CS 395T

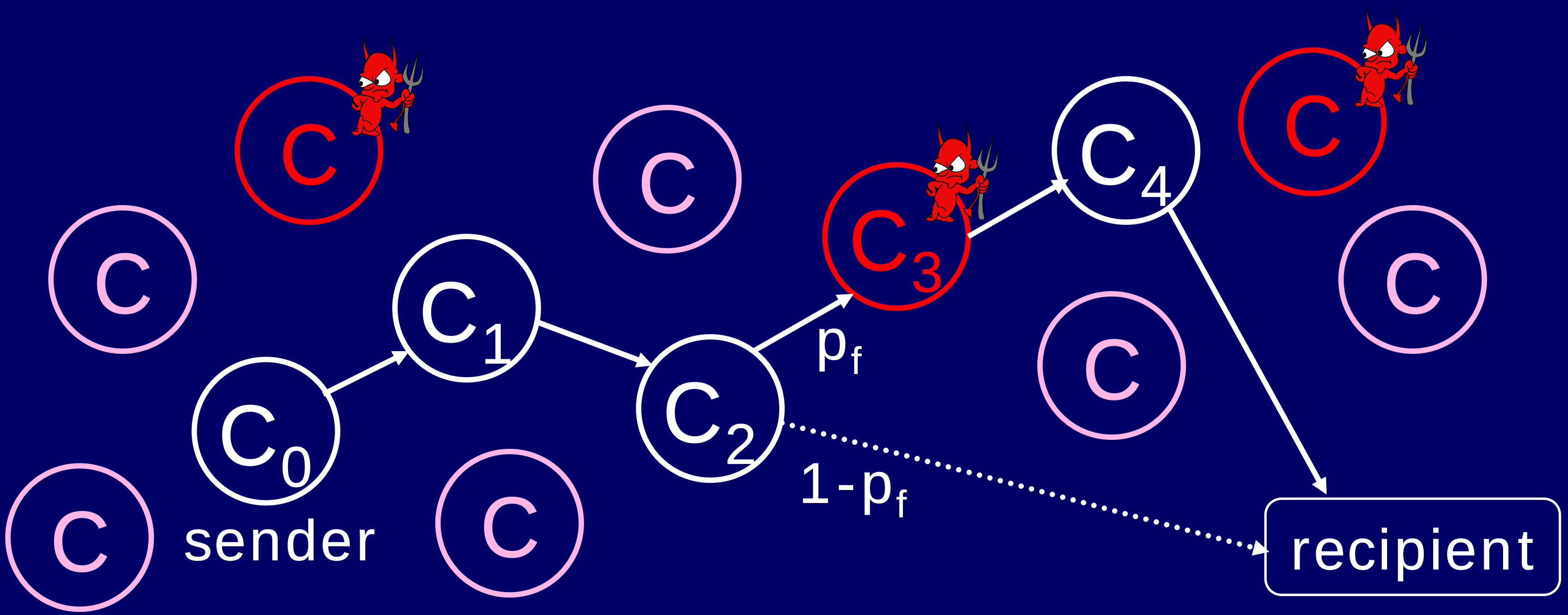
Probabilistic Model Checking

Overview

- Crowds redux
- Probabilistic model checking
  - PRISM model checker
  - PCTL logic
  - Analyzing Crowds with PRISM
- Probabilistic contract signing
  - Rabin's beacon protocol
  - Ben-Or, Goldreich, Rivest, Micali protocol
  - Analyzing probabilistic contract signing protocols with PRISM

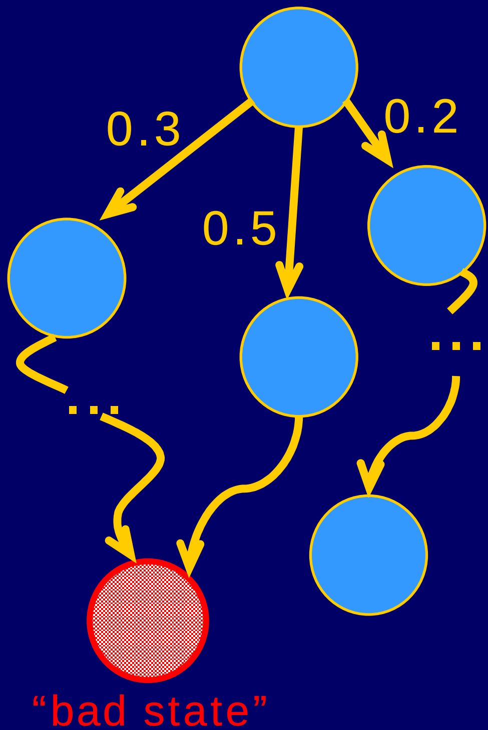
Crowds System

[Reiter,Rubin '98]



