CS 395T

Probabilistic Contract Signing

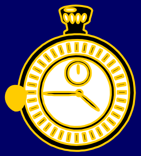
Probabilistic Fair Exchange

- Two parties exchange items of value
  - Signed commitments (contract signing)
  - Signed receipt for an email message (certified email)
  - Digital cash for digital goods (e-commerce)
- Important if parties don't trust each other
  - Need assurance that if one does not get what it wants, the other doesn't get what it wants either
- Fairness is hard to achieve
  - Gradual release of verifiable commitments
  - Convertible, verifiable signature commitments
  - Probabilistic notions of fairness

Properties of Fair Exchange Protocols

- Fairness
  - At each step, the parties have approximately equal probabilities of obtaining what they want

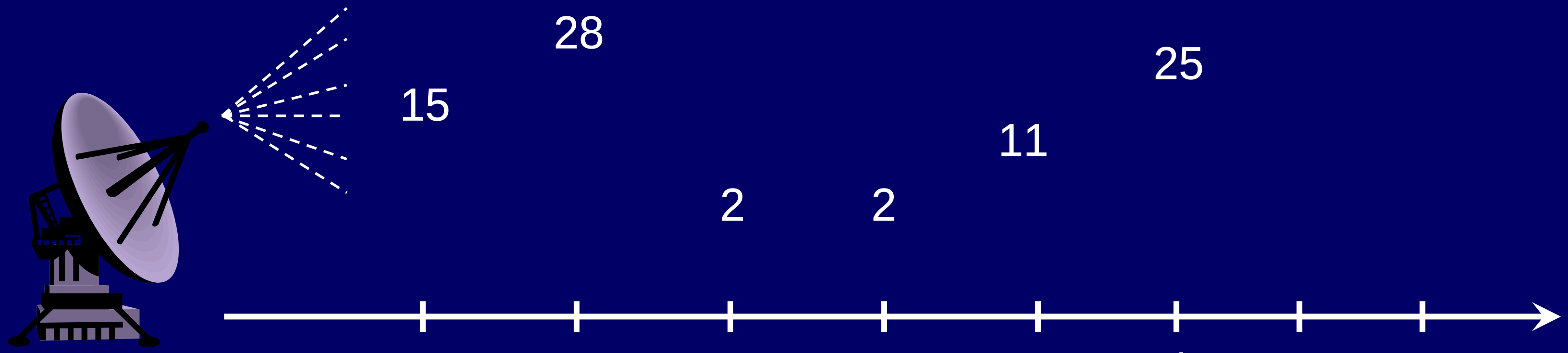
- Optimism
  - If both parties are honest, then exchange succeeds without involving a judge or trusted third party



- Timeliness
  - If something goes wrong, the honest party does not have to wait for a long time to find out whether exchange succeeded or not

