Topic 23
Red Black Trees

"People in every direction
No words exchanged
No time to exchange
And all the little ants are marching
Red and **Black** antennas waving"
-*Ants Marching,* Dave Matthew's Band

"Welcome to L.A.'s Automated Traffic Surveillance and Control Operations Center. See, they use video feeds from intersections and specifically designed algorithms to predict traffic conditions, and thereby control traffic lights. So all I did was come up with my own... **kick ass algorithm** to sneak in, and now we own the place."
-Lyle, the Napster, (Seth Green), *The Italian Job*

Clicker 1

- 2000 elements are inserted one at a time into an initially empty binary search tree using the simplenaive algorithm. What is the maximum possible height of the resulting tree?

A. 1
B. 11
C. 21
D. 500
E. 1999

Binary Search Trees

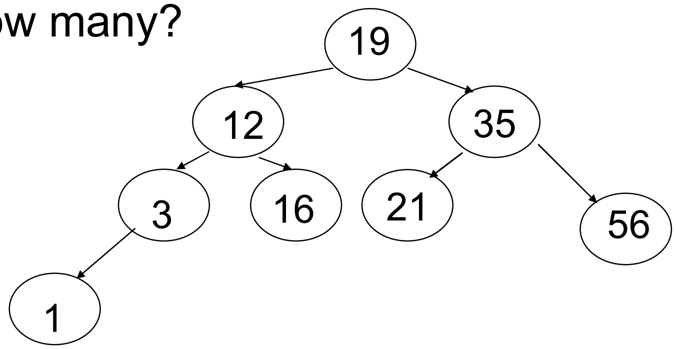
- Average case and worst case Big O for
  - insertion
  - deletion
  - access
- Balance is important. Unbalanced trees give worse than log N times for the basic tree operations
- Can balance be guaranteed?

Red Black Trees

- A BST with more complex algorithms to ensure balance
- Each node is labeled as Red or Black.
- Path: A unique series of links (edges) traverses from the root to each node.
  - The number of edges (links) that must be followed is the path length
- In Red Black trees paths from the root to elements with 0 or 1 child are of particular interest

Paths to Single or Zero Child Nodes

- How many?

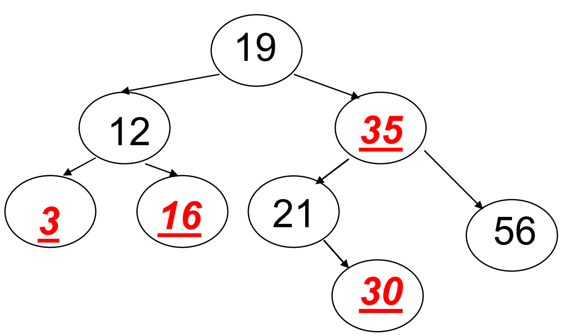


Red Black Tree Rules

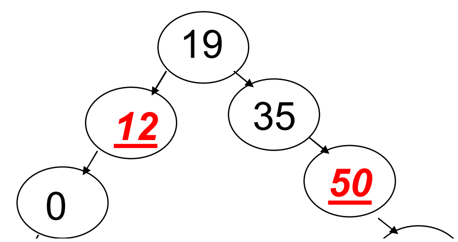
1. Is a binary search tree
2. Every node is colored either red or black
3. The root of the whole tree is black
4. If a node is red its children must be black. (a.k.a. the red rule)
5. Every path from a node to a null link must contain the same number of black nodes (a.k.a. the path rule)

Example of a Red Black Tree

- The root of a Red Black tree is black
- Every other node in the tree follows these rules:
  - Rule 3: If a node is Red, all of its children are Black
  - Rule 4: The number of Black nodes must be the same in all paths from the root node to null nodes



Red Black Tree?

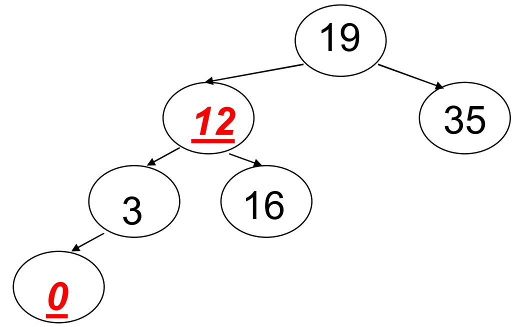


Clicker 2

- Is the tree on the previous slide a binary search tree? Is it a red black tree?