- Routers form a random path when establishing connection
  - In onion routing, random path is chosen in advance by sender
- After receiving a message, honest router flips a biased coin
  - With probability P_f randomly selects next router and forwards msg
  - With probability $1-P_f$ sends directly to the recipient

Probabilistic Model Checking



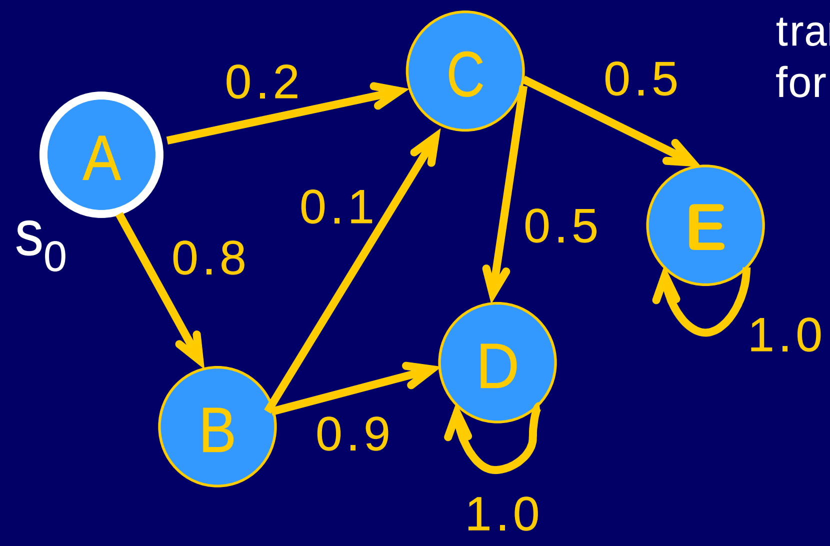
- Participants are finite-state machines
  - Same as Murφ
- State transitions are probabilistic
  - Transitions in Murφ are nondeterministic
- Standard intruder model
  - Same as Murφ: model cryptography with abstract data types
- Murφ question:
  - *Is bad state reachable?*
- Probabilistic model checking question:
  - *What’s the probability of reaching bad state?*

Discrete-Time Markov Chains

(S, s_0, T, L)

- S is a finite set of states
- $s_0 \in S$ is an initial state
- $T : S \times S \rightarrow [0,1]$ is the transition relation
  - $\forall s,s' \in S \;\; \sum_{s'} T(s,s') = 1$
- L is a labeling function

Markov Chain: Simple Example



Probabilities of outgoing transitions sum up to 1.0 for every state

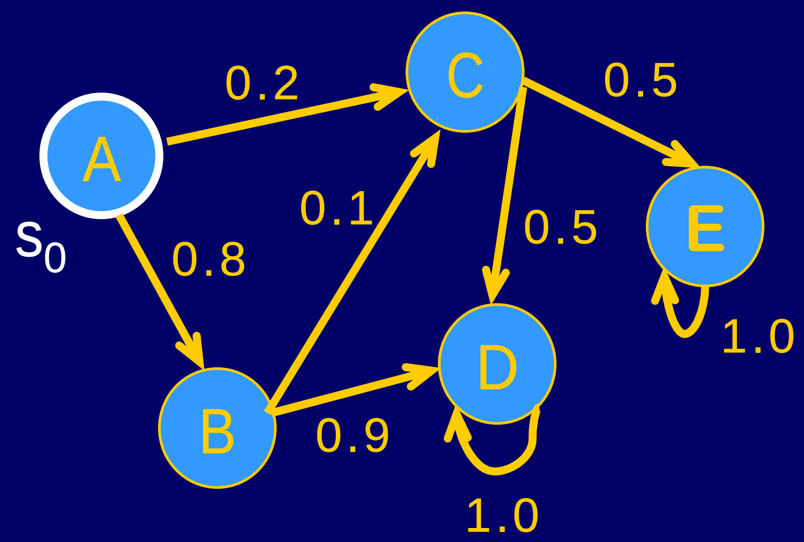
- Probability of reaching E from s_0 is $0.2 \bullet 0.5 + 0.8 \bullet 0.1 \bullet 0.5 = 0.14$
- The chain has infinite paths if state graph has loops
  - Need to solve a system of linear equations to compute probabilities

