Introduction to Secure Multi-Party Computation

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Motivation

- General framework for describing computation between parties who do not trust each other
- Example: elections
  - N parties, each one has a “Yes” or “No” vote
  - Goal: determine whether the majority voted “Yes”, but no voter should learn how other people voted
- Example: auctions
  - Each bidder makes an offer
    - Offer should be committing! (can’t change it later)
  - Goal: determine whose offer won without revealing losing offers

More Examples

- Example: distributed data mining
  - Two companies want to compare their datasets without revealing them
    - For example, compute the intersection of two lists of names
- Example: database privacy
  - Evaluate a query on the database without revealing the query to the database owner
  - Evaluate a statistical query on the database without revealing the values of individual entries
  - Many variations

A Couple of Observations

- In all cases, we are dealing with distributed multi-party protocols
  - A protocol describes how parties are supposed to exchange messages on the network
- All of these tasks can be easily computed by a trusted third party
  - The goal of secure multi-party computation is to achieve the same result without involving a trusted third party

How to Define Security?

- Must be mathematically rigorous
- Must capture all realistic attacks that a malicious participant may try to stage
- Should be “abstract”
  - Based on the desired “functionality” of the protocol, not a specific protocol
  - Goal: define security for an entire class of protocols

Functionality

- K mutually distrustful parties want to jointly carry out some task
- Model this task as a function

$$f: (\{0,1\}^*)^K \rightarrow (\{0,1\}^*)^K$$

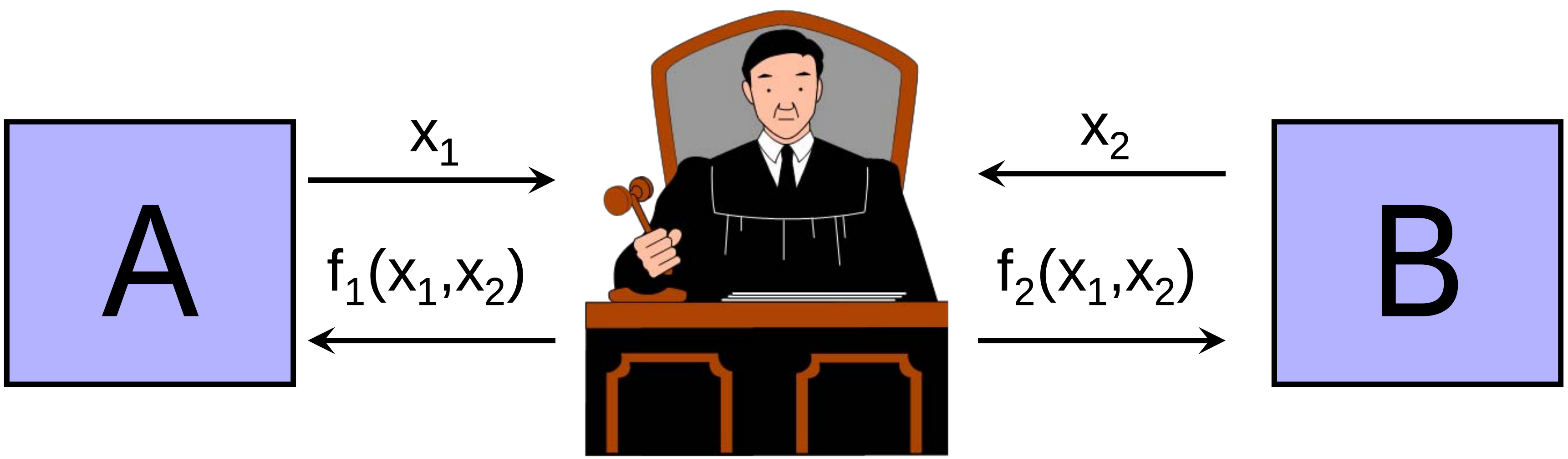
K inputs (one per party); each input is a bitstring