	BST?	Red-Black?
A.	No	No
B.	No	Yes
C.	Yes	No
D.	Yes	Yes

Red Black Tree?



Perfect?
Full?
Complete?

Clicker 3

- Is the tree on the previous slide a binary search tree? Is it a red black tree?

	BST?	Red-Black?
A.	No	No
B.	No	Yes
C.	Yes	No
D.	Yes	Yes

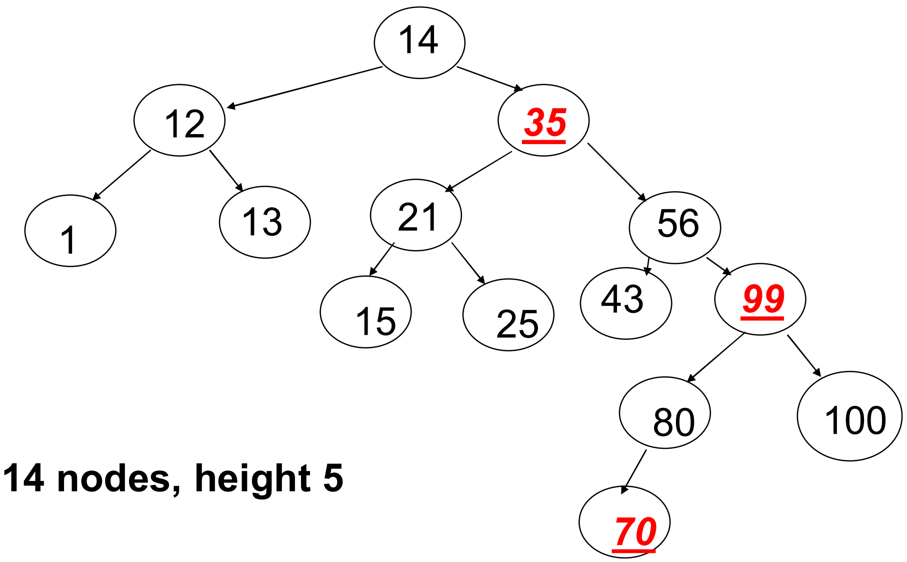
Implications of the Rules

- If a Red node has any children, it must have two children and they must be Black. (Why?)
- If a Black node has only one child that child must be a Red leaf. (Why?)
- Due to the rules there are limits on how unbalanced a Red Black tree may become.
  - on the previous example may we hang a new node off of the leaf node that contains 0?

Properties of Red Black Trees

- If a Red Black Tree is complete, with all Black nodes except for Red leaves at the lowest level the height will be minimal, ~log N
- To get the max height for N elements there should be as many Red nodes as possible down one path and all other nodes are Black
  - This means the max height would b **approximately** 2 * log N (don't use this as a formula)
  - typically less than this
  - see example on next slide
  - interesting exercise, draw max height tree with N nodes

Max Height Red Black Tree



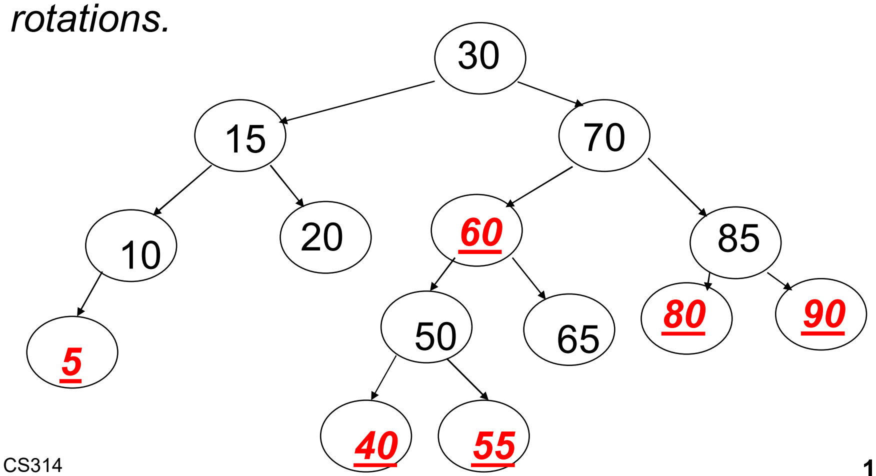
14 nodes, height 5

Maintaining the Red Black Properties in a Tree

- Insertions
- Must maintain rules of Red Black Tree.
- New Node always a leaf
  - can't be black or we will violate rule 4
  - therefore the new leaf must be red
  - If parent is black, done (trivial case)
  - if parent red, things get interesting because a red leaf with a red parent violates rule 3