PRISM

[Kwiatkowska et al., U. of Birmingham]

- Probabilistic model checker
- System specified as a Markov chain
  - Parties are finite-state machines w/ local variables
  - State transitions are associated with probabilities
    - Can also have nondeterminism (Markov decision processes)
  - All parameters must be finite
- Correctness condition specified as PCTL formula
- Computes probabilities for each reachable state
    - Enumerates reachable states
    - Solves system of linear equations to find probabilities

PRISM Syntax




```
module Simple
  state: [1..5] init 1;
  [] state=1 -> 0.8: state'=2 + 0.2: state'=3;
  [] state=2 -> 0.1: state'=3 + 0.9: state'=4;
  [] state=3 -> 0.5: state'=4 + 0.5: state'=5;
endmodule
```

IF state=3 THEN with prob. 50% assign 4 to state,
with prob. 50% assign 5 to state

Modeling Crowds with PRISM

- Model probabilistic path construction
- Each state of the model corresponds to a particular stage of path construction
  - 1 router chosen, 2 routers chosen, ...
- Three probabilistic transitions
  - Honest router chooses next router with probability p_f, terminates the path with probability $1-p_f$
  - Next router is probabilistically chosen from N candidates
  - Chosen router is hostile with certain probability
- Run path construction protocol several times and look at accumulated observations of the intruder

PRISM: Path Construction in Crowds

Next router is corrupt with certain probability

```
module crowds
 . . .
 // N = total # of routers, C = # of corrupt routers
 // badC = C/N, goodC = 1-badC
 [] (!good & !bad & run) ->
     goodC: (good'=true) & (revealAppSender'=true) &
            (run'=false) +
      badC: (badObserve'=true) & (run'=false);

 // Forward with probability PF, else deliver
 [] (good & !deliver) ->
       PF: (pIndex'=pIndex+1) & (forward'=true) &
           (good'=false) +
    notPF: (deliver'=true);
. . .
endmodule
```

Route with probability PF, else deliver

PRISM: Intruder Model

```
module crowds
 . . .
 // Record the apparent sender and deliver
 [] (badObserve & appSender=0) ->
       (observe0'=observe0+1) & (deliver'=true);
 . . .
 // Record the apparent sender and deliver
 [] (badObserve & appSender=15) ->
       (observe15'=observe15+1) & (deliver'=true);
 . . .
endmodule
```

- For each observed path, bad routers record apparent sender
- Bad routers collaborate, so treat them as a single attacker
- No cryptography, only probabilistic inference

PCTL Logic

[Hansson, Jonsson '94]

- Probabilistic Computation Tree Logic
- Used for reasoning about probabilistic temporal properties of probabilistic finite state spaces
- Can express properties of the form "under any scheduling of processes, the probability that event E occurs is at least p"
  - By contrast, Murφ can express only properties of the form "does event E ever occur?"

PCTL Syntax

- **State formulas**
  - First-order propositions over a single state

$$\Phi ::= \text{True} \mid a \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \neg\Phi \mid P_{>p}[\Psi]$$

Predicate over state variables (just like a Murφ invariant)