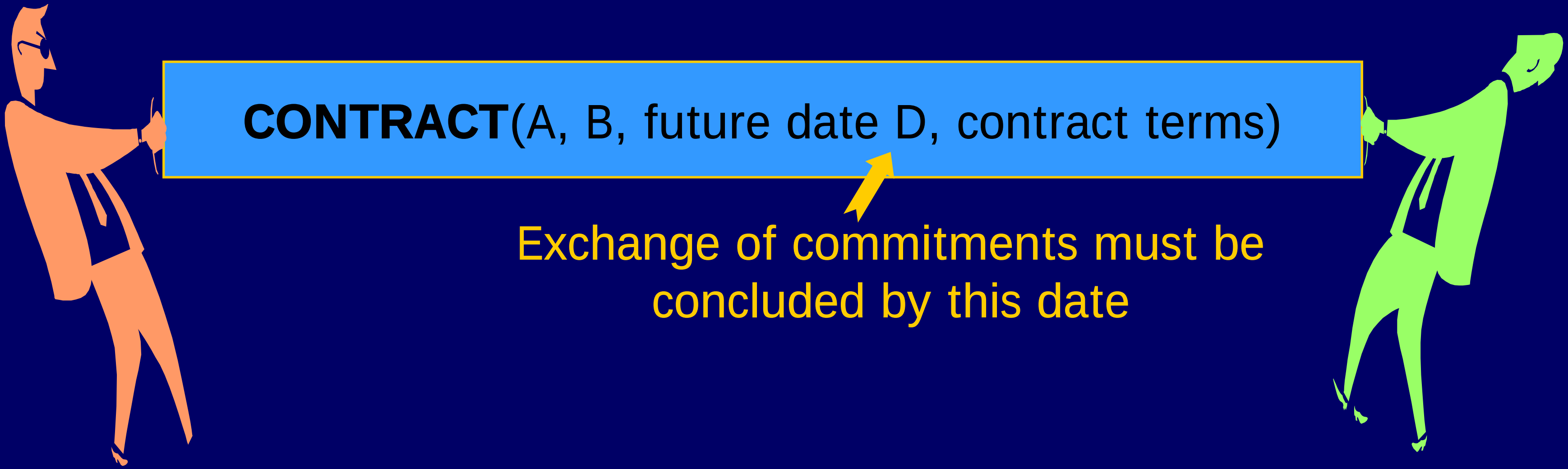
Rabin’s Beacon

- A “beacon” is a trusted party that publicly broadcasts a randomly chosen number between 1 and N every day
  - Michael Rabin. “Transaction protection by beacons”. Journal of Computer and System Sciences, Dec 1983.