K outputs

- Assume that this functionality is computable in probabilistic polynomial time

Ideal Model

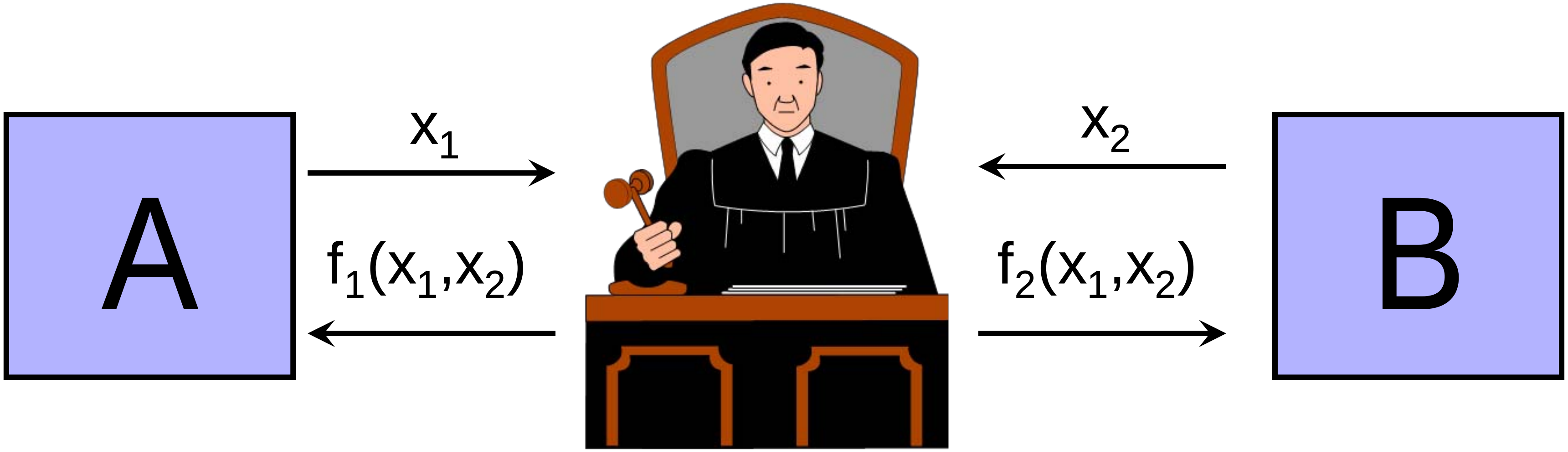
- Intuitively, we want the protocol to behave “as if” a trusted third party collected the parties’ inputs and computed the desired functionality
  - Computation in the ideal model is secure by definition!



Slightly More Formally

- A protocol is secure if it emulates an ideal setting where the parties hand their inputs to a "trusted party," who locally computes the desired outputs and hands them back to the parties

[Goldreich-Micali-Wigderson 1987]



Adversary Models

- Some of protocol participants may be corrupt
  - If all were honest, would not need secure multi-party computation
- Semi-honest (aka passive; honest-but-curious)
  - Follows protocol, but tries to learn more from received messages than he would learn in the ideal model
- Malicious
  - Deviates from the protocol in arbitrary ways, lies about his inputs, may quit at any point
- For now, we will focus on semi-honest adversaries and two-party protocols

Correctness and Security

- How do we argue that the real protocol “emulates” the ideal protocol?
- Correctness
  - All honest participants should receive the correct result of evaluating function f
    - Because a trusted third party would compute f correctly
- Security
  - All corrupt participants should learn no more from the protocol than what they would learn in ideal model
  - What does corrupt participant learn in ideal model?
    - His input (obviously) and the result of evaluating f

Simulation

- Corrupt participant's view of the protocol = record of messages sent and received
  - In the ideal world, view consists simply of his input and the result of evaluating f
- How to argue that real protocol does not leak more useful information than ideal-world view?
- Key idea: simulation
  - If real-world view (i.e., messages received in the real protocol) can be simulated with access only to the ideal-world view, then real-world protocol is secure
  - Simulation must be indistinguishable from real view

Technicalities

- **Distance** between probability distributions A and B over a common set X is

  $\frac{1}{2} * \text{sum}_X(|\Pr(A=x) - \Pr(B=x)|)$

- **Probability ensemble** A_i is a set of discrete probability distributions
  - Index i ranges over some set I
- Function f(n) is **negligible** if it is asymptotically smaller than the inverse of any polynomial

