

CS313K: Logic, Sets, and Functions

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(Lecture 10)

Announcements

Midterm 1 is next Tuesday, in class.

The midterms will be held in WEL 2.224, *not in this room!*

Bring the red book, some pencils, and some scratch paper. Do not bring computers.

Correction to Page 41

γ :

```
(if (and a b)
    (and a b)
    (f (and a b)))
```

γ_1 :

```
(if (and b a)
    (and a b)
    (f (and a b)))
```

...

...

γ_3 :

```
(if (and a b)
    (and a b)
    (f (and b a)))
```

Homework 5 Changed

I deleted the following Questions from HW5: 124, 136, 137, 138, 150, and 155.

This homework will not be due until Wednesday, Feb 25, by 3:30 pm (24 hours later than normal).

The web page for homeworks has been updated appropriately.

I will be out of town next Tuesday (during the midterm) and Wednesday. I will miss my office hours on Wednesday but be available as usual on Thursday.

About Maintaining Equivalence

```
(implies (and (natp e)
```

```
         (mem e x))
```

```
      (natp (if (and (f e) (g x))
```

```
              (h (f x))
```

```
              x))))
```

About Maintaining Equivalence

IFF

(implies (and (natp e)

(mem e x))

(natp (if (and (f e) (g x))

(h (f x))

x)))

About Maintaining Equivalence

IFF

IFF

(implies (and (natp e)

(mem e x))

IFF

(natp (if (and (f e) (g x))

(h (f x))

x)))

About Maintaining Equivalence

IFF

(implies (and (natp e)

IFF IFF

IFF

(mem e x))

IFF

(natp (if (and (f e) (g x))

(h (f x))

x)))

About Maintaining Equivalence

IFF IFF IFF EQ

(implies (and (natp e)

IFF

(mem e x))

IFF

(natp (if (and (f e) (g x))

(h (f x))

x)))

About Maintaining Equivalence

IFF IFF IFF EQ

(implies (and (natp e)

IFF EQ EQ

(mem e x))

IFF

(natp (if (and (f e) (g x))

(h (f x))

x)))

About Maintaining Equivalence

```
IFF      IFF  IFF  EQ
(implies (and (natp e)
              IFF EQ EQ
              (mem e x))
          IFF  EQ
          (natp (if (and (f e) (g x))
                    (h (f x))
                    x))))
```

About Maintaining Equivalence

```
IFF      IFF  IFF  EQ
(implies (and (natp e)
              IFF EQ EQ
              (mem e x))
          IFF  EQ  IFF
(natp (if (and (f e) (g x))
          EQ
          (h (f x))
          EQ
          x)))
```

About Maintaining Equivalence

```
IFF      IFF  IFF  EQ
(implies (and (natp e)
              IFF EQ EQ
              (mem e x))
          IFF  EQ  IFF  IFF  IFF
(natp (if (and (f e) (g x))
          EQ
          (h (f x))
          EQ
          x)))
```

About Maintaining Equivalence

```
IFF      IFF  IFF  EQ
(implies (and (natp e)
              IFF EQ EQ
              (mem e x))
          IFF  EQ  IFF  IFFEQ  IFFEQ
(natp (if (and (f e) (g x))
          EQ EQ EQ
          (h (f x))
          EQ
          x)))
```

Factoring

Here is the example of factoring given in class today.

Suppose we want to prove

$(\text{implies } (\text{and } p \ q) \ r)$

by rewriting q . What assumptions can we make while we rewrite q ?

To answer, factor the formula to put the q part into the conclusion and then reading the hypothesis.

To put the q into the conclusion, we factor:

$$(\text{implies } (\text{and } p \ q) \ r)$$

into

$$(\text{implies } (\text{and } p \ (\text{not } r)) \ (\text{not } q))$$

Thus, when rewriting q , we may assume p and $(\text{not } r)$.

Next, I use the identities on page 44 to prove that these two formulas are equivalent (*iff*), which I'll denote \leftrightarrow .

$(\text{implies } (\text{and } p \ q) \ r)$
 \leftrightarrow {Implicative disjunction}
 $(\text{or } (\text{not } (\text{and } p \ q)) \ r)$
 \leftrightarrow {De Morgan}
 $(\text{or } (\text{or } (\text{not } p) \ (\text{not } q)) \ r)$
 \leftrightarrow {Associativity}
 $(\text{or } (\text{not } p) \ (\text{or } (\text{not } q) \ r))$
 \leftrightarrow {Commutativity}
 $(\text{or } (\text{not } p) \ (\text{or } r \ (\text{not } q)))$
 \leftrightarrow {Associativity}

$(\text{or } (\text{or } (\text{not } p) r) (\text{not } q))$

\leftrightarrow {Double Negation}

$(\text{or } (\text{or } (\text{not } p) (\text{not } (\text{not } r))) (\text{not } q))$

\leftrightarrow {De Morgan}

$(\text{or } (\text{not } (\text{and } p (\text{not } r))) (\text{not } q))$

\leftrightarrow {Implicative disjunction}

$(\text{implies } (\text{and } p (\text{not } r)) (\text{not } q))$

Thus, they're equivalent!

Your Questions

My Questions