15	28	2	2	11	25	
Jan 27	Jan 28	Jan 29	Jan 30	Jan 31	Feb 1	…

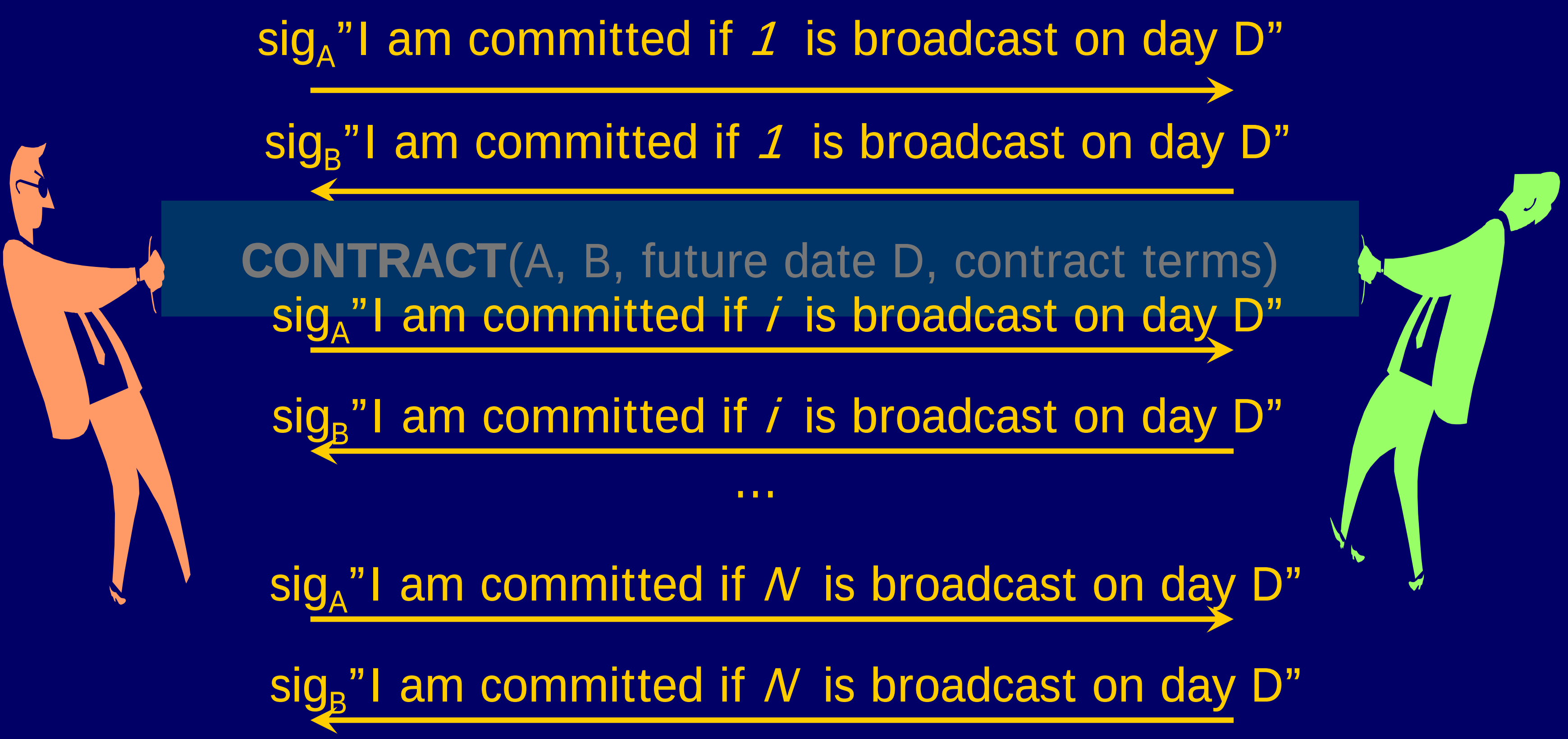
Contract



CONTRACT(A, B, future date D, contract terms)

Exchange of commitments must be concluded by this date

Rabin’s Contract Signing Protocol



CONTRACT(A, B, future date D, contract terms)

sig_A”I am committed if *1* is broadcast on day D”

sig_B”I am committed if *1* is broadcast on day D”

sig_A”I am committed if *i* is broadcast on day D”

sig_B”I am committed if *i* is broadcast on day D”

…

sig_A”I am committed if *N* is broadcast on day D”

sig_B”I am committed if *N* is broadcast on day D”

2N messages are exchanged if both parties are honest

Probabilistic Fairness

- Suppose B stops after receiving A’s i^{th} message
  - B has sig_A”committed if *1* is broadcast”,
    sig_A”committed if *2* is broadcast”,
    ...
    sig_A”committed if i is broadcast”
  - A has sig_B”committed if *1* is broadcast”, …
    sig_B”committed if $i-1$ is broadcast”
- … and beacon broadcasts number b on day D
  - If $b < i$, then both A and B are committed
  - If $b > i$, then neither A, nor B is committed
  - If $b = i$, then only A is committed

This happens only with probability 1 / N

Properties of Rabin’s Protocol

- Fair
  - The difference between A’s probability to obtain B’s commitment and B’s probability to obtain A’s commitment is at most 1/N
    - But communication overhead is 2N messages
- Not optimistic
  - Need input from third party in every transaction
    - Same input for all transactions on a given day sent out as a one-way broadcast. Maybe this is not so bad!
- Not timely
  - If one of the parties stops communicating, the other does not learn the outcome until day D

BGMR Probabilistic Contract Signing

[Ben-Or, Goldreich, Micali, Rivest ’85-90]