Insertions with Red Parent - Child

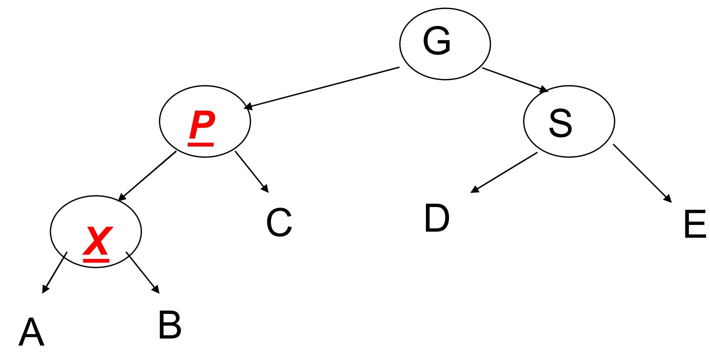
Must modify tree when insertion would result in Red Parent - Child pair using color changes and *rotations.*



Case 1

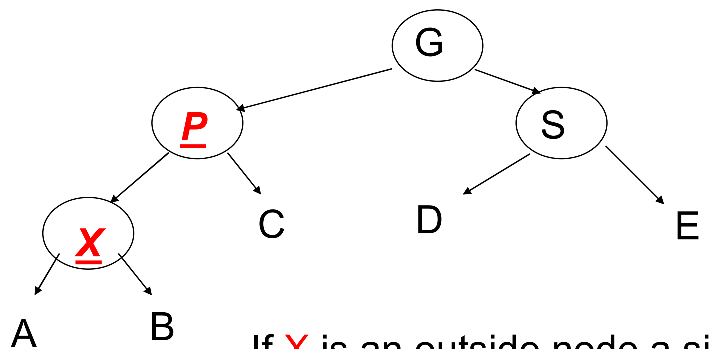
- Suppose sibling of parent is Black.
  - by convention null nodes are black
- In the previous tree, true if we are inserting a 3 or an 8.
  - What about inserting a 99? Same case?
- Let X be the new leaf Node, P be its Red Parent, S the Black sibling and G, P's and S's parent and X's grandparent
  - What color is G?

Case 1 - The Picture



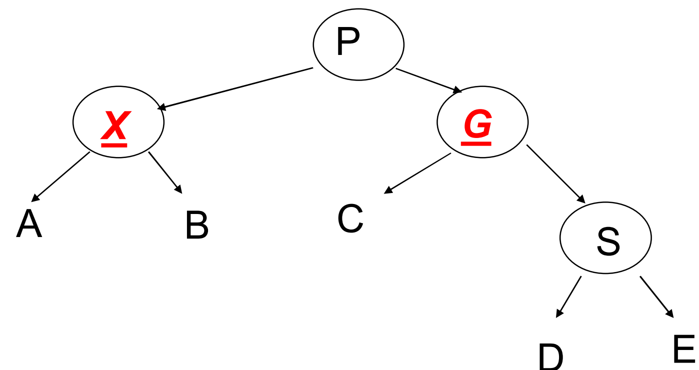
Relative to G, X could be an *inside* or *outside* node.
Outside -> left left or right right moves
Inside -> left right or right left moves

Fixing the Problem



If X is an outside node a single *rotation* between P and G fixes the problem.
A rotation is an exchange of roles between a parent and child node. So P becomes G's parent. Also must recolor P and G.

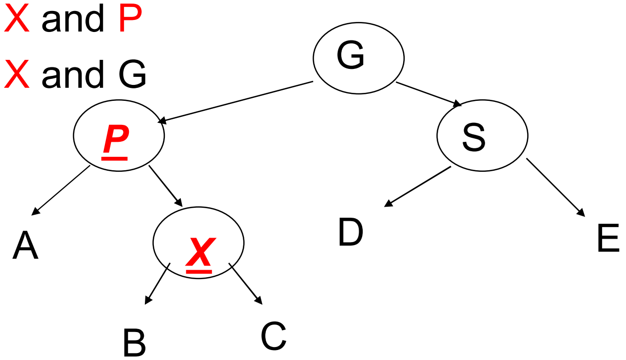
Single Rotation



Apparent rule violation?
Recall, S is null if X is a leaf, so no problem
If this occurs higher in the tree (why?) subtrees A, B, and C will have one more black node than D and E.

