CS313K: Logic, Sets, and Functions

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(Lecture 20)

Announcements

Do not attempt to climb Santa Elena Canyon. The rock there is rotten (meaning it breaks off in your hands and won't support your weight or your pro).

Example Proof

In the following proofs I'll write "$\alpha \Leftarrow \beta$" to mean that α is a theorem if we can prove β.

Example 1 (Question 279)

(p x) $\longrightarrow$ ($\forall$ x : (p x))

$\Leftarrow$ {$\forall$-concl}

(p x) $\longrightarrow$ (p z)

???

not a theorem!

Let (p x) be true for 1 and false for everything else; let x be 1.

Example 2 (Question 280)

```
(∀  x : (p x)) ⟶  (p x)
⇐ {∀-hyp}
(p x) ⟶ (p x)
↔
t
```

Example 3 (Question 303)

In this question, a "location" in a list is a 0-based index into the list and the contents of location `i` in `x` is `(nth i x)`.

Example 3 (Question 303)

Suppose that for every location in `x`, the contents is a symbol.

Suppose that for every object `e` that is in `y`, there is a location in `x` whose contents is `e`.

Suppose `y` is non-empty.

Then `(car y)` is a symbol.

Example 3 (Question 303)

```
(∀j:((natp j)∧(< j (len x)))
      →(symbolp (nth j x)))
∧
(∀e:(mem e y)→(∃i:(natp i)∧(< i (len x))
                          ∧(nth i x)=e))
∧
(consp y)
→
(symbolp (car y))
```

Example 3 (Question 303)

$\Leftarrow$ {$\forall$-Hyp}

(∀j:((natp j)∧(< j (len x)))
→(symbolp (nth j x)))

∧

(mem (car y) y)→(∃i:(natp i)∧(< i (len x))
∧(nth i x)=(car y))

∧

(consp y)

→

(symbolp (car y))

Example 3 (Question 303)

$\leftrightarrow$ {mem, etc}
 (∀j:((natp j)∧(< j (len x)))
       $\rightarrow$(symbolp (nth j x)))
 ∧
(∃i:(natp i)∧(< i (len x))
    ∧(nth i x)=(car y))
 ∧
 (consp y)
$\rightarrow$
 (symbolp (car y))

Example 3 (Question 303)

$\Leftarrow$ {$\exists$-Hyp}

(∀j:((natp j)∧(< j (len x)))
$\rightarrow$(symbolp (nth j x)))

∧

(natp Z)∧(< Z (len x))

∧(nth Z x)=(car y)

∧

(consp y)

$\rightarrow$

(symbolp (car y))

Example 3 (Question 303)

$\Leftarrow$ {$\forall$-Hyp}

((natp Z)$\wedge$(< Z (len x)))

$\rightarrow$(symbolp (nth Z x)))

$\wedge$

(natp Z)$\wedge$(< Z (len x))

$\wedge$(nth Z x)=(car y)

$\wedge$

(consp y)

$\rightarrow$

(symbolp (car y))

Example 3 (Question 303)

$\leftrightarrow$ {Hyps 2,3, forward chain}

(symbolp (nth Z x))

$\wedge$

(natp Z)$\wedge$(< Z (len x))

$\wedge$(nth Z x)=(car y)

$\wedge$

(consp y)

$\rightarrow$

(symbolp (car y))

Example 3 (Question 303)

$\leftrightarrow$ {Hyp 4}

```
  (symbolp (car y))
```

$\wedge$

```
(natp Z)∧(< Z (len x))
```

$\wedge$

```
(nth Z x)=(car y)
```

$\wedge$

```
(consp y)
```

$\longrightarrow$

```
(symbolp (car y))
```

etc.