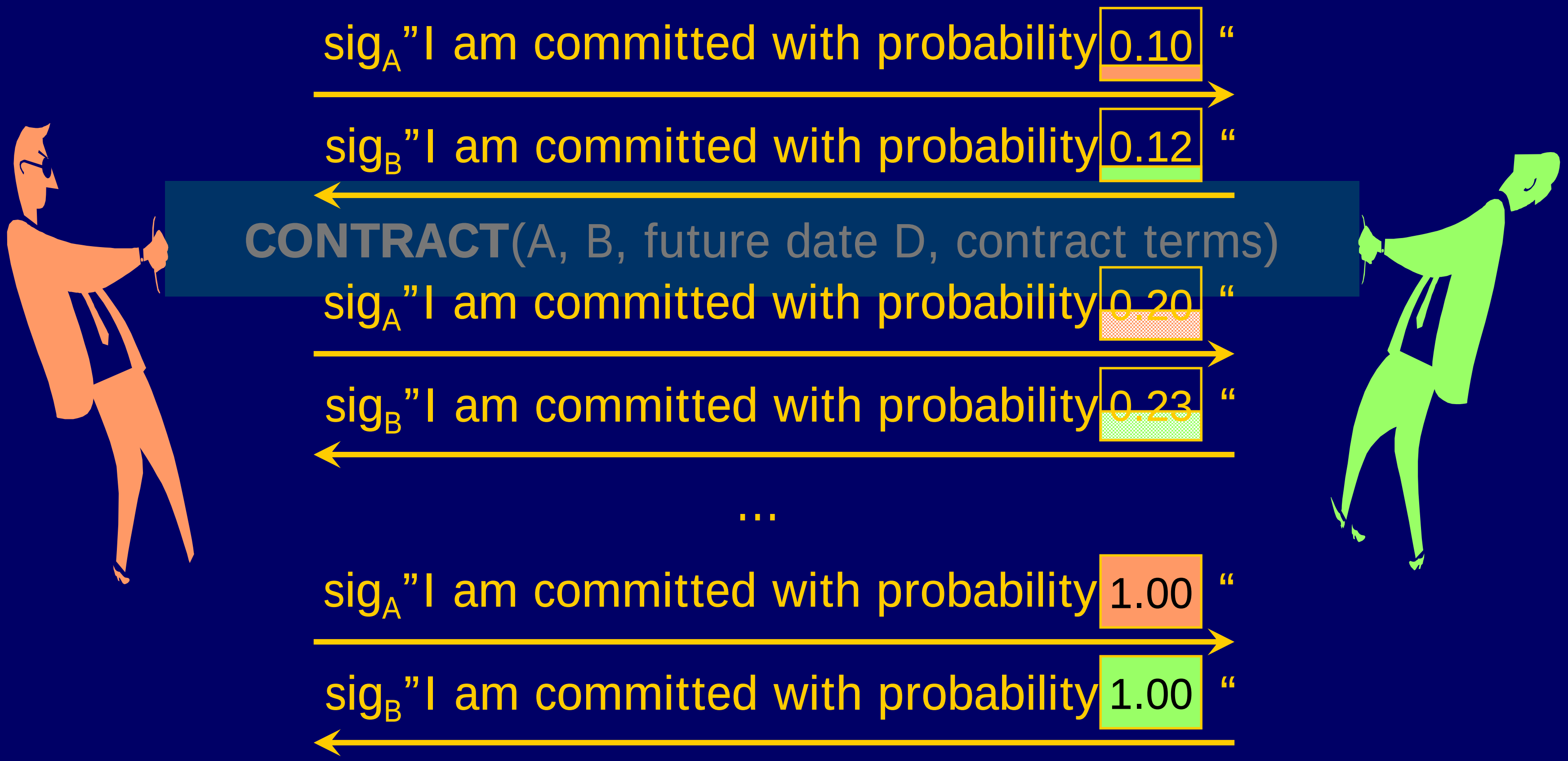
- Doesn’t need beacon input in every transaction
- Uses sig_A”I am committed with probability p_A” instead of

  sig_A”I am committed if *i* is broadcast on day D”
- Each party decides how much to increase the probability at each step
  - A receives sig_B”I am committed with probability p_B” from B
  - Sets $p_A = \min(1, p_B \bullet \alpha)$

    α is a parameter chosen by A
  - Sends sig_A”I am committed with probability p_A” to B