  $\forall$ constant c $\exists m$ such that $|f(n)| < 1/n^c \ \forall n>m$

Notions of Indistinguishability

- Simplest: ensembles A_i and B_i are equal
- Distribution ensembles A_i and B_i are statistically close if dist(A_i,B_i) is a negligible function of i
- Distribution ensembles A_i and B_i are computationally indistinguishable ($A_i \approx B_i$) if, for any probabilistic polynomial-time algorithm D, $|Pr(D(A_i)=1) - Pr(D(B_i)=1)|$ is a negligible function of i
  - No efficient algorithm can tell the difference between A_i and B_i except with a negligible probability

SMC Definition (First Attempt)

- Protocol for computing $f(X_A,X_B)$ betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that for all input pairs (x_A,x_B) ...
- Correctness: $(y_A,y_B) \approx f(x_A,x_B)$
  - Intuition: outputs received by honest parties are indistinguishable from the correct result of evaluating f
- Security: $view_A(\text{real protocol}) \approx S_A(x_A,y_A)$

  $view_B(\text{real protocol}) \approx S_B(x_B,y_B)$
  - Intuition: a corrupt party's view of the protocol can be simulated from its input and output
- This definition does not work! Why?

Randomized Ideal Functionality

- Consider a coin flipping functionality

  f()=(b,-) where b is random bit

  - f() flips a coin and tells A the result; B learns nothing

- The following protocol "implements" f()
  1. A chooses bit b randomly
  2. A sends b to B
  3. A outputs b
- It is obviously insecure (why?)
- Yet it is correct and simulatable according to our attempted definition (why?)

SMC Definition

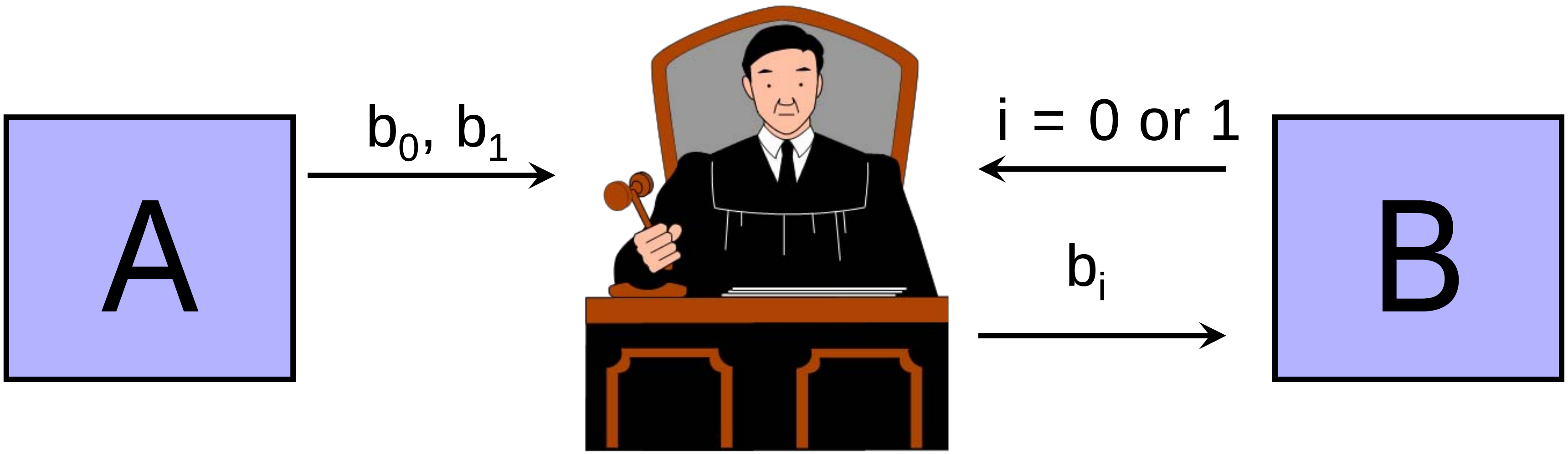
- Protocol for computing $f(X_A,X_B)$ betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that for all input pairs (x_A,x_B) ...
- Correctness: $(y_A,y_B) \approx f(x_A,x_B)$
- Security: $(view_A(\text{real protocol}), y_B) \approx (S_A(x_A,y_A), y_B)$

  $(view_B(\text{real protocol}), y_A) \approx (S_B(x_B,y_B), y_A)$
  - Intuition: if a corrupt party's view of the protocol is correlated with the honest party's output, the simulator must be able to capture this correlation
- Does this fix the problem with coin-flipping f?