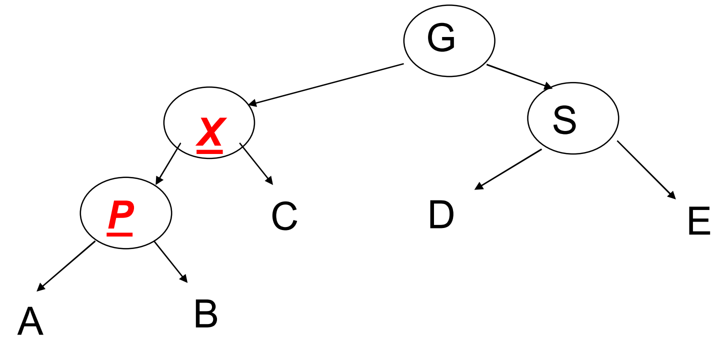
Case 2

- What if X is an inside node relative to G?
  - a single rotation will not work
- Must perform a double rotation
  - rotate X and P
  - rotate X and G



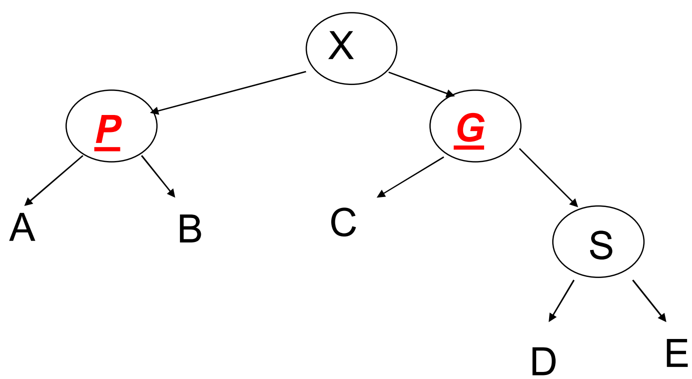
First Rotation

- Rotate P and X, no color change

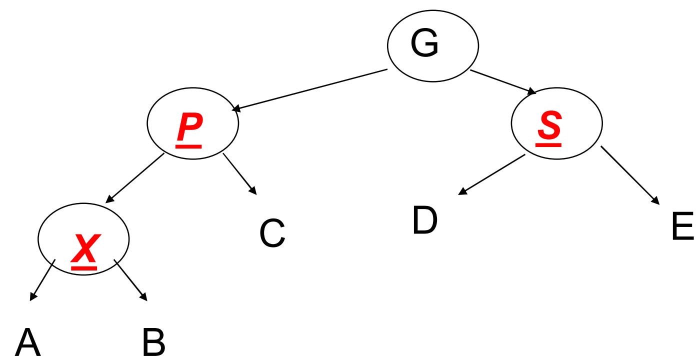


- What does this actually do?

After Double Rotation



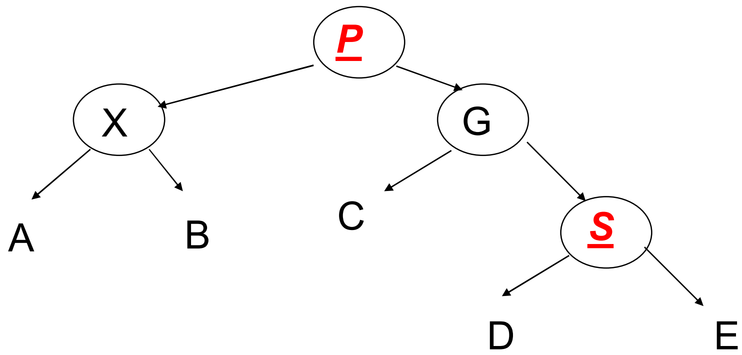
Case 3
Sibling is Red, not Black



Any problems?