... the algorithm for B is symmetric

BGMR Message Flow



sig_A”I am committed with probability 0.10 “

sig_B”I am committed with probability 0.12 “

CONTRACT(A, B, future date D, contract terms)

sig_A”I am committed with probability 0.20 “

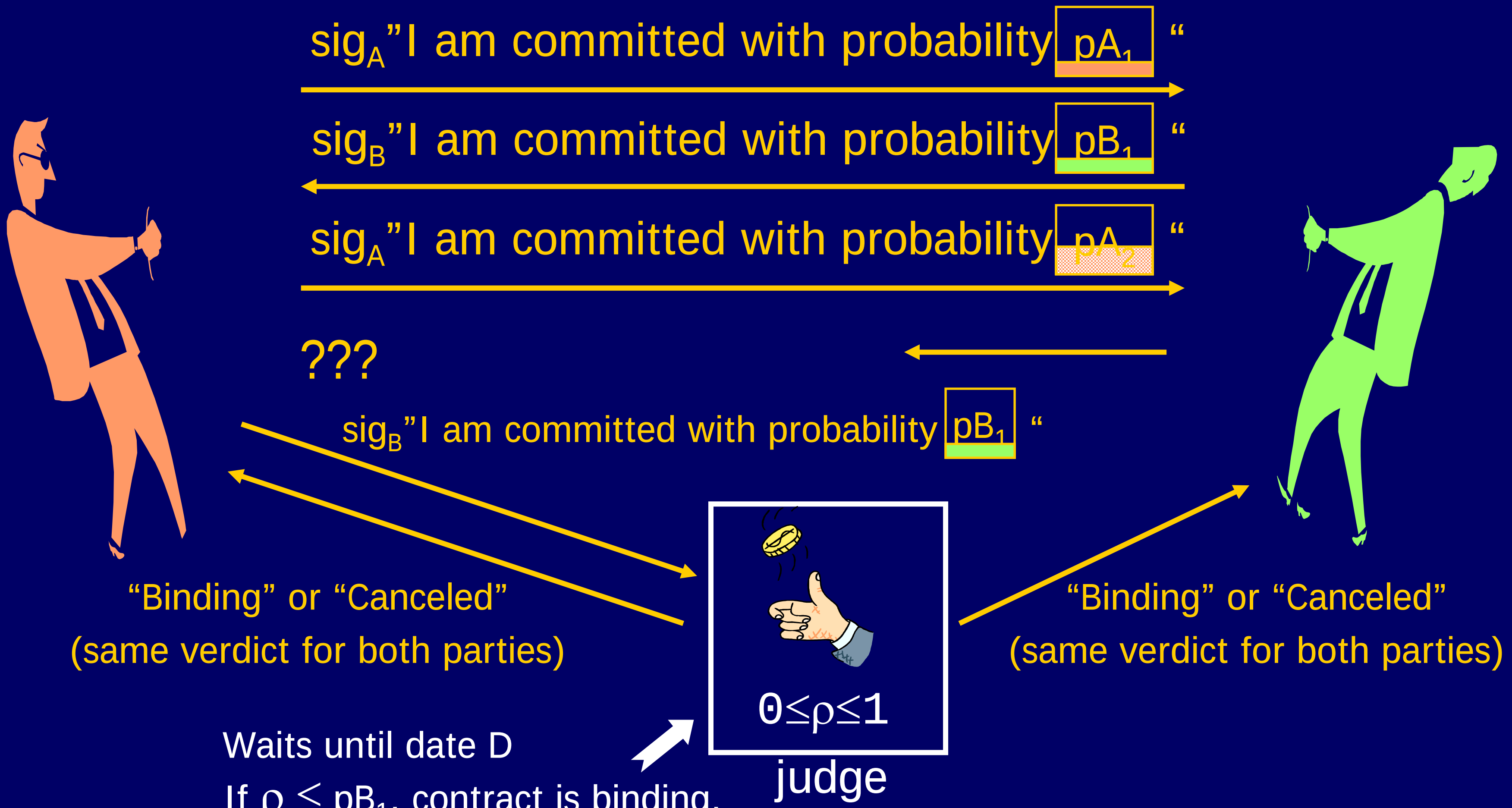
sig_B”I am committed with probability 0.23 “