Oblivious Transfer (OT)

[Rabin 1981]

- Fundamental SMC primitive



- A inputs two bits, B inputs the index of one of A's bits
- B learns his chosen bit, A learns nothing
  - A does not learn <u>which</u> bit B has chosen; B does not learn the value of the bit that he did <u>not</u> choose
- Generalizes to bitstrings, M instead of 2, etc.

One-Way Trapdoor Functions

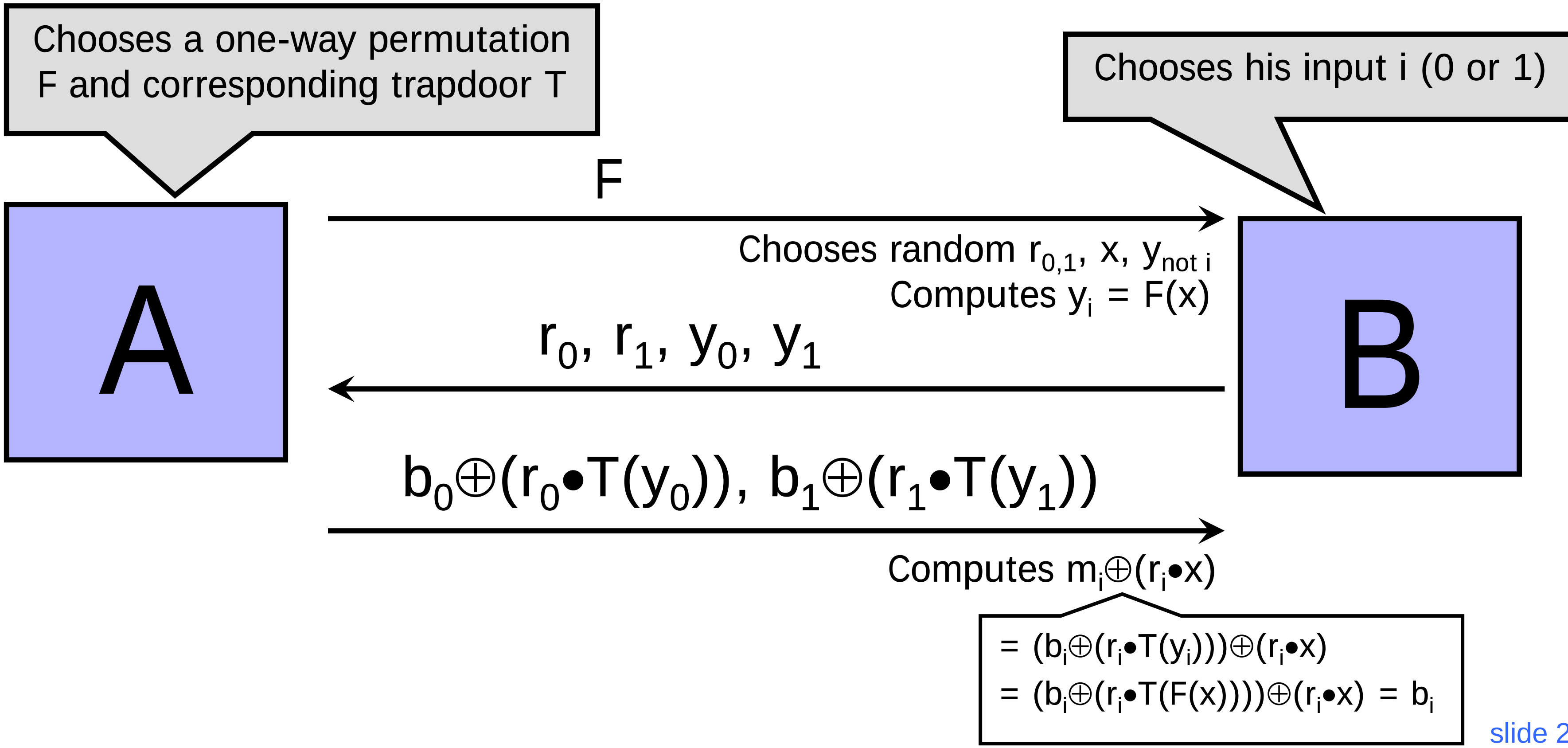
- Intuition: one-way functions are easy to compute, but hard to invert (skip formal definition for now)
  - We will be interested in one-way permutations
- Intution: one-way trapdoor functions are one-way functions that are easy to invert given some extra information called the <u>trapdoor</u>
  - Example: if n=pq where p and q are large primes and e is relatively prime to $\varphi(n)$, $f_{e,n}(m) = m^e \bmod n$ is easy to compute, but it is believed to be hard to invert
  - Given the trapdoor d s.t. de=1 mod $\varphi(n)$, $f_{e,n}(m)$ is easy to invert because $f_{e,n}(m)^d = (m^e)^d = m \bmod n$