Path formula holds with probability > p

- **Path formulas**
  - Properties of chains of states

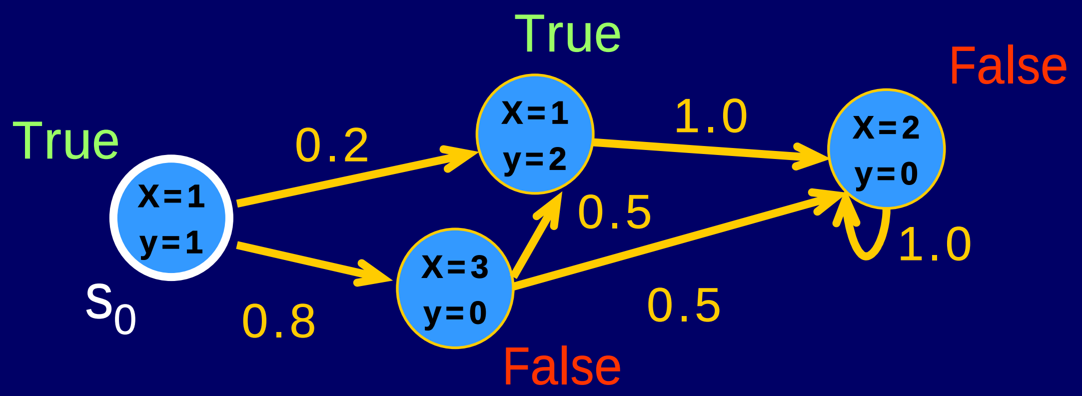
$$\Psi ::= X\ \Phi \mid \Phi\ U^{\leq k}\ \Phi \mid \Phi\ U\ \Phi$$

State formula holds for next state in the chain

First state formula holds for every state in the chain until second becomes true

PCTL: State Formulas

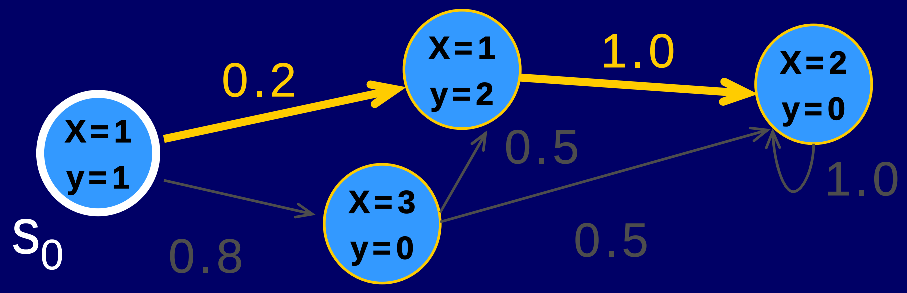
- A state formula is a first-order state predicate
  - Just like non-probabilistic logic



$$\varphi = \texttt{(y>1) | (x=1)}$$

PCTL: Path Formulas

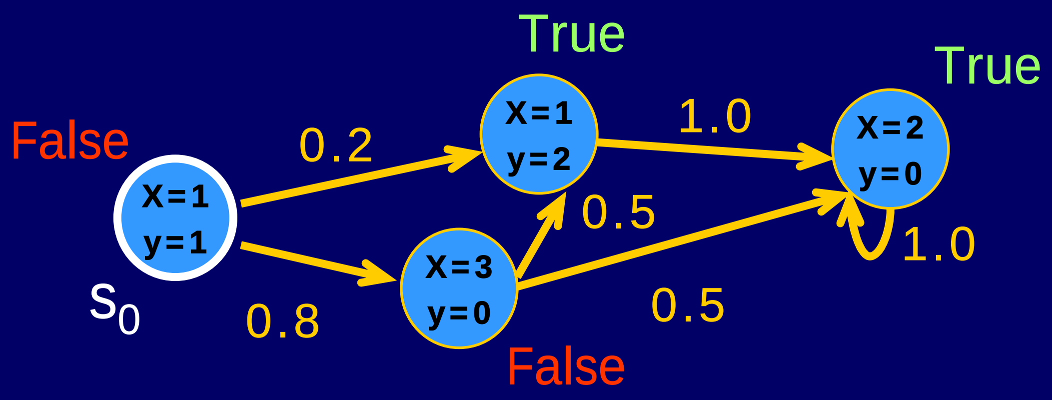
- A path formula is a temporal property of a chain of states
  - $\varphi_1 U \varphi_2$ = "φ_1 is true until φ_2 becomes and stays true"



ψ = **(y>0) U (x>y)** holds for this chain

PCTL: Probabilistic State Formulas

- Specify that a certain predicate or path formula holds with probability no less than some bound



$$\varphi = \mathbf{P}_{>0.5}[(y>0)\ \mathbf{U}\ (x=2)]$$

Intruder Model Redux

```
module crowds
 . . .
 // Record the apparent sender and deliver
 [] (badObserve & appSender=0) ->
       (observe0'=observe0+1) & (deliver'=true);
 . . .
 // Record the apparent sender and deliver
 [] (badObserve & appSender=15) ->
       (observe15'=observe15+1) & (deliver'=true);
 . . .
endmodule
```