…

sig_A”I am committed with probability 1.00 “

sig_B”I am committed with probability 1.00 “

Conflict Resolution



sig$_A$"I am committed with probability pA$_1$ "

sig$_B$"I am committed with probability pB$_1$ "

sig$_A$"I am committed with probability pA$_2$ "

???

sig$_B$"I am committed with probability pB$_1$ "

"Binding" or "Canceled"
(same verdict for both parties)

"Binding" or "Canceled"
(same verdict for both parties)

$0 \le \rho \le 1$

judge

Waits until date D

If $\rho \le pB_1$, contract is binding,
else contract is canceled

Judge

- Waits until date D to decide
- Announces verdict to both parties
- Tosses coin once for each contract
- Remembers previous coin tosses
  - Constant memory: use pseudo-random functions with a secret input to produce repeatable coin tosses for each contract
- Does not remember previous verdicts
  - Same coin toss combined with different evidence (signed message with a different probability value) may result in a different verdict

Privilege and Fairness

Privilege

A party is privileged if it has the evidence to cause the judge to declare contract binding

Intuition: the contract binds either both parties, or neither;
what matters is the ability to make the contract binding

Fairness

At any step where Prob(B is privileged) $> v$,
Prob(A is not privileged | B is privileged) $< \varepsilon$

Intuition: at each step, the parties should have comparable probabilities of causing the judge to declare contract binding (privilege must be symmetric)

Properties of BGMR Protocol

Fair

- Privilege is almost symmetric at each step:
  if Prob(B is privileged) > p_{A_0}, then
  Prob(A is not privileged | B is privileged) < $1-1/\alpha$

Optimistic

- Two honest parties don’t need to invoke a judge

Not timely

- Judge waits until day D to toss the coin
- What if the judge tosses the coin and announces the verdict as soon as he is invoked?

Formal Model

- Protocol should ensure fairness given any possible behavior by a dishonest participant
  - Contact judge although communication hasn’t stopped
  - Contact judge more than once
  - Delay messages from judge to honest participant
- Need nondeterminism
  - To model dishonest participant’s choice of actions
- Need probability
  - To model judge’s coin tosses
- The model is a Markov decision process

Constructing the Model

- Discretize probability space of coin tosses
  - The coin takes any of N values with equal probability
- Fix each party's "probability step"
  - Rate of increases in the probability value contained in the party's messages determines how many messages are exchanged
- A state is unfair if privilege is asymmetric
  - Difference in evidence, <u>not</u> difference in commitments
- Compute probability of reaching an unfair state for different values of the parties' probability steps