Fixing Tree when S is Red

- Must perform single rotation between parent, P and grandparent, G, and then make appropriate color changes

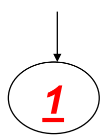


More on Insert

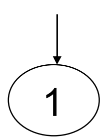
- Problem: What if on the previous example G's parent (GG!) had been red?
- Easier to never let Case 3 ever occur!
- On the way down the tree, if we see a node X that has 2 Red children, we make X Red and its two children black.
  - if recolor the root, recolor it to black
  - the number of black nodes on paths below X remains unchanged
  - If X's parent was Red then we have introduced 2 consecutive Red nodes.(violation of rule)
  - to fix, apply rotations to the tree, same as inserting node

Example of Inserting Sorted Numbers

- 1 2 3 4 5 6 7 8 9 10

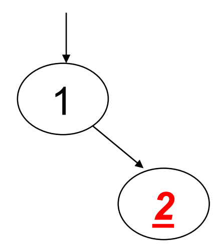


Insert 1. A leaf so red. Realize it is root so recolor to black.



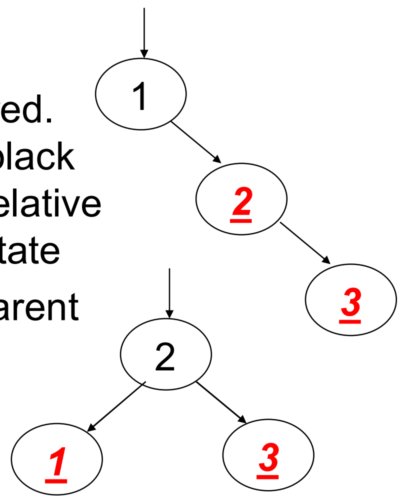
Insert 2

make 2 red. Parent is black so done.



Insert 3

Insert 3. Parent is red. Parent's sibling is black (null) 3 is outside relative to grandparent. Rotate parent and grandparent

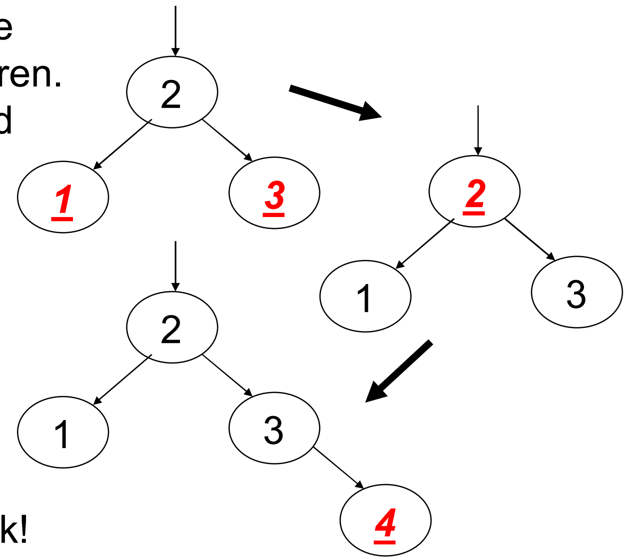


Insert 4

On way down see 2 with 2 red children. Recolor 2 red and children black.

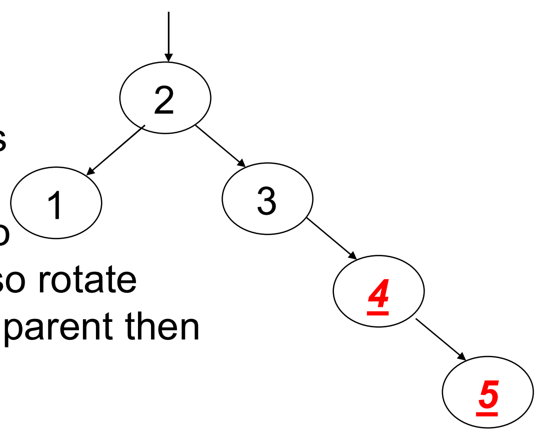
When adding 4 parent is black so done.

Set root to black!

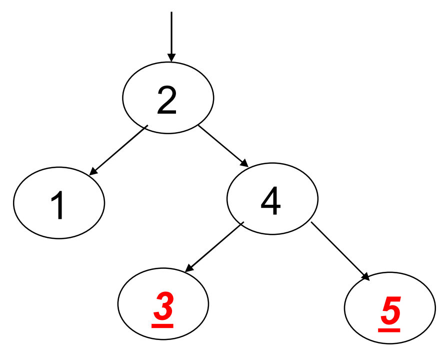


Insert 5

5's parent is red. Parent's sibling is black (null). 5 is outside relative to grandparent (3) so rotate parent and grandparent then recolor