Hard-Core Predicates

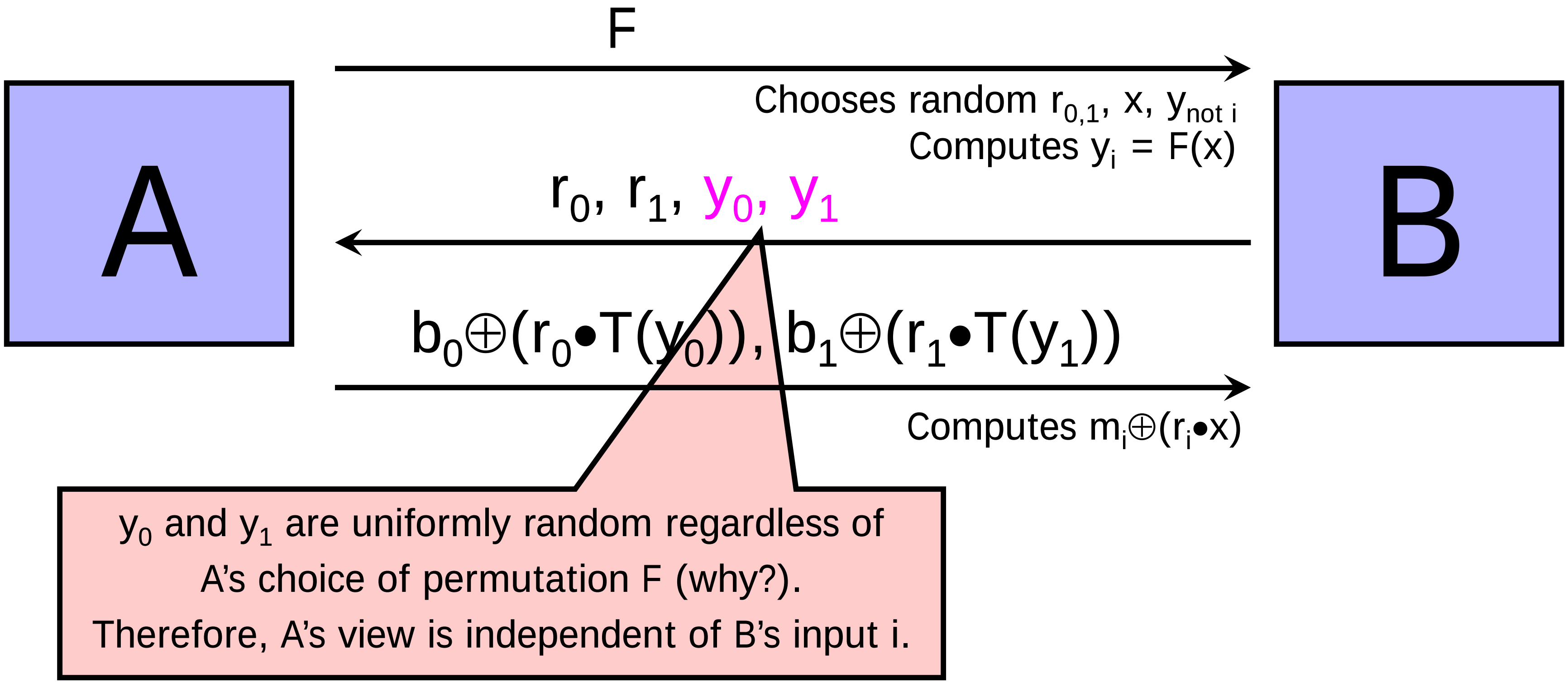
- Let f: $S \to S$ be a one-way function on some set S
- B: $S \to \{0,1\}$ is a hard-core predicate for f if
  - B(x) is easy to compute given $x \in S$
  - If an algorithm, given only f(x), computes B(x) correctly with prob > $\frac{1}{2}+\varepsilon$, it can be used to invert f(x) easily
    - Consequence: B(x) is hard to compute given only f(x)
  - Intuition: there is a bit of information about x s.t. learning this bit from f(x) is as hard as inverting f
- Goldreich-Levin theorem
  - $B(x,r)=r \bullet x$ is a hard-core predicate for $g(x,r) = (f(x),r)$
    - f(x) is any one-way function, $r \bullet x = (r_1 x_1) \oplus \ldots \oplus (r_n x_n)$