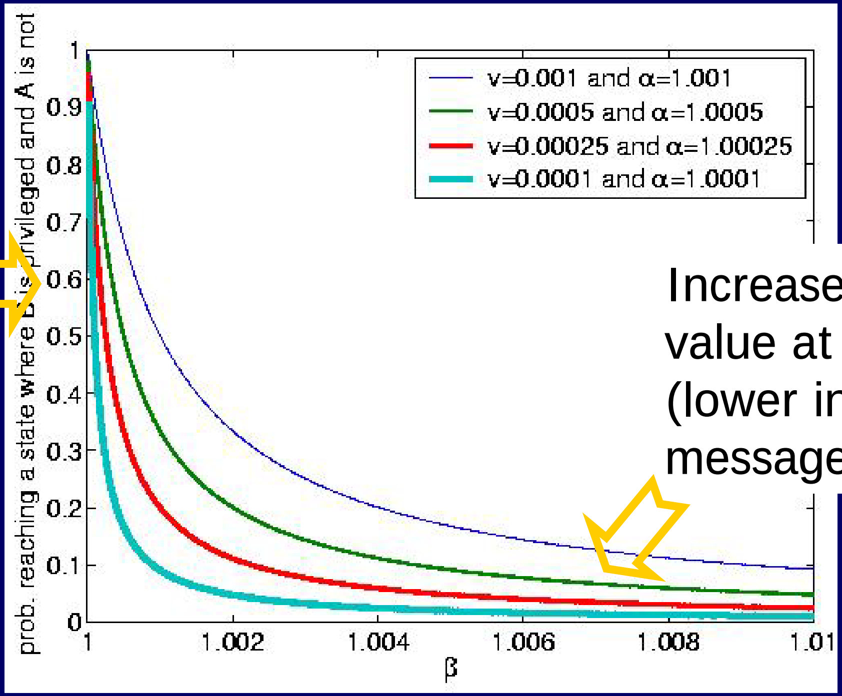
Defines state space

Use PRISM

Attack Strategy

- Dishonest B's probability of driving the protocol to an unfair state is maximized by this strategy:
    1. Contact judge as soon as first message from A arrives
    2. Judge tries to send verdict to A (the verdict is probably negative, since A's message contains a low probability value)
    3. B delays judge's verdicts sent to A
    4. B contacts judge again with each new message from A until a positive verdict is obtained
- This strategy only works in the timely protocol
    - In the original protocol, coin is not tossed and verdict is not announced until day D
- Conflict between optimism and timeliness

Analysis Results



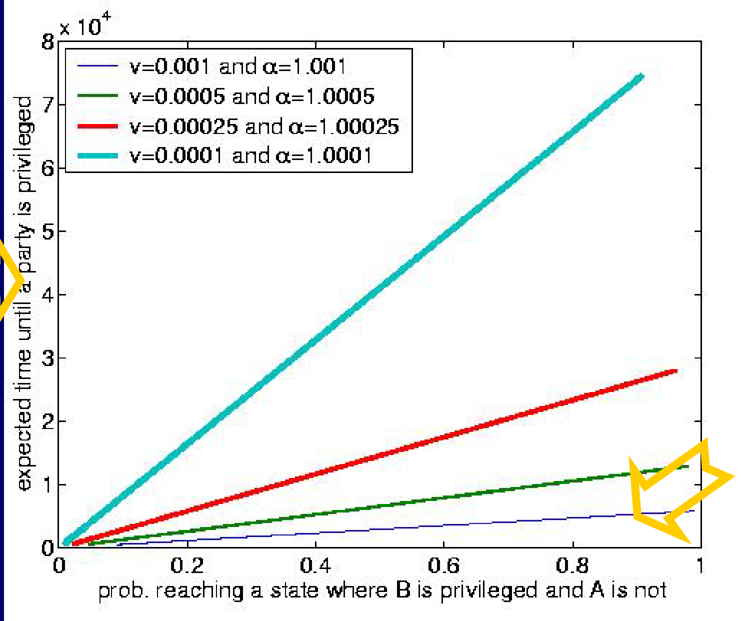
Probability of reaching a state where B is privileged and A is not

Increase in B’s probability value at each step
(lower increase means more messages must be exchanged)

For a higher probability of winning, dishonest B must exchange more messages with honest A

Attacker’s Tradeoff

Expected number of messages before unfair state is reached



Probability of reaching a state where B is privileged and A is not

- Linear tradeoff for dishonest B between probability of winning and ability to delay judge’s messages to A
- Without complete control of the communication network, B may settle for a lower probability of winning

Summary

- Probabilistic contract signing is a good testbed for probabilistic model checking techniques
  - Standard formal analysis techniques not applicable
  - Combination of nondeterminism and probability
  - Good for quantifying tradeoffs
- Probabilistic contract signing is subtle
  - Unfairness as asymmetric privilege
  - Optimism cannot be combined with timeliness, at least not in the obvious way