Tuning the Wall-Following Controller

Benjamin Kuipers

September 24, 1999

We would like to tune the parameters of the wall-following controller so that it is as close to critically-damped as possible. That is, it converges to the setpoint as quickly as possible without overshooting.

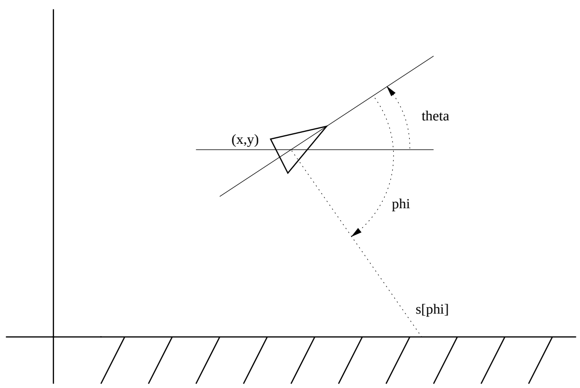

Figure 1: The robot is at position (x, y) and orientation θ. The range sensor in direction ϕ senses distance s_ϕ, but in this paper we assume that y and θ are sensed directly.

The dynamical model of the robot (Figure 1) is $\mathbf{x}' = f(\mathbf{x}, \mathbf{u})$, with constant forward velocity v and exogenously specified angular velocity ω.

$$\mathbf{x}' = \begin{bmatrix} x' \\ y' \\ \theta' \\ v' \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ 0 \end{bmatrix} \tag{1}$$

The Wall-Following Controller

We use a slightly simplified version of the wall-following controller from [2].

The wall-following control law (3) sets angular velocity ω, responding to positional error $e = y - y_{set}$ and orientation error θ, and assuming that forward velocity v is constant.

We want to specify the control law so that the behavior of the system will be described by

$$\ddot{e} + k_\theta \dot{e} + k_e e = 0. \tag{2}$$

where the constants k_e and k_θ are tuned to make the system behave well (i.e., critically damped convergence $e \to 0$).

For small values of θ,

$$\dot{e} = v \sin\theta \approx v\theta$$

and

$$\ddot{e} = v \cos\theta \, \dot{\theta} \approx v\omega,$$

so we can transform the general system description (2) into a control law: a rule for specifying the value of the controlled variable ω as a function of the values of the observed variables e, θ and v:

$$\omega = \frac{1}{v}[-k_\theta v\theta - k_e e]. \tag{3}$$

Qualitative Behavior

Any elementary discussion of differential equations and dynamical systems (e.g. [1]) provides the qualitative framework we need to tune this controller.

When obeying the wall-following control law, the robot's behavior approximates the linear harmonic oscillator, which is a special case of the general linear second-order system:

$$a\ddot{x} + b\dot{x} + cx = 0. \tag{4}$$

The behavior of this system is determined by the roots of its characteristic equation:

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The qualitative behavior $x(t)$ is determined by the qualitative properties of the roots, which are determined in important part by the sign of the *discriminant*:

$$D = b^2 - 4ac.$$

- If the roots have non-zero imaginary part (i.e., $D < 0$), the behavior oscillates. If the roots are purely imaginary (i.e., $D < 0$ and $b = 0$), the oscillation is periodic.

- If either root has a positive real part, the behavior diverges. In a usefully controlled system, the behavior must converge, so both roots must have negative real parts. If $D < 0$, we require only that $b > 0$. If $D > 0$, we also need to have $b > \sqrt{b^2 - 4ac}$, which therefore requires that
$$0 < c < \frac{b^2}{2a}.$$

- *Critical damping* occurs at the boundary between oscillatory and non-oscillatory behavior; that is, where $D = 0$, so:
$$c = \frac{b^2}{4a}. \tag{5}$$

Tuning the Controller

To make the behavior of the system (2) critically damped, we apply (5) and require that

$$k_e = \frac{k_\theta^2}{4} \text{ or equivalently, } k_\theta = \sqrt{4k_e}. \tag{6}$$

Experimental Results

The model `rwall2.m` implements this controller, with some modifications. In order to avoid divergence when $v \approx 0$ during starting and stopping, the $1/v$ term in (3) is replaced by $min(v^2, 1/max(0.01, v))$.

Setting k_e to values in the range $[0.1, 0.6]$ controlls how aggressively the controller seeks the setpoint. Setting $k_\theta = \sqrt{4k_e}$ makes the resulting controller critically damped, as expected. High gains at lower velocities results in turns that might be overly aggressive.

Next Steps

- Set the gain as a function of forward velocity, to see whether this gives better performance.

- What are the applicability conditions for this controller? How close is $\theta \approx 0$? What happens if the robot is close to facing the wall?

References

[1] M. Hirsch and S. Smale. *Differential Equations, Dynamical Systems, and Linear Algebra.* Academic Press, New York, NY, 1974.

[2] P. van Turennout, G. Honderd, and L. J. van Schelven. Wall-following control of a mobile robot. In *IEEE International Conference on Robotics and Automation*, pages 280–285, 1992.