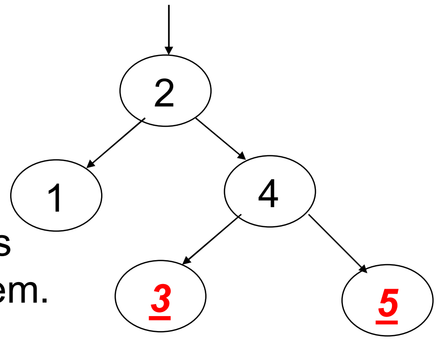


Finish insert of 5



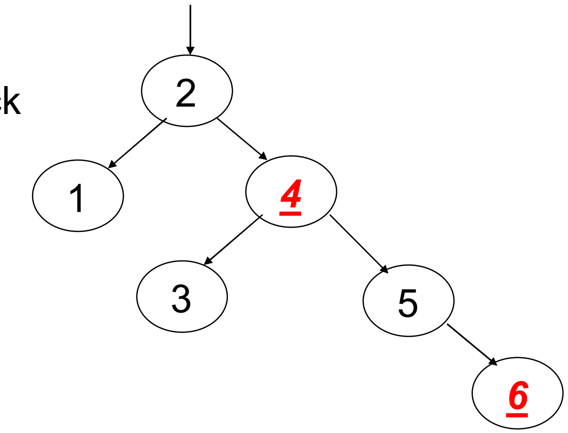
Insert 6

On way down see 4 with 2 red children. Make 4 red and children black. 4's parent is black so no problem.



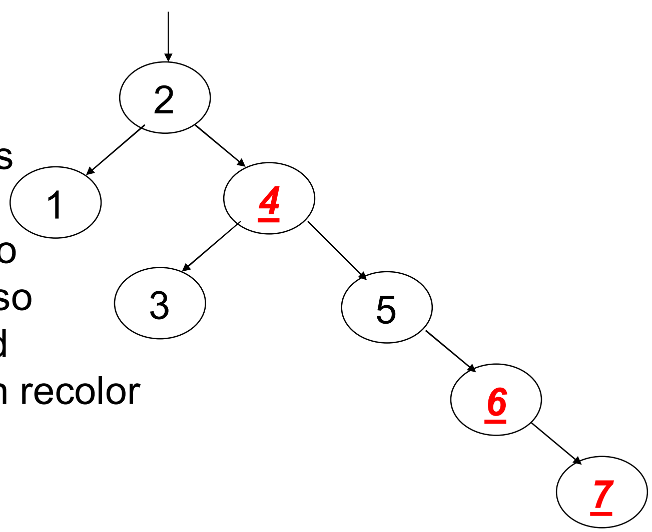
Finishing insert of 6

6's parent is black so done.

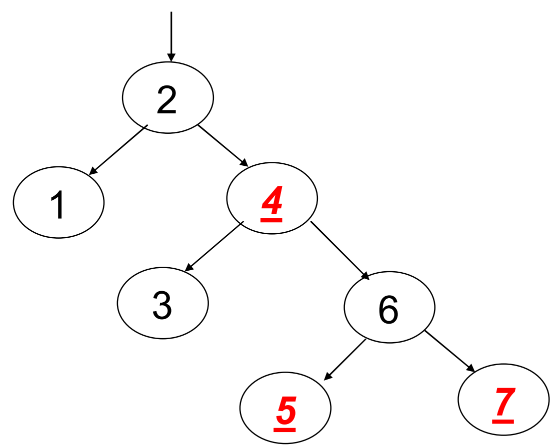


Insert 7

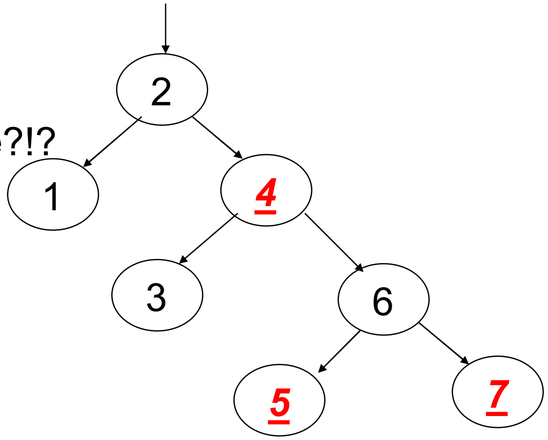
7's parent is red. Parent's sibling is black (null). 7 is outside relative to grandparent (5) so rotate parent and grandparent then recolor



