CS313K: Logic, Sets, and Functions

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(Lecture 28)

The Final

The final exam will be like the midterms, but will cover propositional proofs, inductive proofs, quantifiers, and set theory (including relations and functions). It will be more about the later material than the earlier material.

The Final

It will be 3 hours.

Open book.

Bring scratch paper.

There will be 15–20 questions, worth a varying number of points each, totaling 300 points.

The Final

Some “yes/no” type questions have “negative points” for wrong answers!

E.g., 10 points if you get it right, -10 if you get it wrong, 0 if you don’t answer.

The Final – Sample Yes/No Question

Let $R = \ldots$.

Check the properties R has (on $\mathbf{N}$). A right answer (whether "yes" or "no") is worth 2 points, but a wrong answer will cost 2 points! A missing answer will not add or subtract points.

		yes	*no*
1.	R is a relation		
2.	R is a function		
3.	R is reflexive		
... 11.	... R is an equivalence relation		
12.	R is a partial order		
13.	R is a total order		

The Final

The final is Friday, May 15, 2:00 – 5:00 pm.

There is alternative final, Thursday, May 14, 9:00 – 12:00 noon.

The two tests will be completely different.

Email me if you have to take the alternative final. I will tell you the room number.

Tutoring

The TIP Learning Lab is closed this week. No tutoring for Lon today!

An Observation

On page 148 of the notes, my formal definition of partition included $(P \subseteq \wp(A))$. A student observed after class that this is implied by $(\bigcup P = A)$.

Theorem: $(\bigcup P = A) \rightarrow (P \subseteq \wp(A))$.

Proof. Use the hypothesis by substituting $\bigcup P$ for A into the conclusion and prove: $(P \subseteq \wp(\bigcup P))$.

$P \subseteq \wp(\bigcup P))$

$\leftrightarrow$ {def $\subseteq$}

$(\forall x : x \in P \rightarrow x \in \wp(\bigcup P))$

$\Leftarrow$ {$\forall$-concl}

$x \in P \rightarrow x \in \wp(\bigcup P)$

$\leftrightarrow$ {def $\wp$ and $\in$}

$x \in P \rightarrow x \subseteq \bigcup P$

$\leftrightarrow$ {def $\subseteq$}

$x \in P \rightarrow (\forall y : y \in x \rightarrow y \in \bigcup P)$

$$x \in P \rightarrow (\forall y : y \in x \rightarrow y \in \bigcup P)$$

$\Leftarrow$ {$\forall$-concl}

$$x \in P \rightarrow (y \in x \rightarrow y \in \bigcup P)$$

$\leftrightarrow$ {promotion}

$$(x \in P \wedge y \in x) \rightarrow y \in \bigcup P$$

$\leftrightarrow$ {def $\bigcup$}

$$(x \in P \wedge y \in x) \rightarrow y \in \{e : (\exists d : d \in P \wedge e \in d)\}$$

$(x \in P \land y \in x) \rightarrow y \in \{e : (\exists d : d \in P \land e \in d)\}$

$\leftrightarrow$ {def $\in$}

$(x \in P \land y \in x) \rightarrow (\exists d : d \in P \land y \in d)$

$\Leftarrow$ {$\exists$-concl}

???

$$(x \in P \land y \in x) \to y \in \{e : (\exists d : d \in P \land e \in d)\}$$

$\leftrightarrow$ {def $\in$}

$$(x \in P \land y \in x) \to (\exists d : d \in P \land y \in d)$$

$\Leftarrow$ {$\exists$-concl}

$$(x \in P \land y \in x) \to (x \in P \land y \in x)$$

$\square$

Correction

On page 152, the conditions of the Theorem at the bottom of the page are wrong. The correct statement of the theorem is:

Theorem If f, g, and h are functions and $ran(g) \subseteq dom(f)$ and $ran(h) \subseteq dom(g)$, then $f \circ (g \circ h) = (f \circ g) \circ h$.

Note: I made this mistake because I did not do the proof in Question 429!