sells at a profit of $0.40 a gallon and gasoline sells at a profit of $0.50 a gallon.
- Both products are processed in three stages, though the time in each stage differs for each product.
- For each gallon of product, the following chart describes the time required at each stage.

	Stage A	Stage B	Stage C
Fuel Oil	1 min.	5 min.	3 min.
Gasoline	2 min.	4min.	1 min.

Problem Continued

- During each production run:
  - Stage A is available for 720 minutes
  - Stage B is available for 1800 minutes
  - Stage C is available for 900 minutes
- If stages can be allocated to the making of either type of product at all available times, what volume of fuel oil and gasoline production maximizes profit?

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  - x = gallons of fuel oil
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- Profit = 40x + 50y

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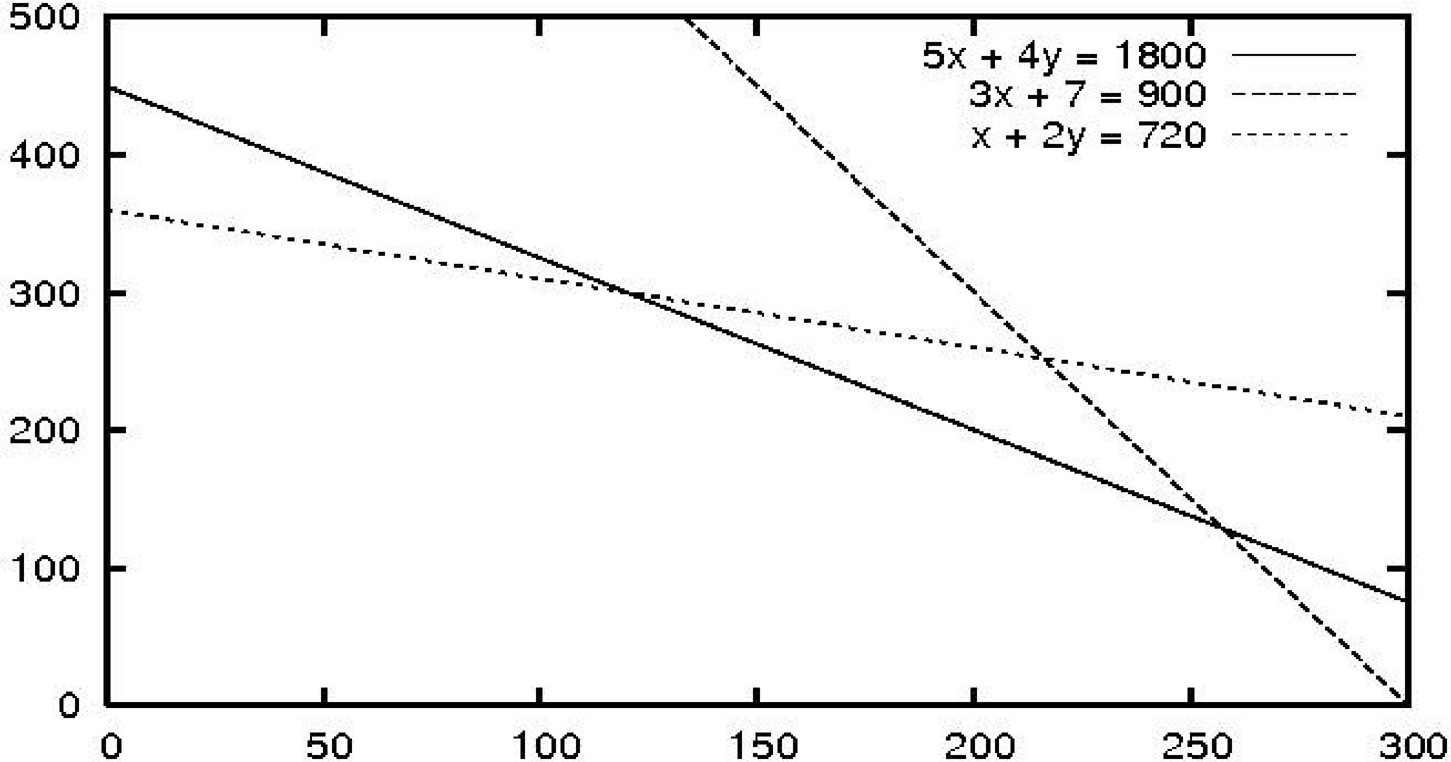
- Stage C: $3x + 1y \leq 900$

A Geometric View



- What points satisfy the constraints?

How might this apply to TAC-SCM?

A TAC SCM Example

- Suppose we have three orders for PCs
  - An order consists of (Quantity, Price)
  - { (30, 2000), (80, 1800), (100, 1500) }
- Suppose we only have 130 computers
- How might we formulate this as an linear programming problem?

Thanks to David Pardoe for the example!

Example Continued

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- Revenue function:
    - (2000 * 30) x + (1800 * 80) y + (1500 * 100) z

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- We want to maximize the value of the orders we choose to fill
- Suppose we denote an order by a variable that is 1 if we fill the order and 0 otherwise
- Revenue function:
    - $(2000 * 30)\ x + (1800 * 80)\ y + (1500 * 100)\ z$
- Constraints:
    - x, y, z must be 0 or 1 (integer programming problem)
    - $30x + 80y + 100z \leq 130$

Some Questions to Think About

- Do optimal LP solutions always occur at vertices in the solution set?
- The LP presented here may remind you of solving for equilibria in mixed strategy games. Is there a connection?
- Are integer programming problems computationally harder or easier to solve than linear programming problems?

References

The first linear programming problem presented here was adapted from:

An Introduction to Linear Programming and the Theory of Games by A.M. Glicksman