Finish insert of 7



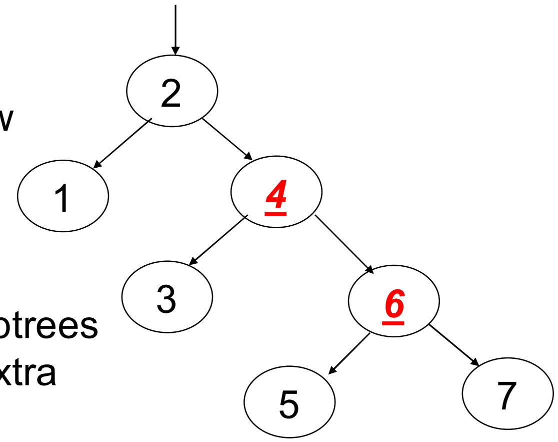
Insert 8



The caveat!!!
Getting unbalanced on that right subtree?!?

On way down see 6 with 2 red children. Make 6 red and children black. This creates a problem because 6's parent, 4, is also red. Must perform rotation.

Still Inserting 8

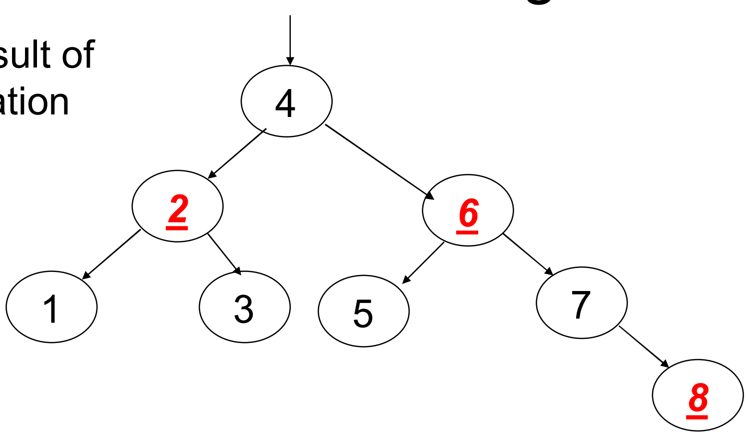


Recolored now need to rotate.

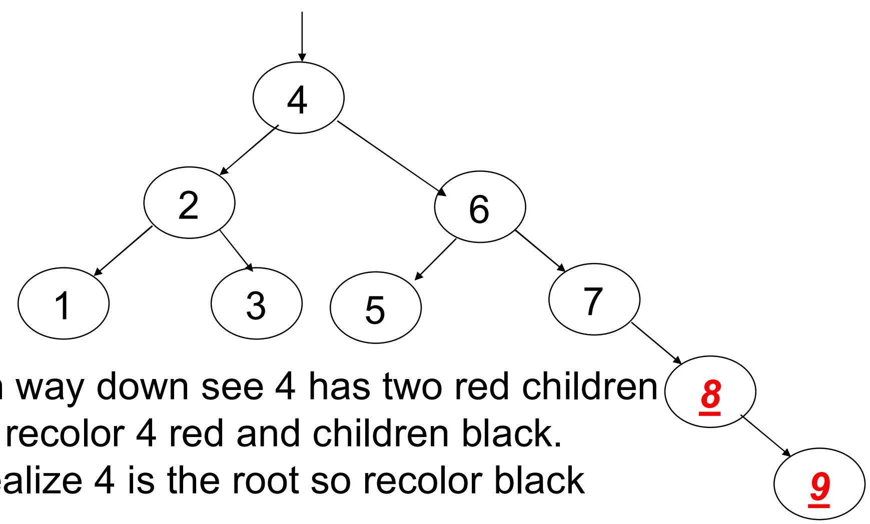
Recall, the subtrees and the one extra black node.