Every time a hostile crowd member receives a message from some honest member, he records his observation (increases the count for that honest member)

Negation of Probable Innocence

```
launch ->
 [true U (observe0>observe1) & done] > 0.5
 …
launch ->
 [true U (observe0>observe9) & done] > 0.5
```

"The probability of reaching a state in which hostile crowd members completed their observations and observed the true sender (crowd member #0) more often than any of the other crowd members (#1 … #9) is greater than 0.5"

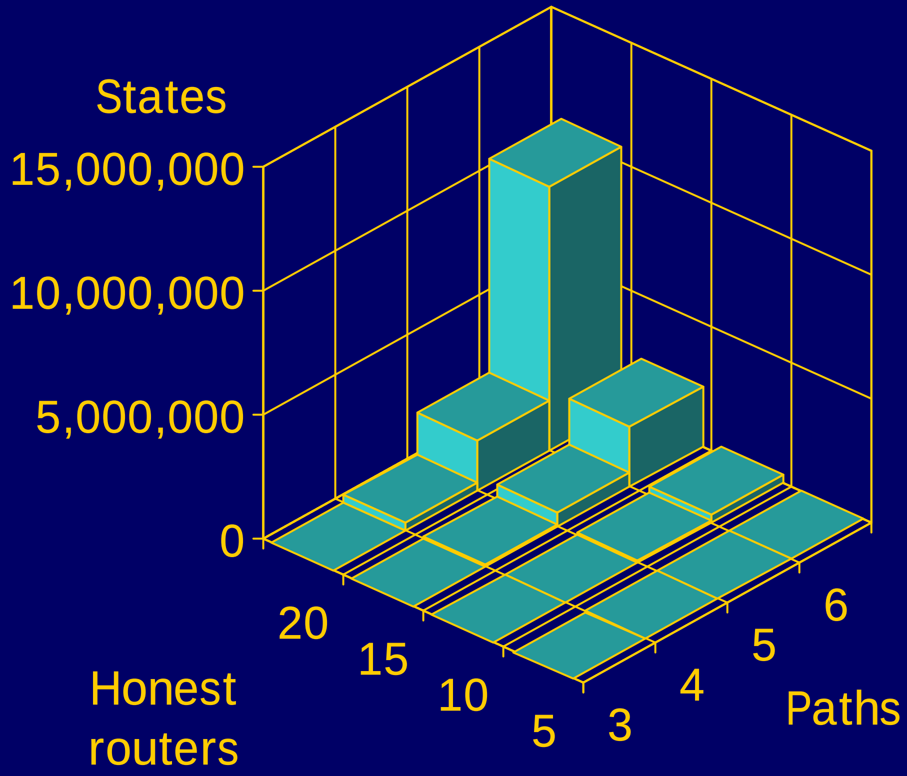
Analyzing Multiple Paths with PRISM

Use PRISM to automatically compute interesting probabilities for chosen finite configurations

- "Positive": $P(K_0 > 1)$
  - Observing the true sender more than once
- "False positive": $P(K_{i\neq 0} > 1)$
  - Observing a wrong crowd member more than once
- "Confidence": $P(K_{i\neq 0} \leq 1 \mid K_0 > 1)$
  - Observing <u>only</u> the true sender more than once