Oblivious Transfer Protocol

- Assume the existence of some family of one-way trapdoor permutations

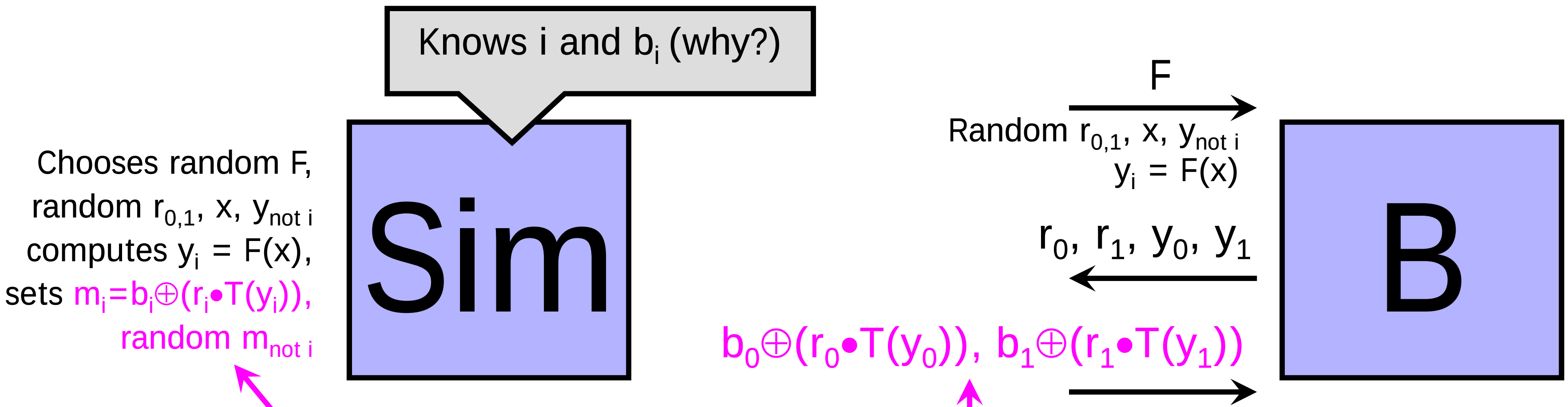


Proof of Security for B



Proof of Security for A (Sketch)

- Need to build a simulator whose output is indistinguishable from B's view of the protocol



The only difference between simulation and real protocol:

In simulation, $m_{not\ i}$ is random (why?)

In real protocol, $m_{not\ i}=b_{not\ i}\oplus(r_{not\ i} \bullet T(y_{not\ i}))$

Proof of Security for A (Cont'd)

- Why is it computationally infeasible to distinguish random m and $m' = b \oplus (r \bullet T(y))$?
  - b is some bit, r and y are random, T is the trapdoor of a one-way trapdoor permutation
- $(r \bullet x)$ is a hard-core bit for $g(x,r) = (F(x), r)$
  - This means that $(r \bullet x)$ is hard to compute given $F(x)$
- If B can distinguish m and $m' = b \oplus (r \bullet x')$ given only $y = F(x')$, we obtain a contradiction with the fact that $(r \bullet x')$ is a hard-core bit
  - Proof omitted