Finish inserting 8

Result of rotation

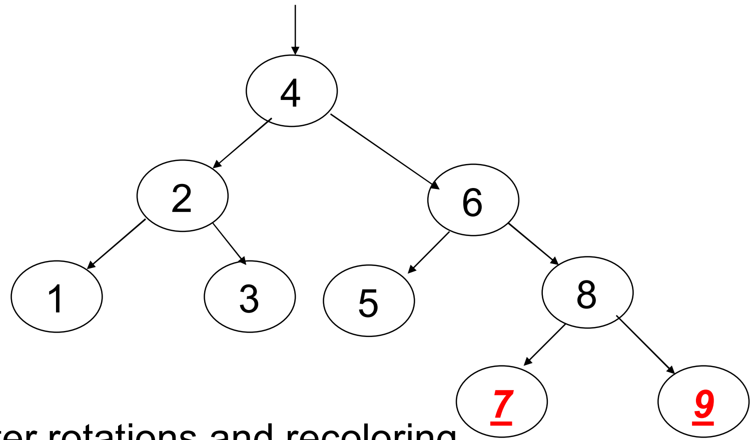


Insert 9



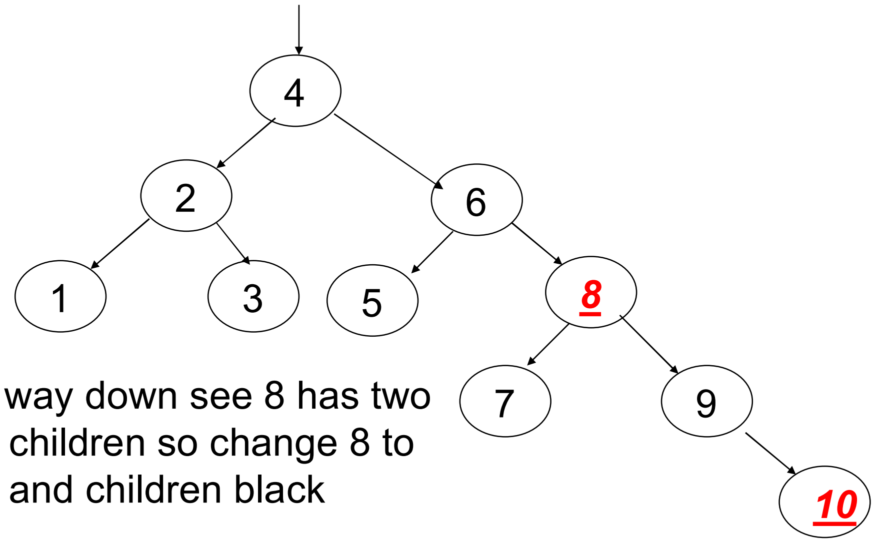
On way down see 4 has two red children so recolor 4 red and children black. Realize 4 is the root so recolor black

Finish Inserting 9



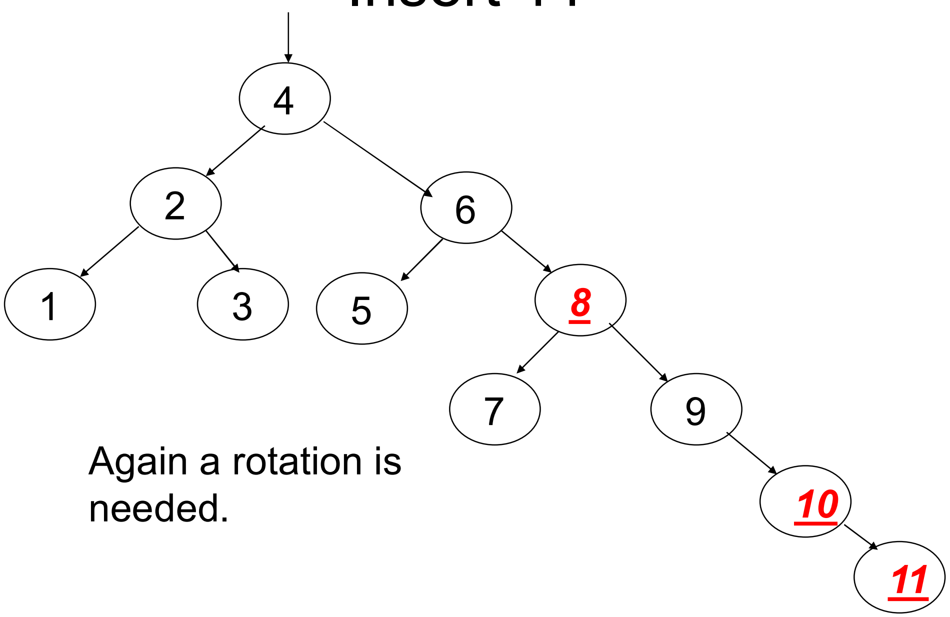
After rotations and recoloring

Insert 10



On way down see 8 has two red children so change 8 to red and children black

Insert 11



Again a rotation is needed.

Finish inserting 11