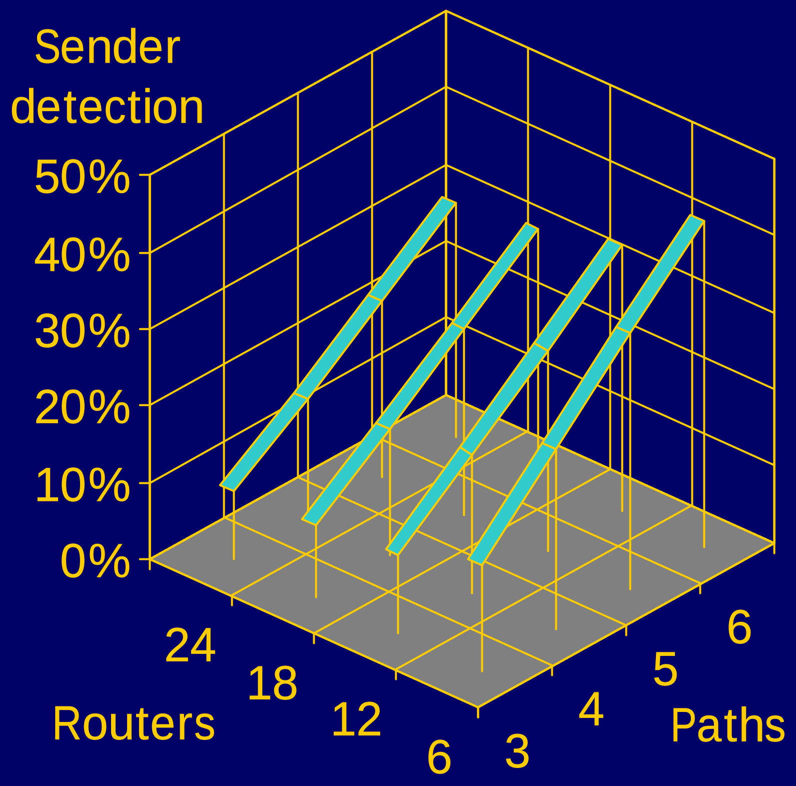
K_i = how many times crowd member *i* was recorded as apparent sender

Size of State Space



All hostile routers are treated as a single router, selected with probability 1/6

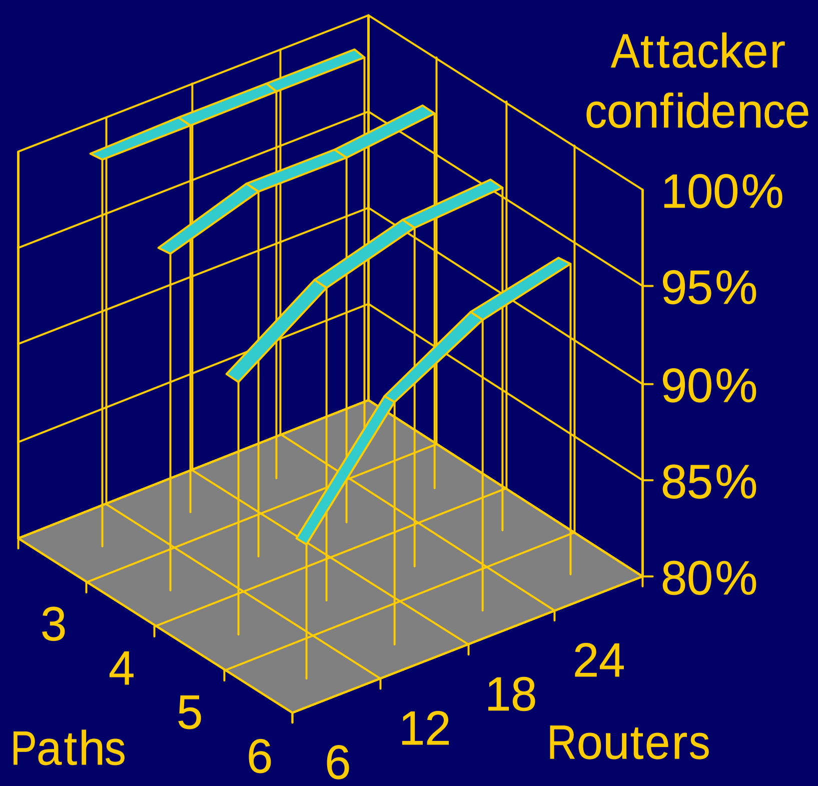
Sender Detection (Multiple Paths)



1/6 of routers are hostile

- All configurations satisfy probable innocence
- Probability of observing the true sender increases with the number of paths observed...
- ... but decreases with the increase in crowd size
- Is this an attack?
  - Can’t avoid building new paths
  - Hard to prevent attacker from correlating same-sender paths

Attacker's Confidence



1/6 of routers are hostile

- "Confidence" = probability of detecting only the true sender
- Confidence grows with crowd size
- Maybe this is not so strange
  - True sender appears in every path, others only with small probability
  - Once attacker sees somebody twice, he knows it's the true sender
- Is this an attack?
  - Large crowds: lower probability to detect senders, but higher confidence that the detected user is the true sender